P12 #1

1. Propositions must have clearly defined truth values, so a proposition must be a declarative sentence with no free variables.
   a) This is a true proposition.
   b) This is a false proposition (Tallahassee is the capital).
   c) This is a true proposition.
   d) This is a false proposition.
   e) This is not a proposition (it contains a variable; the truth value depends on the value assigned to $x$).
   f) This is not a proposition, since it does not assert anything.

P13 #13bcdg

13. a) This is just the negation of $p$, so we write $\neg p$.
   b) This is a conjunction (“but” means “and”): $p \land \neg q$.
   c) The position of the word “if” tells us which is the antecedent and which is the consequence: $p \rightarrow q$.
   d) $\neg p \rightarrow \neg q$
   e) The sufficient condition is the antecedent: $p \rightarrow q$.
   f) $q \land \neg p$
   g) “Whenever” means “if”: $q \rightarrow p$.

P14 #16

16. a) This is $T \leftrightarrow T$, which is true.
 b) This is $T \leftrightarrow F$, which is false.
 c) This is $F \leftrightarrow F$, which is true.
 d) This is $F \leftrightarrow T$, which is false.

P15 #31c

31. To construct the truth table for a compound proposition, we work from the inside out. In each case, we will show the intermediate steps. In part (d), for example, we first construct the truth table for $p \lor q$, then the truth table for $p \land q$, and finally combine them to get the truth table for $(p \lor q) \rightarrow (p \land q)$. For parts (a) and (b) we have the following table (column three for part (a), column four for part (b)).

<table>
<thead>
<tr>
<th>$p$</th>
<th>$\neg p$</th>
<th>$p \land \neg p$</th>
<th>$p \lor \neg p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

For part (c) we have the following table.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$\neg q$</th>
<th>$p \lor \neg q$</th>
<th>$(p \lor \neg q) \rightarrow q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
</tbody>
</table>
9. We construct a truth table for each conditional statement and note that the relevant column contains only T’s. For parts (a) and (b) we have the following table (column four for part (a), column six for part (b)).

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p \land q )</th>
<th>( (p \land q) \rightarrow p )</th>
<th>( p \lor q )</th>
<th>( p \rightarrow (p \lor q) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

For parts (c) and (d) we have the following table (columns five and seven, respectively).

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( \neg p )</th>
<th>( p \rightarrow q )</th>
<th>( \neg p \rightarrow (p \rightarrow q) )</th>
<th>( p \land q )</th>
<th>( (p \land q) \rightarrow (p \rightarrow q) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

33. To show that these are not logically equivalent, we need only one assignment of truth values to \( p, q, r \), and \( s \) for which the truth values of \( (p \rightarrow q) \rightarrow (r \rightarrow s) \) and \( (p \rightarrow r) \rightarrow (q \rightarrow s) \) differ. Let us try to make the first one false. That means we have to make \( r \rightarrow s \) false, so we want \( r \) to be true and \( s \) to be false. If we let \( p \) and \( q \) be false, then each of the other three simple conditional statements \( (p \rightarrow q), (p \rightarrow r), \) and \( q \rightarrow s \) will be true. Then \( (p \rightarrow q) \rightarrow (r \rightarrow s) \) will be \( T \rightarrow F \), which is false; but \( (p \rightarrow r) \rightarrow (q \rightarrow s) \) will be \( T \rightarrow T \), which is true.

P53 #15

15. Recall that the integers include the positive and negative integers and 0.

a) This is the well-known true fact that the square of a real number cannot be negative.

b) There are two real numbers that satisfy \( n^2 = 2 \), namely \( \pm \sqrt{2} \), but there do not exist any integers with this property, so the statement is false.

c) If \( n \) is a positive integer, then \( n^2 \geq n \) is certainly true; it’s also true for \( n = 0 \); and it’s trivially true if \( r \) is negative. Therefore the universally quantified statement is true.

d) Squares can never be negative; therefore this statement is false.

P56 #61ac

61. a) This is asserting that every person who is a baby is necessarily not logical: \( \forall x(P(x) \rightarrow \neg Q(x)) \).

b) If a person can manage a crocodile, then that person is not despised: \( \forall x(R(x) \rightarrow \neg S(x)) \).

c) Every person who is not logical is necessarily despised: \( \forall x(\neg Q(x) \rightarrow S(x)) \).

d) Every person who is a baby cannot manage a crocodile: \( \forall x(P(x) \rightarrow \neg R(x)) \).

e) The conclusion follows. Suppose that \( x \) is a baby. Then by the first premise, \( x \) is illogical, and hence, by the third premise, \( x \) is despised. But the second premise says that if \( x \) could manage a crocodile, then \( x \) would not be despised. Therefore \( x \) cannot manage a crocodile. Thus we have proved that babies cannot manage crocodiles.
9. We construct a truth table for each conditional statement and note that the relevant column contains only T’s. For parts (a) and (b) we have the following table (column four for part (a), column six for part (b)).

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p ∧ q</th>
<th>(p ∧ q) → p</th>
<th>p ∨ q</th>
<th>p → (p ∨ q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
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</tr>
<tr>
<td>F</td>
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<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

For parts (c) and (d) we have the following table (columns five and seven, respectively).

