ECONOMICS 3304:
Statistics

Brief Statistics Review!

- Basics
- Sampling
- Estimation
- Hypothesis Testing

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ECONOMICS 3304:
Basics: Random Variables

- X is a random variable if it represents a random draw from some population.
  - Discrete random variable can take on only selected, countable values (e.g., coin toss, rainy day).
  - Continuous random variable can take on any value in a real interval (e.g., time measured outcomes, height, etc.)
- Associated with each random variable is a probability distribution.

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Basics: Standardized Variables

- Sometimes random variables represent the culmination of random events.
  - Life Expectancy (genes, nutrition, exercise, etc.)
  - Intelligence (heredity, parenting, schooling, etc.)
  - Stock Price (economy, dividends, forecasts, etc.)
- Standardizing these types of random variables can be important because
  - Common unit of measurement.
  - Similar probability distributions.

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Standardized Variables: Common Measurement Unit

- How to standardize random variables?

$$Z = \frac{X - \mu}{\sigma}$$

- The new standardized random variable Z has a mean of 0 and standard deviation equal to 1.
- Ex: The IQ of a randomly selected person can be considered a random variable drawn from a population with mean 100 and std dev of 15.

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Standardized Variables: Central Limit Theorem

- The ***central limit theorem*** states that a random variable X will tend to have a normal distribution, if it reflects the culmination of a large number of independent random variables.
- Culmination typically is interpreted as the summation or average of random variables.

$$\sum X_i = X_1 + X_2 + \ldots + X_N$$

$$\overline{X} = \tfrac{1}{N}(X_1 + X_2 + \ldots + X_N)$$

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Standardized Variables: The Central Limit Theorem

Let $X_1, X_2, \ldots, X_N$ be independent random variables each with a distribution that has a mean of μ and variance of σ^2. Let $\overline{X}$ be the average of the random variables: $\overline{X} = \frac{1}{N}\sum X_i$.

Then $\overline{X} \sim N\left(\mu, \frac{\sigma^2}{N}\right)$ approximately.

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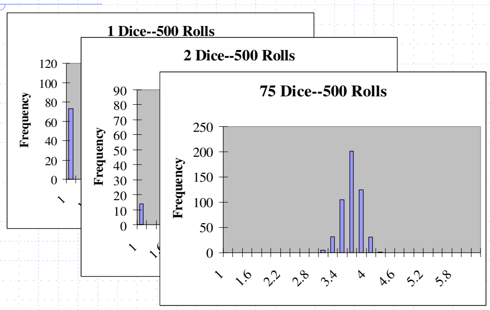
Central Limit Theorem: Histograms for Six-Sided Dice



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Standardized Variables: The Central Limit Theorem

Suppose that $\overline{\mathrm{X}} = \frac{1}{N}\sum X_i$ is standardized with its mean μ and standard deviation $\sigma / \sqrt{\mathrm{N}}$ then the resulting standardized Z statistic will converge to a normal distribution with mean 0 and variance of 1.

Then $\mathrm{Z} = \dfrac{\overline{\mathrm{X}} - \mu}{\sigma / \sqrt{N}} \sim N(0,1)$ approximately.

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Statistics Review: Sampling

- A **sample** is the part of the data the econometrician actually observes. It can be contrasted with the **population**, which is the complete set of data.
- Why bother taking samples?
  - Expensive to collect data on the entire population.
  - Sample data can be used to make statistical inferences about the population.

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Statistics Review: Sampling

- **Selection Bias** refers to any sample systematically differing from the population it is supposed to represent.
- **Self-selection Bias** is a similar form of bias that can occur when examining data where individuals have chosen to be in a particular group.
- **Survivor Bias** occurs when choosing a sample from a current population to make inferences about a past population.
- **Nonresponse bias** refers to the systematic refusal of some groups to participate in an experiment or survey.

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Random Samples and Sampling

- For a random variable X, repeated draws from the same population can be labeled as $X_1, X_2, \ldots, X_N$
- If every combination of N sample points has an equal chance of being selected, this is a random sample.
- A random sample is a set of independent, identically distributed (i.i.d.) random variables.

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Statistics Review: Estimators and Estimates

- Typically, we can't observe the full population, so we must make inferences base on estimates from a random sample
- An ***estimator*** is just a mathematical formula for estimating a population *parameter* from sample data
- An ***estimate*** is the actual number the formula produces from the sample data

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Examples of Estimators

- Suppose we want to estimate the parameter for mean of a population. Can calculate the sample average for our sample:

$$\overline{X} = \frac{1}{N}\sum_{i=1}^{n} X_i$$

- Similar estimate can be made for the median parameter.

$$Med\left[X\right] = Med\left[X_1,\ldots,X_N\right]$$

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Estimators & Estimates: Average Value of Single-Family Homes in Diamond Bar, CA

- A random sample of 22 single family homes in Diamond Bar, California is drawn from a population of 20,000 homes.

$$\overline{X} = \frac{\$425{,}000 + \ldots + \$788{,}888}{22}$$
$$= \$565{,}829$$

- Another random sample will likely yield an estimate with a different value.

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Estimators: Sampling Distributions

- The **sampling distribution** of a statistic is the probability distribution that describes the population of all possible values of the estimator.
- For sample average, the sampling distribution will tend to be normal due to the central limit theorem.
- For the sample average, also …

$$\text{Mean of } \overline{X} = \mu$$

$$\text{Std.Dev.of } \overline{X} = \frac{\sigma}{\sqrt{N}}$$

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Estimators: Sampling Distributions

- The Mean of the Sampling Distribution
  - Gauges how close the sample statistic is to the population parameter. An **unbiased estimator** occurs when the mean of the sampling distribution equals the population parameter.
- Standard Deviation of the Sampling Distribution
  - The accuracy of an estimator can be gauged by its standard deviation. The smaller the standard deviation, the higher probability the estimate will be close to the mean.

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Statistics Review: Hypothesis Tests

- Sample data can be used to support or refute theories about the value of population parameters.
- For example, whether the average price of single family homes in Diamond Bar, CA is $400,000.
- A null hypothesis and alternative hypothesis are proposed.

$$H_0 : \mu = \$400{,}000$$
$$H_A : \mu \neq \$400{,}000$$

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Hypothesis Tests: The *t*-Distribution

- The central limit theorem shows a standardized variable converges to a normal distribution with mean 0 and variance 1.

$$Z = \frac{\overline{X} - \mu}{\sigma / \sqrt{N}} \sim N(0,1)$$

- The t-distribution does not rely on knowing the parameter for standard deviation.

$$t = \frac{\overline{X} - \mu}{s / \sqrt{N}} \sim t_{(N-1)}$$

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Hypothesis Tests: The *t*-Distribution

- We have a good estimate of μ, would like a good estimate of σ^2
- Can use the sample variance given below – note division by n-1, not n, since mean is estimated too – if know μ can use n

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$$

Hypothesis Tests

- Constructing the Test Statistic
  - An estimator is used to test the null hypothesis, which is called a test statistic.

$$t = \frac{\overline{X} - \mu}{s / \sqrt{N}}$$
$$= \frac{\$565{,}829 - \$400{,}000}{\$116{,}596 / \sqrt{22}} \sim t_{(N-1)}$$
$$= 6.671$$