Assigment 7

1) Let $\rho_i : G \to End(V_i)$, $\theta_i : H \to End(W_i)$ be representations for $i \in \{1,2\}$ and $H \leq G$.

If each ρ_i is induced by θ_i, then for C a set of coset representatives of H in G,
$$V_i = \bigoplus_{c \in C} c \cdot W_i$$

Thus $V_1 \oplus V_2 = \bigoplus_{c\in C} c \cdot W_1 \oplus \bigoplus_{c \in C} c \cdot W_2 \cong \bigoplus_{c \in C} c \cdot (W_1 \oplus W_2)$

Thus $\rho_1 \oplus \rho_2$ is induced by $\theta_1 \oplus \theta_2$.

2) $\sigma_i \sigma_{i+1} \sigma_i^{-1} = (\sigma_{i+1}^{-1} \sigma_{i+1}) \sigma_i \sigma_{i+1} \sigma_i^{-1} = \sigma_{i+1}^{-1} (\sigma_{i+1} \sigma_i \sigma_{i+1}) \sigma_i^{-1}$
$= \sigma_{i+1}^{-1} (\sigma_i \sigma_{i+1} \sigma_i) \sigma_i^{-1} = \sigma_{i+1}^{-1} \sigma_i \sigma_{i+1} (\sigma_i \sigma_i^{-1}) = \sigma_{i+1}^{-1} \sigma_i \sigma_{i+1}$

3) let $\imath_n : B_n \to B_{n+1}$ be "inclusion" by adding a strand on the left, i.e. $\sigma_i \to \sigma_{i+1}$.

If $j > 1$, $\sigma_j \Delta_n = \sigma_j \imath_n(\Delta_{n-1}) \sigma_1 \cdots \sigma_{n-1}$
$= \imath_n(\sigma_{j-1} \Delta_{n-1}) \sigma_1 \cdots \sigma_{n-1}$
$= \imath_n(\Delta_{n-1} \sigma_{n-j}) \sigma_1 \cdots \sigma_{n-1}$
$= \imath_n(\Delta_{n-1}) \sigma_{n-j+1} \sigma_1 \cdots \sigma_{n-1}$
$= \imath_n(\Delta_{n-1}) \sigma_1 \cdots \sigma_{n-j-1} \sigma_{n-j+1} \sigma_{n-j} \sigma_{n-j+1} \sigma_{n-j+2} \cdots \sigma_{n-1}$
$= \imath_n(\Delta_{n-1}) \sigma_1 \cdots \sigma_{n-j-1} (\sigma_{n-j} \sigma_{n-j+1} \sigma_{n-j}) \sigma_{n-j+2} \cdots \sigma_{n-1}$
$= \imath_n(\Delta_{n-1}) \sigma_1 \cdots \sigma_{n-1} \sigma_{n-j} = \Delta_n \sigma_{n-j}$

If $j = 1$, $\sigma_1 \Delta_n = \sigma_1 \imath_n(\Delta_{n-1}) \sigma_1 \cdots \sigma_{n-1}$
$= \sigma_1 \imath_n(\imath_{n-1}(\Delta_{n-2}) \sigma_1 \cdots \sigma_{n-2}) \sigma_1 \cdots \sigma_{n-1}$
$= \sigma_1 \imath_n(\imath_{n-1}(\Delta_{n-2})) \sigma_2 \cdots \sigma_{n-1} \sigma_1 \cdots \sigma_{n-1}$

far commutativity on σ_1, $\iota_n(\iota_{n-1}(\Delta_{n-2}))$

$= \iota_n(\iota_{n-1}(\Delta_{n-2}))\,\sigma_1\sigma_2\cdots\sigma_{n-1}\,\sigma_1\cdots\sigma_{n-1}$

$= \iota_n(\iota_{n-1}(\Delta_{n-2}))\,\sigma_1\sigma_2\,\sigma_1\sigma_3\,\sigma_2\sigma_4\sigma_3\cdots\sigma_{n-3}\sigma_{n-1}\sigma_{n-2}\,\sigma_{n-1}$

(by applications of far commutativity)

$= \iota_n(\iota_{n-1}(\Delta_{n-2}))\,\sigma_2\sigma_1\sigma_3\sigma_2\sigma_4\sigma_3\cdots\sigma_{n-1}\sigma_{n-2}\sigma_{n-1}\,\sigma_{n-1}$

(by a "wave" of R-III moves)

$= \iota_n(\iota_{n-1}(\Delta_{n-2}))\,\sigma_2\cdots\sigma_{n-1}\,\sigma_1\cdots\sigma_{n-1}\,\sigma_{n-1}$

(by far comm.)

$= \iota_n(\iota_{n-1}(\Delta_{n-2})\,\sigma_1\cdots\sigma_{n-2})\,\sigma_1\cdots\sigma_{n-1}\,\sigma_{n-1}$

$= \iota_n(\Delta_{n-1})\,\sigma_1\cdots\sigma_{n-1}\,\sigma_{n-1}$

$= \Delta_n\,\sigma_{n-1}$.