# HOMEWORK 3, STAT 6331

1. Let $(X, Y)$ denote an iid sample from Geometric($p$), that is, the pmf of $X$ is

$$P(X = k|p) = p(1-p)^{k-1}, \quad k = 1, 2, \ldots$$

Consider the statistic $T = X - Y$, and show that it is not sufficient.

Hint: It suffices to consider a particular value of $T$, say $T = 0$ for the sake of simplicity, and prove that $P((X, Y) = (i, j)|T = 0, p)$ depends on $p$.

2. Suppose that a family $\{f_\theta(\underline{x}), \theta \in \Omega\}$ of pdfs or pmfs is specified. It is known that $\Omega := \{\theta_0, \theta_1\}$, that is, the parameter $\theta$ takes on only two values.

The *likelihood ratio* statistic (typically written as $\Lambda(\underline{x})$ or $\lambda(\underline{x})$) is defined as

$$T(\underline{X}) = \frac{f_{\theta_1}(\underline{X})}{f_{\theta_0}(\underline{X})}.$$

Prove that the likelihood ratio is SS (Sufficient Statistic).

3. Let $\underline{X} := (X_1, X_2, \ldots, X_n)$ be iid sample from a uniform distribution on $(\theta - 1/2, \theta + 1/2)$, that is, $X_1 \sim U(\theta - 1/2, \theta + 1/2)$ or we can say that $X_1$ has the uniform pdf $f(x|\theta) = I(x \in (\theta - 1/2, \theta + 1/2))$. Find a two-dimensional SS.

4. Prove that if a sufficient statistic $T$ is a function of $U$, then $U$ is sufficient. And if $T$ is a function of $U$ which is not sufficient, neither is $T$.

Problems below are from the text, p.300

5. Exerc. 6.1
6. Exerc. 6.2
7. Exerc. 6.3
8. Exerc. 6.5
9. Exerc. 6.6
10. Exerc. 6.7