How to Read an Equation

Duncan MacFarlane

The University of Texas at Dallas

Background

- There is a huge volume of reading associated with engineering
  - Need to learn to keep up efficiently
- When you’re starting out it’s tempting to focus on the words
  - May even ignore the equations
  - Or, may feel the need to memorize the math
- The reverse is better:
  - Leap from equation to equation
  - Read the words trying to predict what the author will say.

Goal of this talk

- Learn to look at an equation and draw lessons from it.

Look at the parts of the equation

- What functions are used?
- What constants are in the equation
- What are the units of everything in the equation?
  - I overwhelmingly prefer mks units, but ymmv

What functions are in the equation?

- Polynomials and fractional powers
- Transcendental functions
  - Familiar: cosine, exponentials …
  - Unfamiliar: Bessel, Neumann, Airy …
    - Plot these! (and save the plots)
- Aggressive vs Gentle
  - What's the strongest term in the equation?

UTD

What constants are in the equation?

- They tell the story behind the equation
  - π: cycles or circles
  - k: Boltzman's constant, thermodynamics
  - h: Planck's constant, quantum mechanics
  - c: electromagnetics or relativity
  - 2.404: Bessel functions
  - i,j: complex quantity used to track in-phase vs quadrature

Look *where* the parts are in the equation

- What’s in the numerator?
- What’s in the denominator?

As basic as this is, looking at what makes the equation bigger or smaller is good way to start developing your engineering judgment and intuition about the equation

Play with the equation

- Limits
  - What does the equation do as variables get very large and very small
- Put in the (mks) units for each term in the equation

Play with the equation

- Take its derivative
  - Units?
  - Maximums, minimums
- Perform a sensitivity analysis
  - x goes to x+Δx for all terms
- Take its integral
  - Over what range?

Plot the equation

- Plot it
  - This forces you to look at the range of values of every part
  - What choices are you making about the axes
    - May be multiple plots
    - Does it make sense to take the FFT?
  - Save the plots, eventually you will have a nice catalog
- Eventually you'll be able to mentally plot the equation
  - But all that will do is save time

UTD

More complicated equations

- Perform the analysis term by term
- Which term is most significant
  - For large values of the variable(s)
  - For small values of the variable(s)

Differential Equations

- First step is to classify the differential equation
  - Order?
  - Ordinary or Partial?
  - Forced or unforced?
  - Linear or nonlinear?
  - Constant coefficients?
- Wide range of notations: f', f_t, df/dt, $\dot{f}$

Differential equations

- Is it a familiar form?
  - Diffusion equation
  - Wave equation
- Is it a variation on a familiar form?
  - Solve the equation without solving it!

An Example:

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{a}_r$$

- What is the goal of the equation?
  - The force between two charges
- What are the constituents of the equation?
  - The charges
  - Distance
  - Some constants

$$\vec{F} = \vec{F}(q_1, q_2 r)$$

Example:

Take stock in the specific quantities that comprise the equation

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

Quantity	Meaning, thoughts	Units (mks)
F, Force	Attraction, Repulsion *It's a vector what is the direction?* Takes a mass and accelerates it … motion	N, kg m/sec^2
q, Charges,	Electrons, ions *Mixing electricity with mechanics*	Coulomb
r, distance	Separation of the charges *Wait, will it change bc of the force?*	m
ε, dielectric constant	Choice of material *What's a capacitor doing here?*	Farads/m
4 π	Constant, π comes from circles, spheres, angles, trig	

Example

Now look at functional forms

The numerator. Notice the way the charges multiply. Can interchange subscripts (why should the electrons care about our numbering system) The individuality of the charges is lost. Can't tell which one is bigger.

Forces are vector … where does this point?

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{a}_r$$

Constants. Gives Units, and in those units helps give the physical context of the equation

Main functional dependence. In denominator. Quadratic … faster decrease than linear. *Why quadratic, and why exactly a power of 2?*

How to study a derivation

- Write down in words what you are doing on each step of the derivation
- What about first skipping the steps of a derivation that are obvious to you
- On the next pass, write a very short description of each step you leave out
- The idea is for you to go from following a derivation to being able to do one
  - My old advisor described it as "pretending to do the math."
- Don’t worry about not understanding the physical situation at each and every step of the equation.
  - Math can guide you through foreign territory.

Summary

- Big picture look at the equation
- Inventory the constituents one at a time
- Look at functional forms
- What makes the end quantity bigger or smaller? How fast?