TAKE HOME FINAL FOR M6390 FALL 08

Submit solutions for up to five of the following problems. (I will comment on additional solutions submitted, but will choose five at random for grading if you do not specify which problems to grade).

(1) Prove that if $f : V \to W$ is a G-equivariant map, then the map $f^* : W^* \to V^*$, defined as $(Id_{V^*} \otimes ev_W) \circ (Id_{V^*} \otimes f \otimes Id_{W^*}) \circ (coev_V \otimes Id_{W^*})$ is also G-equivariant.

(2) (S_n on a twisted ribbon): For any $n \in \mathbf{N}^+$ one can define a group T_n generated by elements of S_n and a new element f such that $ff = e$ and for any cycle $(a_1, a_2, \ldots, a_k) \in S_n$, $(a_1, a_2, \ldots, a_k)f = f(n+1-a_1, n+1-a_2, \ldots, n+1-a_k)$. This group has order $2n!$. Compute the character table for T_3.

(3) For arbitrary n, describe the irreps of T_n in terms of the irreps of S_n.

(4) Prove that for any $n \in \mathbf{N}^+$ and any $0 \le k \le n-1$ there exists an irreducible representation of S_n with dimension $\binom{n-1}{k}$.

(5) Prove that for the partition $\lambda = (n-1, 1)$, c_λ generates the standard (irreducible) representation of S_n as a right ideal. (One way: $Im(c_\lambda)$ has a basis obtained from standard tableau. Examine the action of S_n on this basis.)

(6) Prove that any two positive braid words $b_1^+, b_2^+ \in B_n^+$ which give the same braid are connected by a series of Reidemeister III moves (another way to say this is that they are connected by a sequence of moves such that the intermediate steps lie within B_n^+).

(7) Find an irreducible representation of the group of Rubik's cube twists not of degree one (Let's allow all physically possible motions which send the cube shape back to itself).