NAME:			

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Math 2415 (Spring 2014) Exam 2 4/3/14
Dr. Minkoff

No calculators, books or notes! Show all work and give **complete explanations**. Don't spend too much time on any one problem.

[15 pts](1a) If $f(x, y) = 16 - 4x^2 - y^2$, find $f_x(1, 2)$.

$f_x(x,y) = -8x$

$f_x(1,2) = -8$

(b) Sketch the surface given in Part (a) above, and interpret $f_x(1, 2)$ as a slope by drawing the appropriate tangent line onto your graph.

① traces in xz planes:

$z = 16 - 4x^2 - k^2$ (parabolas pting down)

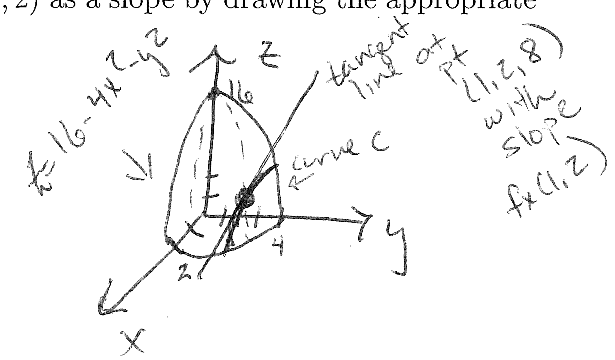
② traces in yz planes:

$z = 16 - 4k^2 - y^2$ (parabolas pting down)

③ xy traces:

$k = 16 - 4x^2 - y^2$

$\Rightarrow 4x^2 + y^2 = 16 - k$
which are ellipses for $16 > k$



C = curve of intersection of plane $y = 2$ and surface $z = 16 - 4x^2 - y^2$

$f_x(1,2)$ = slope of tangent at (1,2,8) to curve C

[8 pts](2) For the function f in the previous problem, find the equation of the plane tangent to the graph $z = f(x, y)$ at the point $(1, 2, 8)$.

$$z = z_0 + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$$f_x(1, 2) = -8$$

$$f_y(x, y) = -2y, \quad f_y(1, 2) = -4$$

tangent plane is

$$z = 8 - 8(x - 1) - 4(y - 2)$$

or $\boxed{z = -8x - 4y + 24}$

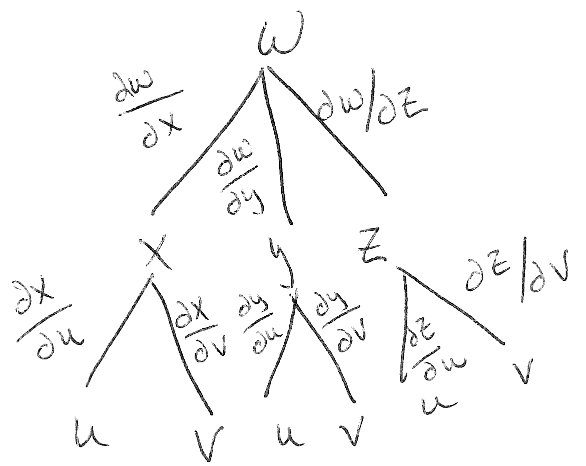
[7 pts](3) A mountain climber's oxygen mask is leaking. If the surface of the mountain is represented by $z = 5 - x^2 - 2y^2$ and the climber is at location $(x, y) = (\frac{1}{2}, \frac{-1}{2})$, in what direction should the climber turn to descend most rapidly?

Should descend in negative gradient direction:

$$\nabla f(x,y) = (-2x, -4y)$$
$$\nabla f(1/2, -1/2) = (-1, 2)$$

Climber should descend in
$-\nabla f = (1, -2)$ direction

[8 pts](4) Draw the tree diagram, write down the appropriate version of the chain rule, and calculate $\partial w/\partial u$ if $w = x^2 - 2y - 7z$; $x = v\cos(\pi - u)$, $y = u\sin(\pi - v)$, $z = uv$.



