The following almost obvious result forms the basis of what is known as the *principle of superposition.*

1.19. Theorem. *If $Lu = g$ is a linear equation and u_1 and u_2 are solutions of this equation with $g = g_1$ and $g = g_2$, respectively, then $u_1 + u_2$ is a solution of the equation with $g = g_1 + g_2$; in other words, if*

$$Lu_1 = g_1, \quad Lu_2 = g_2,$$

then

$$L(u_1 + u_2) = g_1 + g_2.$$

Exercises

In (1)–(22) find the general solution of the given equation.

(1) $(x^2+1)y' = 2xy$.
(2) $y' - 3x^2(y+1) = 0$.
(3) $(x-1)y' + 2y = x,\ x \neq 1$.
(4) $x^2y' - 2xy = x^5e^x$.
(5) $2y' + 5y = 0$.
(6) $3y' - 2y = 0$.
(7) $y'' - 4y' + 3y = 0$.
(8) $2y'' - 5y' + 2y = 0$.
(9) $4y'' + 4y' + y = 0$.
(10) $y'' - 6y' + 9y = 0$.
(11) $y'' + 2y' + 5y = 0$.
(12) $y'' - 6y' + 13y = 0$.
(13) $y' + 2y = 2x + e^{4x}$.
(14) $2y' - 3y = -3x - 4 + e^x$.
(15) $2y' - y = e^{x/2}$.
(16) $y' + y = -x + 2e^{-x}$.
(17) $y'' - y = x^2 - x + 2$.
(18) $y'' - 2y' - 8y = 4 + 4x - 8x^2$.
(19) $y'' - 25y = 30e^{-5x}$.
(20) $4y'' + y = 8\cos(x/2)$.
(21) $2x^2y'' + xy' - 3y = 0$.
(22) $x^2y'' + 2xy' - 6y = 0$.

In (23)–(26) verify whether the given ODE is linear or nonlinear.

(23) $xy'' - y' \sin x = xe^x$.

(24) $y' + 2x \sin y = 1$.

(25) $y'y'' - xy = 2x$.

(26) $y'' + \sqrt{x}\, y = \ln x$.