QUIZ 3 STAT4351; Name (Please Print):

1. Consider a binomial random variable $Binom(\theta, n)$. Let $p(x|\theta, n)$ denotes its probability mass function. Use $\sum_{x=0}^{n} p(x|\theta, n) = 1$, differentiate both sides with respect to $\theta$, and then use the result to prove that the expectation of $X$ is $n\theta$. Show all steps of the proof.

2. For the binomial random variable in Problem 1, prove that $p(x+1|\theta, n) = \theta(n-x)[(x+1)(1-\theta)]^{-1}p(x|\theta, n)$.

3. Moment generating function uniquely defines exponential and Gamma random variables. Using this fact, prove that the sum of independent and identically distributed exponential random variables has a Gamma distribution. Write down the proof and the corresponding Gamma distribution.

4. A manufacturer claims that poor quality of supplies yields 70% of defected TVs. Find the probability that 5 of the next 6 TVs will be defected due to the poor quality of supplies. Use both the formula and Table.

5. Among the 300 employees of a company, 240 are union members. If six of the employees are chosen to serve on a committee, what is the probability that four of the six will be union members?

6.If 18 defective glass bricks include 10 that have cracks but no discoloration, five that have discoloration but no cracks, and three that have cracks and discoloration, what is the probability that among six of the bricks (chosen at random for further checks) three will have cracks but no discoloration, one will have discoloration but no cracks, and two will have cracks and discoloration?

7. Consider an exponential random variable with parameter $\theta$. (a) Find the probability that it will take on value smaller than $-\theta \ln(1-p)$. (b) Find its hazard function $f(x)/(1-F(x))$ where $f$ is the density and $F$ is the cdf.

8. Consider a probability density function $f_X(x) = kx^{\beta-1}e^{-\alpha x^{\beta}}I(x > 0)$. Find $k$ and then the expected value.

9. Let $X$ and $Y$ have a bivariate normal distribution. Find the correlation coefficient between $U = 2X + Y$ and $V = X + 2Y$.