21. Find a way of accurately computing $f(x) = x + e^x - e^{2x}$ for small values of x.

22. a. Find a way to calculate accurate values near 0 for the function

$$f(x) = (e^{\tan x} - e^x)/x^3$$

b. Determine $\lim_{x\to 0} f(x)$. *Hint:* See Problem **1.2.4** (p. 25).

23. Explain why loss of significance due to subtraction is not serious in using the approximation

$$x - \sin x \approx (x^3/6)(1 - (x^2/20)(1 - x^2/42))$$

24. In computing the sum of an infinite series $\sum_{n=1}^{\infty} x_n$, suppose that the answer is desired with an absolute error less than ε. Is it safe to stop the addition of terms when their magnitude falls below ε? Illustrate with the series $\sum_{n=1}^{\infty} (0.99)^n$.

25. (Continuation) Repeat the preceding problem under the additional assumptions that the terms x_n are alternately positive and negative and that $|x_n|$ converges monotonically downward to 0. (Use a theorem in calculus about alternating series.)

26. Show that if x is a machine number on the `Marc-32` and if $x > 2^{25}\pi$, then $\cos x$ can be computed with *no* significant digits.

COMPUTER PROBLEMS 2.2

1. Write and execute a program to compute

$$f(x) = \sqrt{x^2 + 1} - 1$$
$$g(x) = x^2/\left(\sqrt{x^2 + 1} + 1\right)$$

for a succession of values of x, such as $8^{-1}, 8^{-2}, 8^{-3}, \ldots$. Although $f = g$, the computer produces different results. Which results are reliable and which are not?

2. Write and test a subroutine that accepts a machine number x and returns the value $y = x - \sin x$ with nearly full machine precision.

3. Using your computer, print the values of the functions

$$f(x) = x^8 - 8x^7 + 28x^6 - 56x^5 + 70x^4 - 56x^3 + 28x^2 - 8x + 1$$
$$g(x) = (((((((x - 8)x + 28)x - 56)x + 70)x - 56)x + 28)x - 8)x + 1$$
$$h(x) = (x - 1)^8$$

at 101 equally spaced points covering the interval [0.99, 1.01]. Calculate each function in a straightforward way without rearranging or factoring. Observe that the three functions are identical. Account for the fact that the printed values are not all positive as they should be. If a plotter is available, plot these functions near 1.0 using a magnified scale for the function values to see the variations involved. (See Rice [1992, p. 43].)

4. Write and test a code to supply accurate values of $1 - \cos x$ for $-\pi \le x \le \pi$. Use a Taylor series near 0 and the subprogram for cosine otherwise. Determine carefully the range where each method should be used to lose at most one bit.

5. Write and test a function subprogram for $f(x) = x^{-2}(1 - \cos x)$. Avoid loss of significance in subtraction for all arguments x and (of course) take care of the difficulty at $x = 0$.

6. An interesting numerical experiment is to compute the dot product of the following two vectors: