Two Hanging Masses

Description: Two masses hanging from a string and connected by another string. Find tension when the masses are at rest or accelerating upward (with constant acceleration).

Two blocks with masses $M_1$ and $M_2$ hang one under the other. For this problem, take the positive direction to be upward, and use $g$ for the magnitude of the acceleration due to gravity.

Case 1: The blocks are at rest.

Part A

Find $T_2$, the tension in the lower rope.

Hint A.1  Free-body diagram
Isolate the lower block (mass $M_2$) by considering just the forces that act on it. Use Newton's 2nd law while noting that the acceleration of this block is zero.

Hint A.2  Sum of forces
Write down the sum of the $y$ components of all the forces acting on the lower block.

Express your answer in terms of some or all of the variables $M_1$, $M_2$, $T_2$ and $g$.

ANSWER:

$$\sum F_{2y} = M_2a_2 = 0 \quad T_2 - M_2g$$
Express your answer in terms of some or all of the variables $M_1$, $M_2$, and $g$.

ANSWER: 

$$T_2 = M_2g$$

Part B
Find $T_1$, the tension in the upper rope.

Hint B.1  **Sum of forces**
Write down the sum of the $y$ components of all the forces acting on the upper block.

Express your answer in terms of some or all of the variables $M_1$, $M_2$, $T_1$, $T_2$, and $g$.

ANSWER: 

$$\sum F_{1y} = 0 \quad T_1 - T_2 - M_1g$$

Express your answer in terms of some or all of the variables $M_1$, $M_2$, and $g$.

ANSWER: 

$$T_1 = (M_1 + M_2)g$$

Case 2: The blocks are now accelerating upward (due to the tension in the strings) with acceleration of magnitude $a$.

Part C
Find $T_2$, the tension in the lower rope.

Hint C.1  **Sum of forces**
Apply Newton's 2nd law, $\sum F_y = M_2a$, to the lower block. Write the sum of the $y$ components of the forces.

Express your answer in terms of some or all of the variables $M_1$, $M_2$, $T_2$ and $g$.

ANSWER: 

$$\sum F_{2y} = M_2a \quad T_2 - M_2g$$

Express your answer in terms of some or all of the variables $M_1$, $M_2$, $a$, and $g$.

ANSWER: 

$$T_2 = M_2 (g + a)$$
Good! Now notice that acceleration acts in the tension equation the same way that gravity does. If we look at a limiting case in which the block of mass $M_2$ is in free fall, accelerating downward with $a = -g$, then the tension goes to zero.

Part D

Find $T_1$, the tension in the upper rope.

**Hint D.1  Sum of forces**

Consider the block of mass $M_1$ as an isolated system and apply Newton's 2nd law, $\sum F_y = M_1a$, with the value of $T_2$ found in the previous part.

Express your answer in terms of some or all of the variables $M_1$, $M_2$, $a$, and $g$.

**ANSWER:**

$$T_1 = (M_1 + M_2)(a + g)$$

It should be noted that, even though tension has units of force, it is a scalar quantity and not a vector like force is. A vector has a magnitude and direction whereas a scalar just has a magnitude. The force exerted by a rope is a result of the rope's tension and points along the rope.

---

**Understanding Newton's Laws**

**Description:** Conceptual questions on Newton's first and second laws.

**Part A**

An object cannot remain at rest unless which of the following holds?

**Hint A.1  How to approach the problem**

This problem describes a situation of static equilibrium (i.e., a body that remains at rest). Hence, it is appropriate to apply Newton's 1st law.

**Hint A.2  Newton's 1st law: a body at rest**

According to Newton's 1st law, a body at rest remains at rest if the net force acting on it is zero.

**ANSWER:**

- The net force acting on it is zero.
- The net force acting on it is constant and nonzero.
- There are no forces at all acting on it.
- There is only one force acting on it.

If there is a net force acting on a body, regardless of whether it is a constant force, the body accelerates. If the body is at rest and the net force acting on it is zero, then it will remain at rest. The net
Part B

If a block is moving to the left at a constant velocity, what can one conclude?

Hint B.1  **How to approach the problem**

This problem describes a situation of dynamic equilibrium (i.e., a body that moves at a constant velocity). Hence, it is appropriate to apply Newton's 1st law.

Hint B.2  **Newton's 1st law: a body in motion**

According to Newton's 1st law, a body initially in motion continues to move with constant velocity if the net force acting on it is zero.

**ANSWER:**

- There is exactly one force applied to the block.
- The net force applied to the block is directed to the left.
- The net force applied to the block is zero.
- There must be no forces at all applied to the block.

If there is a net force acting on a body, regardless of whether the body is already moving, the body accelerates. If a body is moving with constant velocity, then it is not accelerating and the net force acting on it is zero. The net force could be zero either because there are no forces acting on the body at all or because several forces are acting on the body but they all cancel out.

Part C

A block of mass $2\,\text{kg}$ is acted upon by two forces: $3\,\text{N}$ (directed to the left) and $4\,\text{N}$ (directed to the right). What can you say about the block's motion?

Hint C.1  **How to approach the problem**

This problem describes a situation of dynamic motion (i.e., a body that is acted on by a net force). Hence, it is appropriate to apply Newton's 2nd law, which allows you to relate the net force acting on a body to the acceleration of the body.

Hint C.2  **Newton's 2nd law**

Newton's 2nd law states that a body accelerates if a net force acts on it. The net force is proportional to the acceleration of the body and the constant of proportionality is equal to the mass of the body. In other words,

$$F = ma,$$

where $F$ is the net force acting on the body, and $m$ and $a$ are the mass and the acceleration of the body, respectively.

Hint C.3  **Relating acceleration to velocity**

Acceleration is defined as the change in velocity per unit time. Keep in mind that both acceleration and velocity are vector quantities.
The acceleration of an object tells you nothing about its velocity—the direction and speed at which it is moving. In this case, the net force on (and therefore the acceleration of) the block is to the right, but the block could be moving left, right, or in any other direction.

Part D
A massive block is being pulled along a horizontal frictionless surface by a constant horizontal force. The block must be __________.

Hint D.1  How to approach the problem
This problem describes a situation of dynamic motion (i.e., a body that is acted on by a net force). Hence, it is appropriate to apply Newton's 2nd law, which allows you to relate the net force acting on a body to the acceleration of the body.

Hint D.2  Newton's 2nd law
Newton's 2nd law states that a body accelerates if a net force acts on it. The net force is proportional to the acceleration of the body and the constant of proportionality is equal to the mass of the body. In other words,

\[ F = ma, \]

where \( F \) is the net force acting on the body, and \( m \) and \( a \) are the mass and the acceleration of the body, respectively.

Since there is a net force acting, the body does not move at a constant velocity, but it accelerates instead. However, the force acting on the body is constant. Hence, according to Newton's 2nd law of motion, the acceleration of the body is also constant.

Part E
Two forces, of magnitude 4 N and 10 N, are applied to an object. The relative direction of the forces is unknown. The net force acting on the object __________.

Hint E.1  How to approach the problem
By definition, the net force is the vector sum of all forces acting on the object. To find the magnitude of the net force you need to add the components of the two forces acting. Try adding the two forces graphically (by connecting the head of one force to the tail of the other). The directions of the two forces are arbitrary, but by trying different possibilities you should be able to determine the maximum and minimum net forces that could act on the object.
**Hint E.2**  
Find the net force when the two forces act on the object in opposite directions

Find the magnitude of the net force if both the forces acting on the object are horizontal and the 10-N force is directed to the right, while the 4-N force is directed to the left.

**Hint E.2.1**  
**Vector addition**

The magnitude of the vector sum of two parallel forces is the sum of the magnitudes of the forces. The magnitude of the vector sum of two antiparallel forces is the absolute value of the difference in magnitudes of the forces.

**Express your answer in newtons.**

ANSWER:

\[
6.0 \text{ N}
\]

Is there any other orientation of the two forces that would lead to a net force with a smaller magnitude than what you just calculated?

**Hint E.3**  
Find the direction of the net force when the two forces act in opposite directions

If both the forces acting on the object are horizontal and the 10-N force is directed to the right, while the 4-N force is directed to the left, the net force is horizontal and directed _________.

ANSWER:

- in the same direction as the 10-N force
- in the opposite direction to the 10-N force

Check all that apply.

ANSWER:

- cannot have a magnitude equal to 5 N
- cannot have a magnitude equal to 10 N
- cannot have the same direction as the force with magnitude 10 N
- must have a magnitude greater than 10 N

---

**Newton's 3rd Law**

**Description:** Multiple choice questions about Newton's Third Law.

Mark each of the following statements as true or false. If a statement refers to "two bodies" interacting via some force, you are not to assume that these two bodies have the same mass.

**Part A**

Every force has one and only one 3rd law pair force.

ANSWER:
Part B
The two forces in each pair act in opposite directions.
ANSWER: ○ true ○ false

Part C
The two forces in each pair can either both act on the same body or they can act on different bodies.
ANSWER: ○ true ○ false

Part D
The two forces in each pair may have different physical origins (for instance, one of the forces could be due to gravity, and its pair force could be due to friction or electric charge).
ANSWER: ○ true ○ false

Part E
The two forces of a 3rd law pair always act on different bodies.
ANSWER: ○ true ○ false

Part F
Given that two bodies interact via some force, the accelerations of these two bodies have the same magnitude but opposite directions. (Assume no other forces act on either body.)

Hint F.1 \[ \vec{F} = m\vec{a} \]
Remember \[ \vec{F} = m\vec{a} \]: If the forces are equal in magnitude, must the accelerations also be of equal magnitude?
ANSWER: ○ true
Newton's 3rd law can be summarized as follows: A physical interaction (e.g., gravity) operates between two interacting bodies and generates a pair of opposite forces, one on each body. It offers you a way to test for real forces (i.e., those that belong on the force side of \( \sum F = ma \))—there should be a 3rd law pair force operating on some other body for each real force that acts on the body whose acceleration is under consideration.

Part G

According to Newton's 3rd law, the force on the (smaller) moon due to the (larger) earth is

ANSWER:  
- greater in magnitude and antiparallel to the force on the earth due to the moon.  
- greater in magnitude and parallel to the force on the earth due to the moon.  
- equal in magnitude but antiparallel to the force on the earth due to the moon.  
- equal in magnitude and parallel to the force on the earth due to the moon.  
- smaller in magnitude and antiparallel to the force on the earth due to the moon.  
- smaller in magnitude and parallel to the force on the earth due to the moon.

Block on an Incline

Description: A block is at rest on an inclined plane with friction. Find the normal force in terms of the block's weight.

A block lies on a plane raised an angle \( \theta \) from the horizontal. Three forces act upon the block: \( \vec{F}_w \), the force of gravity; \( \vec{F}_n \), the normal force; and \( \vec{F}_f \), the force of friction. The coefficient of friction is large enough to prevent the block from sliding.

Part A

Consider coordinate system a, with the x axis along the plane. Which forces lie along the axes?

ANSWER: \( \vec{F}_f \) only
Part B

Which forces lie along the axes of the coordinate system b, in which the y axis is vertical?

ANSWER:

- $F_t$ only
- $F_n$ only
- $F_w$ only
- $F_t$ and $F_n$
- $F_t$ and $F_w$
- $F_n$ and $F_w$
- $F_t$ and $F_n$ and $F_w$

Now you are going to ignore the general rule (actually, a strong suggestion) that you should pick the coordinate system with the most vectors, especially unknown ones, along the coordinate axes. You will find the normal force, $F_n$, using vertical coordinate system b. In these coordinates you will find the magnitude $F_n$ appearing in both the x and y equations, each multiplied by a trigonometric function.

Part C

Because the block is not moving, the sum of the y components of the forces acting on the block must be zero. Find an expression for the sum of the y components of the forces acting on the block, using coordinate system b.

Hint C.1

Find the y component of $F_n$

Write an expression for $F_{ny}$, the y component of the force $F_n$, using coordinate system b.

Hint C.1.1

Some geometry help - a useful angle
The smaller angle between $\vec{F}_n$ and the $y$-axis is also $\theta$, as shown in the figure.

Express your answer in terms of $F_n$ and $\theta$.

**ANSWER:**

$F_{ny} = F_n \cos(\theta)$

**Hint C.2**  
Find the $y$ component of $\vec{F}_f$.

Write an expression for $F_{fy}$, the $y$ component of the force $\vec{F}_f$, using coordinate system b.

**Hint C.2.1**  
Some geometry help - a useful angle

The smaller angle between $\vec{F}_f$ and the $x$-axis is also $\theta$, as shown in the figure.

Express your answer in terms of $F_f$ and $\theta$.

**ANSWER:**

$F_{fy} = F_f \sin(\theta)$
Now simply add the $y$ components of all forces. Do not forget the force of gravity $\vec{F}_w$, and remember that in coordinate system b the positive vertical direction points upward.

Express your answer in terms of some or all of the variables $F_n$, $F_l$, $F_w$, and $\theta$.

**ANSWER:**

$$\sum F_y = 0 \quad F_w \cos(\theta) + F_l \sin(\theta) - F_w$$

Part D

Because the block is not moving, the sum of the $x$ components of the forces acting on the block must be zero. Find an expression for the sum of the $x$ components of the forces acting on the block, using coordinate system b.

**Hint D.1** Find the $x$ component of $\vec{F}_n$

Write an expression for $F_{nx}$, the $x$ component of the force $\vec{F}_n$, using coordinate system b.

Express your answer in terms of $F_n$ and $\theta$.

**ANSWER:**

$$F_{nx} = -F_n \sin(\theta)$$

Express your answer in terms of some or all of the variables $F_n$, $F_l$, $F_w$, and $\theta$.

**ANSWER:**

$$\sum F_x = 0 \quad -F_n \sin(\theta) + F_l \cos(\theta)$$

Part E

To find the magnitude of the normal force, you must express $F_n$ in terms of $F_w$ since $F_l$ is an unknown. Using the equations you found in the two previous parts, find an expression for $F_n$ involving $F_w$ and $\theta$ but not $F_l$.

**Hint E.1** How to approach the problem

From your answers to the previous two parts you should have two force equations ($\sum F_y = 0$ and $\sum F_x = 0$). Combine these equations to eliminate $F_l$. The key is to multiply the equation for the $y$ components by $\cos \theta$ and the equation for the $x$ components by $\sin \theta$, then add or subtract the two equations to eliminate the term $F_l \cos(\theta) \sin(\theta)$.

An alternative motivation for the algebra is to eliminate the trig functions in front of $F_n$ by using the trig
identity \( \sin^2(\theta) + \cos^2(\theta) = 1 \). At the very least this would result in an equation that is simple to solve for \( F_n \).

ANSWER:

\[
F_n = F_\text{w} \cos(\theta)
\]

Congratulations on working this through. Now realize that in coordinate system a, which is aligned with the plane, the y-coordinate equation is \( \sum F_y = F_n - F_\text{w} \cos(\theta) = 0 \), which leads immediately to the result obtained here for \( F_n \).

CONCLUSION: A thoughtful examination of which coordinate system to choose can save a lot of algebra.

---

### Pushing a Block

**Description:** Simple quantitative and qualitative questions about kinetic and static friction.

A block of mass \( m \) lies on a horizontal table. The coefficient of static friction between the block and the table is \( \mu_s \). The coefficient of kinetic friction is \( \mu_k \), with \( \mu_k < \mu_s \).

**Part A**

If the block is at rest (and the only forces acting on the block are the force due to gravity and the normal force from the table), what is the magnitude of the force due to friction?

**Hint A.1** Consider the type of friction at rest

What type of friction is acting in this case?

**ANSWER:** neither static nor kinetic

**ANSWER:**

\( F_{\text{friction}} = 0 \)

**Part B**

Suppose you want to move the block, but you want to push it with the least force possible to get it moving. With what force \( F \) must you be pushing the block just before the block begins to move?

**Hint B.1** Consider the type of friction to start movement

What type of friction is acting in this case?

**ANSWER:** static friction

Express the magnitude of \( F \) in terms of some or all the variables \( \mu_s \), \( \mu_k \), and \( m \), as well as the
acceleration due to gravity $g$.

**ANSWER:**

$$ F = \mu_s mg $$

---

### Part C

Suppose you push horizontally with half the force needed to just make the block move. What is the magnitude of the friction force?

**Hint C.1 What level of force is required?**

In this situation, the force of static friction prevents the object from moving. Therefore, the magnitude of the static friction force must equal the magnitude of the net horizontal applied force acting on the object, up to a certain maximum value. In this case,

$$ F_{\text{static friction}} \leq \mu_s N, $$

where $\mu_s$ is the coefficient of static friction and $N$ is the normal force that the surface exerts on the object.

**Express your answer in terms of some or all of the variables $\mu_s$, $\mu_k$, and $m$, as well as the acceleration due to gravity $g$.**

**ANSWER:**

$$ F_{\text{friction}} = \frac{1}{2} \mu_s mg $$

---

### Part D

Suppose you push horizontally with precisely enough force to make the block start to move, and you continue to apply the same amount of force even after it starts moving. Find the acceleration $a$ of the block after it begins to move.

**Hint D.1 Calculate applied force**

What is the magnitude $F$ of the force that you are applying to make the block move?