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>¬p</th>
<th>p → q</th>
<th>¬p → (p → q)</th>
<th>p ∧ q</th>
<th>(p ∧ q) → (p → q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
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<tr>
<td>T</td>
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<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

13. In each case we set up the proof in two columns, with reasons, as in Example 6.

a) Let \( c(x) \) be “\( x \) is in this class,” let \( j(x) \) be “\( x \) knows how to write programs in JAVA,” and let \( h(x) \) be “\( x \) can get a high paying job.” We are given premises \( c(Doug) \), \( j(Doug) \), and \( \forall x (j(x) \rightarrow h(x)) \), and we want to conclude \( \exists x (c(x) \land h(x)) \).

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( \forall x (j(x) \rightarrow h(x)) ) Hypothesis</td>
</tr>
<tr>
<td>2.</td>
<td>( j(Doug) \rightarrow h(Doug) ) Universal instantiation using (1)</td>
</tr>
<tr>
<td>3.</td>
<td>( j(Doug) ) Hypothesis</td>
</tr>
<tr>
<td>4.</td>
<td>( h(Doug) ) Modus ponens using (2) and (3)</td>
</tr>
<tr>
<td>5.</td>
<td>( c(Doug) ) Hypothesis</td>
</tr>
<tr>
<td>6.</td>
<td>( c(Doug) \land h(Doug) ) Conjunction using (4) and (5)</td>
</tr>
<tr>
<td>7.</td>
<td>( \exists x (c(x) \land h(x)) ) Existential generalization using (6)</td>
</tr>
</tbody>
</table>

b) Let \( c(x) \) be “\( x \) is in this class,” let \( w(x) \) be “\( x \) enjoys whale watching,” and let \( p(x) \) be “\( x \) cares about ocean pollution.” We are given premises \( \exists x (c(x) \land w(x)) \) and \( \forall x (w(x) \rightarrow p(x)) \), and we want to conclude \( \exists x (c(x) \land p(x)) \). In our proof, \( y \) represents an unspecified particular person.
c) Let \( c(x) \) be “\( x \) is in this class,” let \( p(x) \) be “\( x \) owns a PC,” and let \( w(x) \) be “\( x \) can use a word processing program.” We are given premises \( c(Zeke) \), \( \forall x (c(x) \rightarrow p(x)) \), and \( \forall x (p(x) \rightarrow w(x)) \), and we want to conclude \( w(Zeke) \).

\[
\text{Step} \\
1. \exists x (c(x) \land w(x)) \quad \text{Reason} \\
2. c(y) \land w(y) \quad \text{Hypothesis} \\
3. w(y) \quad \text{Existential instantiation using (1)} \\
4. c(y) \quad \text{Simplification using (2)} \\
5. \forall x (w(x) \rightarrow p(x)) \quad \text{Hypothesis} \\
6. w(y) \rightarrow p(y) \quad \text{Universal instantiation using (5)} \\
7. p(y) \quad \text{Modus ponens using (3) and (6)} \\
8. c(y) \land p(y) \quad \text{Conjunction using (4) and (7)} \\
9. \exists x (c(x) \land p(x)) \quad \text{Existential generalization using (8)}
\]

\[
\text{Step} \\
1. \forall x (c(x) \rightarrow p(x)) \quad \text{Hypothesis} \\
2. c(Zeke) \rightarrow p(Zeke) \quad \text{Universal instantiation using (1)} \\
3. c(Zeke) \quad \text{Hypothesis} \\
4. p(Zeke) \quad \text{Modus ponens using (2) and (3)} \\
5. \forall x (p(x) \rightarrow w(x)) \quad \text{Hypothesis} \\
6. p(Zeke) \rightarrow w(Zeke) \quad \text{Universal instantiation using (5)} \\
7. w(Zeke) \quad \text{Modus ponens using (4) and (6)}
\]

d) Let \( j(x) \) be “\( x \) is in New Jersey,” let \( f(x) \) be “\( x \) lives within fifty miles of the ocean,” and let \( s(x) \) be “\( x \) has seen the ocean.” We are given premises \( \forall x (j(x) \rightarrow f(x)) \) and \( \exists x (j(x) \land \neg s(x)) \), and we want to conclude \( \exists x (f(x) \land \neg s(x)) \). In our proof, \( y \) represents an unspecified particular person.

\[
\text{Step} \\
1. \exists x (j(x) \land \neg s(x)) \quad \text{Reason} \\
2. j(y) \land \neg s(y) \quad \text{Hypothesis} \\
3. j(y) \quad \text{Existential instantiation using (1)} \\
4. \forall x (j(x) \rightarrow f(x)) \quad \text{Simplification using (2)} \\
5. j(y) \rightarrow f(y) \quad \text{Hypothesis} \\
6. f(y) \quad \text{Universal instantiation using (4)} \\
7. \neg s(y) \quad \text{Modus ponens using (3) and (5)} \\
8. f(y) \land \neg s(y) \quad \text{Simplification using (2)} \\
9. \exists x (f(x) \land \neg s(x)) \quad \text{Conjunction using (6) and (7)} \\
\]

P108 #19

19. We are being asked to solve \( n = (k - 2) + (k + 3) \) for \( k \). Using the usual, reversible, rules of algebra, we see that this equation is equivalent to \( k = (n - 1)/2 \). In other words, this is the one and only value of \( k \) that makes our equation true. Since \( n \) is odd, \( n - 1 \) is even, so \( k \) is an integer.