Week 7: Differentiation using the Chain Rule and Implicit Differentiation

Welcome to the Weekly Review for MATH 2413. This week’s review talks about Differentiation using the Chain Rule and Implicit Differentiation. We would like to thank Patrick Bourque and the Fall 2014 MATH 2413 students for allowing us to film the Weekly Reviews.

The following problems are presented in the Week 7 videos. Thank you!

Part A: Differentiation using the Chain Rule

1. Differentiation Rules
2. Find the Derivative for the following functions:

(a) $f(x) = \sin(6x^3 + 3x^2)$

(b) $g(x) = x \tan(x^2)$

(c) $f(x) = \sqrt{x^3 + 12x^4}$

(d) $f(x) = \sec^3(x^2 + 1)$
(e) \( f(x) = x^3 e^x \)

(f) \( f(x) = \cot\left(\frac{x^2 + x}{e^x}\right) \)

(g) \( f(x) = e^x \sec(x^3) \)
3. Find $g'(0)$, given

$$y(x) = f(\cos(g(x))) \quad g(0) = \frac{\pi}{2} \quad y'(0) = 16 \quad f'(0) = 4$$
Part B: Implicit Differentiation

1. Find \( \frac{dy}{dx} \) for each of the following functions

(a) \( y = x^2 \)

(b) \( \sin(x + y) = x^2 + y^3 \)

(c) \( \frac{x^2 + y^3}{x+y} = x \)
(d) \((x^2 + y^4)^3 = xy\)

2. Find \(\frac{d^3 y}{dx^3}\) for \(x^2 + y^2 + y = 1\)
3. Find two equations of the tangent lines to \((x^2 + 1)y^2 + 3(x + 1)y + 2 = 0\) when \(x = 0\).

4. Patrick’s Interesting Problem - The Circle Tangent Theorem!!!

The Circle Tangent Theorem: The line connecting the center of the circle to a point \(p\) is perpendicular to the circle’s tangent line at \(p\).