$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u}$$

$$\frac{\partial w}{\partial x} = 2x \qquad \frac{\partial w}{\partial y} = -2 \qquad \frac{\partial w}{\partial z} = -7$$

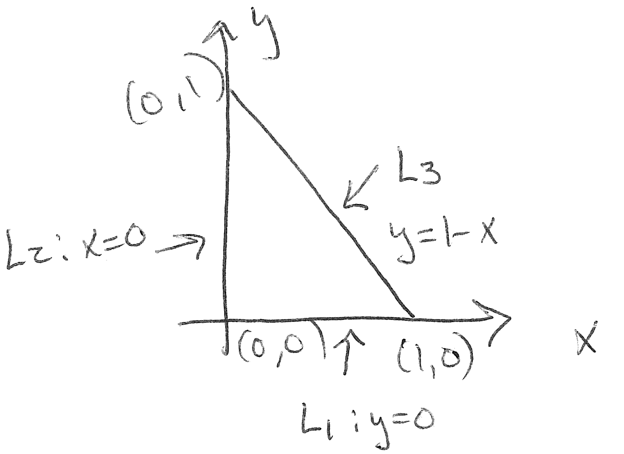
$$\frac{\partial x}{\partial u} = -v\sin(\pi - u)(-1) = v\sin(\pi - u)$$

$$\frac{\partial y}{\partial u} = \sin(\pi - v) \qquad \frac{\partial z}{\partial u} = v$$

so $$\frac{\partial w}{\partial u} = 2x\left(v\sin(\pi - u)\right) - 2\sin(\pi - v) - 7v$$

so $$\boxed{\frac{\partial w}{\partial u} = 2v^2\cos(\pi - u)\sin(\pi - u) - 2\sin(\pi - v) - 7v}$$

[12 pts](5) Find the absolute maximum and minimum values of the function $f(x,y) = x^2 - y^2$ on the triangle with vertices $(x,y) = (0,0), (1,0), (0,1)$. (Also state at what (x,y) points these extreme values occur.)



$f_x = 2x$, $f_y = -2y$

so only critical pt of f is $(x,y) = (0,0)$.

$f_{xx} = 2$, $f_{yy} = -2$, $f_{xy} = 0$

$D = f_{xx} f_{yy} - f_{xy}^2 = 2(-2) - 0^2 = -4 < 0$

so f has a saddle point at $(0,0)$. However, this point is actually on the boundary.

$f(0,0) = 0$

check boundary:

on $L_1: y=0$, so $\tilde{f}(x,0) = x^2$ $\quad \frac{d\tilde{f}}{dx} = 2x$ which has critical pt at $(0,0)$.

At other endpt $(x,y) = (1,0)$ so $f(1,0) = 1$

on $L_2: x=0$: so $\tilde{f} = -y^2$ $\quad \frac{d\tilde{f}}{dy} = -2y$ (critical pt at $(0,0)$).

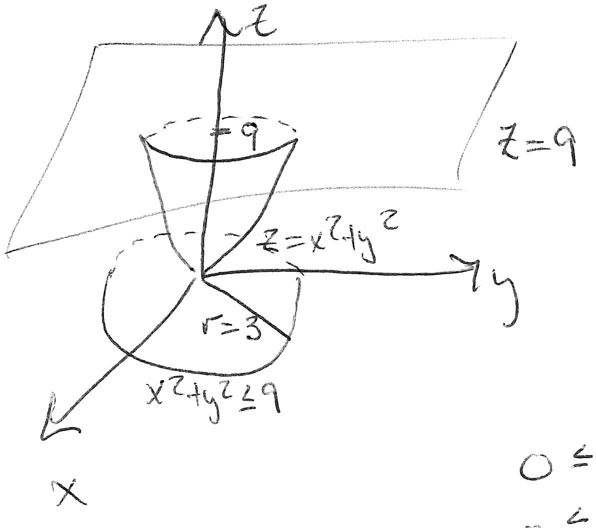
$f(0,1) = -1$

on $L_3: y = 1-x$ $\quad \tilde{f}(x,1-x) = x^2 - (1-x)^2 = x^2 - (1-2x+x^2)$

$= -1 + 2x$ $\quad \frac{d\tilde{f}}{dx} = 2 \Rightarrow$ no critical pts

$\boxed{\text{so min is at } (0,1), \text{ value} = -1 \text{ and max is at } (1,0), \text{ value} = 1}$

[12 pts](6) Find the volume of the solid under the paraboloid $z = x^2 + y^2$ and above the disk $x^2 + y^2 \leq 9$.



$0 \leq r \leq 3$
$0 \leq \theta \leq 2\pi$

$$\iint f(x,y)\,dx\,dy = \iint x^2+y^2\,dx\,dy = \int_0^{2\pi}\int_0^3 r^2(r\,dr\,d\theta)$$

$$= \int_0^{2\pi}\int_0^3 r^3\,dr\,d\theta = \int_0^{2\pi} \frac{r^4}{4}\Big|_{r=0}^{3} d\theta$$

$$= \int_0^{2\pi} \frac{81}{4}\,d\theta = \frac{81}{4}\theta\Big|_{\theta=0}^{2\pi} = \boxed{\frac{81\pi}{2}}$$

Please sign the following honor statement:

On my honor, I pledge that I have neither given nor received any aid on this exam.

Signature: ______________________________