**Express your answer in terms of some or all of the variables $\mu_s$, $\mu_k$, and $m$, as well as the acceleration due to gravity $g$.**

**ANSWER:**

$$ F = \mu_s mg $$

**Hint D.2 Consider applied force and kinetic friction**

When the block is moving, there is a force of kinetic friction acting on it, with magnitude

$$ |F_{\text{kinetic friction}}| = \mu_k n, $$

where $\mu_k$ is the coefficient of kinetic friction and $n$ is the magnitude of the normal force.

**Hint D.3 Calculate net horizontal force**

What is the magnitude of the net horizontal force acting on the block? Remember that the friction force is...
directed \textit{opposite} to the motion of the object.

Express your answer in terms of some or all of the variables \( \mu_a, \mu_b, \) and \( m, \) as well as the acceleration due to gravity \( g. \)

\textbf{ANSWER:}
\[ F_{\text{horizontal}} = (\mu_a - \mu_k) mg \]

Express your answer in terms of some or all of the variables \( \mu_a, \mu_b, \) and \( m, \) as well as the acceleration due to gravity \( g. \)

\textbf{ANSWER:}
\[ \alpha = (\mu_a - \mu_k) g \]

\begin{center}
\textbf{Moment of Inertia and Center of Mass for Point Particles}
\end{center}

\textbf{Description:} Two massive balls are connected by a massless rod. Given the ratio of moments of inertia through two different axes, find the ratio of the masses of the balls and the position of the center of mass of the system.

Ball a, of mass \( m_a, \) is connected to ball b, of mass \( m_b, \) by a massless rod of length \( L. \) The two vertical dashed lines in the figure, one through each ball, represent two different axes of rotation, axes \( a \) and \( b. \) These axes are parallel to each other and perpendicular to the rod. The moment of inertia of the two-mass system about axis \( a \) is \( I_a, \) and the moment of inertia of the system about axis \( b \) is \( I_b. \) It is observed that the ratio of \( I_a \) to \( I_b \) is equal to 3:

\[ \frac{I_a}{I_b} = 3 \]

Assume that both balls are pointlike; that is, neither has any moment of inertia about its own center of mass.

\textbf{Part A}

Find the ratio of the masses of the two balls.

\textbf{Hint A.1} \textbf{How to approach the problem}

Find an expression for \( I_a \) and for \( I_b \) in terms of the masses \( m_a \) and \( m_b. \) Substitute these expressions into the formula given in the problem introduction and then solve for the ratio of the masses.

\textbf{Hint A.2} \textbf{Find} \( I_a \)

Find \( I_a, \) the moment of inertia of the system about axis \( a. \)
Hint A.2.1  Formula for the moment of inertia

The formula for the moment of inertia $I$ of an object consisting of particles with masses $m_i$, at distances $r_i$ from the rotation axis is

$$I = \sum_i m_i r_i^2.$$ 

Express your answer in terms of any or all of the following quantities: $L$, $m_a$, and $m_b$.

**ANSWER:**

$$I_a = m_b L^2$$

Express your answer numerically.

**ANSWER:**

$$\frac{m_a}{m_b} = \frac{1}{3}$$

Part B

Find $d_a$, the distance from ball A to the system's center of mass.

Hint B.1  How to approach the problem

To find $d_a$, compute the position of the center of mass of the system, using a coordinate system in which ball a is at the origin. In these coordinates, ball a is at distance zero from the origin, ball b is at distance $L$ from the origin, and the center of mass is at distance $d_a$ from the origin. Using these values in the equation for center of mass, you obtain

$$d_a = \frac{0 \cdot m_a + L m_b}{m_a + m_b}.$$ 

Use the result from Part A to eliminate $m_a$ and $m_b$ from this equation and obtain $d_a$ in terms of $L$.

Hint B.2  Find $m_b$ in terms of $m_a$

What is $m_b$ in terms of $m_a$?

**ANSWER:**

$$m_b = 3m_a$$

Now substitute $m_b = 3m_a$ in your formula for $d_a$.

Express your answer in terms of $L$, the length of the rod.

**ANSWER:**

$$d_a = \frac{3L}{4}$$
A 1520-N crate is to be held in place on a ramp that rises at 30.0° above the horizontal (see in the figure). The massless rope attached to the crate makes a 22.0° angle above the surface of the ramp. The coefficients of friction between the crate and the surface of the ramp are $\mu_k = 0.450$ and $\mu_s = 0.650$. The pulley has no appreciable mass or friction. What is the maximum weight $w$ needed to hold this crate stationary on the ramp?

ANSWER: 1380 N

Score Summary:
Your score on this assignment is 0%.
You received 0 out of a possible total of 15 points.