Week 11: Concavity, Curve Sketching, The Second Derivative Test and L’Hôpital’s Rule

Welcome to the Weekly Review for MATH 2413. This week’s review talks about concavity, curve sketching, the second derivative test and L’Hôpital’s Rule. We would like to thank Patrick Bourque and the Fall 2014 MATH 2413 students for allowing us to film the Weekly Reviews.

The following problems are presented in the Week 11 videos. Thank you!

Part A: Concavity and Curve Sketching

1. Introduction to Concavity

2. Find where the function is concave up, concave down and find any points of inflection for $f(x) = 3x^5 + 5x^4 - 20x^3$. 
3. Find where the function is concave up, concave down and find any points of inflection for $f(x) = 4\sin(x) - \sin(2x)$ in the interval $[0, 2\pi]$.

4. Sketch the following function by finding asymptotes, relative extrema and points of inflection, if any.

$$f(x) = \frac{x^3}{x^2 - 4}.$$
6. Use the Second Derivative Test to find all relative extrema for \( f(x) = 3x^4 - 16x^3 + 6x^2 - 48x \)
Part B: L’Hôpital’s rule

1. Introduction to L’Hôpital’s rule

2. \( \lim_{x \to 0} \left( \frac{e^{2x} - 2x - 1}{xe^x - x} \right) = \)
3. \( \lim_{x \to \infty} \left( \frac{x \ln(x)}{x^2 + 1} \right) = \)

4. \( \lim_{x \to \infty} \left( \sqrt{x^2 + 2x - x} \right) = \)
5. \( \lim_{x \to \infty} \left( x \sin \left( \frac{2}{x} \right) \right) = \)

6. \( \lim_{x \to \infty} (e^x + x)^\frac{2}{3} = \)
7. \( \lim_{x \to 0^+} (\sin(x))^x = \)

8. \( \lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^{3x} = \)