HOMEWORK 6, ACTS 4306

For each problem, you need to choose a correct answer among 5 given answers.

1. Let X be a continuous random variable with density function $f(x) = \frac{|x|}{10} I(-2 \leq x \leq 4)$. Calcualte the expected value of X.

[A] 1/5 [B] 3/5 [C] 1 [D] 28/15 [E] 12/5

2. An insurer's annual weather repated loss, X, is a random variable with density function

$$f(x) = \frac{2.5(200)^{2.5}}{x^{3.5}} I(x \geq 200).$$

Calculate the difference between the 30th and 70th percentiles of X.

[A] 35 [B] 93 [C] 124 [D] 131 [E] 298

3. A recent study indicates that the annual cost of maintaining and repairing a car averages 200 with a variance of 260. If a tax of 20% is introduced on all items associated with the maintenance and repair of cars (i.e., everything is made 20% more expansive), what will be the variance of the annual cost of maintaining and repairing a car?

[A] 208 [B] 260 [C] 270 [D] 312 [E] 374

4. A probability distribution of the claim sizes for an auto insurance policy is given in the table below:

Claim Size	20	30	40	50	60	70	80
Probability	.15	.10	.05	.20	.10	.10	.30

What percentage of claims are within one standard deviation of the mean claim size?

[A] 45% [B] 55% [C] 68% [D] 85% [E] 100%

5. An actuary determines that the claim size for a certain class accidents is a random variable, X, with moment generating function $M_X(t) = (1 - 2500t)^{-4}$. determine the standard deviation of the claim size for this class of accidents.

[A] 1,340 [B] 5,000 [C] 8,660 [D] 10,000 [E] 11,180

6. A company insurers homes in three cities: J, K, and L. It is reasonable to assume that the losses occurring in the cities are independent. The moment generating functions for the loss distributions of the cities are:

$$M_J(t) = (1 - 2t)^{-3}, \quad M_K(t) = (1 - 2t)^{-2.5}, \quad M_L(t) = (1 - 2t)^{-4.5}.$$

Let X represent the combined losses from the three cities. Calculate $E(X^3)$.

[A] 1,320 [B] 2,082 [C] 5,760 [D] 8,000 [E] 10,560

7. Let X_1, X_2, X_3 be a random sample from a discrete distribution with probability mass function $p(0) = 1/3$ and $p(1) = 2/3$. Determine the moment generating function $M_Y(t)$ of $Y = X_1X_2X_3$.

[A] $(19/27) + (8/27)e^t$ [B] $1 + 2e^t$ [C] $[(1/3) + (2/3)e^t]^3$ [D] $(1/27) + (8/27)e^{3t}$ [E] $(1/3) + (2/3)e^{3t}$

8. An insurance company's monthly claims are modeked by a continuous, positive random variable X, whose probability density function is proportional to $(1+x)^{-4}$, where $0 < x < \infty$. Determine the company's expeted monthly claims.

[A] 1/6 [B] 1/3 [C] 1/2 [D] 1 [E] 3

9. A random variable has the cumulative distribution function

$$F(x) = \begin{cases} 0 & \text{for } x < 1 \\ \frac{x^2-2x+2}{2} & \text{for } 1 \le x < 2 \\ 1 & \text{for } x > 2 \end{cases}$$

Calculate the variance of X.

[A] 7/72 [B] 1/8 [C] 5/36 [D] 4/32 [E] 23/12

10. The loss amount, X, for a fire damage, has the cumulative distribution function

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ (1/9)[2x^2 - x^3/3] & \text{for } 0 \le x \le 3 \\ 1 & \text{for } x > 3. \end{cases}$$

Calculate the mode of the distribution.

[A] 2/3 [B] 1 [C] 3/2 [D] 2 [E] 3