Week 3: Evaluating Limits

Welcome to the Weekly Review for MATH 2413. This week’s review talks about evaluating limits. We would like to thank Patrick Bourque and the Fall 2014 MATH 2413 students for allowing us to film the Weekly Reviews.

There are three cases for evaluating limits:

1. \( \lim_{x \to c} f(x) = L \).

2. \( \lim_{x \to c} f(x) = \frac{\text{non-zero number}}{0} = \pm \infty \).

3. \( \lim_{x \to c} f(x) = \frac{0}{0} \) and \( \lim_{x \to c} f(x) = \frac{\infty}{\infty} \).

The third case is referred to as the indeterminate form. When dealing with the indeterminate form, there are a couple of ways to approach the problem. The different approaches include: (1) Factor, (2) Multiply by the Conjugate, (3) Perform Long Division, (4) Get a Common Denominator and Simplify.

The following problems are presented in the Week 3 videos. Thank you!

1. \( \lim_{x \to 4} \left( \frac{x^2 - 16}{x - 4} \right) = \)

2. \( \lim_{x \to 1} \left( \frac{x^5 - 1}{x - 1} \right) = \)
3. \( \lim_{x \to 2} \left( \frac{\sqrt{x + 2} - 2}{x - 2} \right) = \)

4. \( \lim_{x \to 3} \left( \frac{x^2 - 6x + 9}{\sqrt{x^2 + 16} - 5} \right) = \)

5. \( \lim_{x \to 4} \left( \frac{\sqrt{x + 5} - 3}{\sqrt{x} - 2} \right) = \)
6. \( \lim_{{x \to 2}} \left( \frac{x+1}{{x^2} - 4} \right) = \)

7. \( \lim_{{x \to \frac{\pi}{4}}} \left( \frac{\cos(x) - \sin(x)}{1 - \tan(x)} \right) = \)

8. \( \lim_{{x \to \frac{\pi}{4}}} \left( \frac{\cos(2x)}{\cos(x) - \sin(x)} \right) = \)
9. Find the \( \lim_{x \to 1} f(x) \), given
\[
 f(x) = \begin{cases} 
 \frac{x^2-1}{x-1} & x < 1 \\
 2x & x \geq 1 
\end{cases}
\]

10. Find the value of \( a \) so that the \( \lim_{x \to a} f(x) \) exists, given
\[
 f(x) = \begin{cases} 
 \frac{x^2-a^2}{x-a} & x < a \\
 ax + 1 & x \geq a 
\end{cases}
\]

11. Find all vertical asymptotes of \( f(x) = \frac{x}{x-3} \).
12. Find all vertical asymptotes of \( f(x) = \frac{\sin(x)}{\sin(2x)+\sin(x)} \).

13. For what values of \( A \) does \( f(x) \) have 0, 1, 2 vertical asymptotes, given \( f(x) = \frac{1}{x^2+Ax+9} \).

Want some more practice? The following problems were provided to you by the Math Lab Learning Specialists. Please feel free to come and visit the UT Dallas Math Lab if you need any help. Thank you!!

1. \( \lim_{x \to 5} \left( \frac{x^2 - 25}{x - 5} \right) = \)

2. \( \lim_{x \to 3} \left( \frac{4x^4 - 12x^3 + 2x^2 - x - 15}{x - 3} \right) = \)

3. \( \lim_{x \to 5} \left( \frac{\sqrt{4x + 16} - 6}{x - 5} \right) = \)

4. \( \lim_{x \to 5} \left( \frac{\frac{1}{x+1} - \frac{1}{5}}{x - 5} \right) = \)

5. Find the value of \( a \) so that the \( \lim_{x \to 2} f(x) \) exists, given

\[
f(x) = \begin{cases} 
  x + 5 & x \leq 2 \\
  ax - 2 & x > 2 
\end{cases}
\]
6. Find the values of $a$, $b$, and $c$ so that $f(x)$ has the following properties.

\[
\lim_{x \to 1} f(x) \text{ exists} \quad \lim_{x \to 4} f(x) \text{ exists} \quad f(2) = 3
\]

\[
f(x) = \begin{cases} 
\frac{x^2 - 1}{x - 1} & x < 1 \\
ax^2 + bx + c & 1 \leq x \leq 4 \\
\frac{x^2 + 3x - 28}{x - 4} & x > 4
\end{cases}
\]

7. Find all vertical asymptotes of $f(x) = \frac{\sqrt{x-1}}{16-x^2}$. 