The Transshipment Problem: An Example

In a transportation problem, shipments are allowed only between source-sink pairs. In many applications, this assumption is too strong. For example, it is often the case that shipments may be allowed between sources and between sinks. Moreover, there may also exist points through which units of a product can be transshipped from a source to a sink. Models with these additional features are called *transshipment problems*. Interestingly, it turns out that any given transshipment problem can be converted easily into an equivalent transportation problem. The availability of such a conversion procedure significantly broadens the applicability of our algorithm for solving transportation problems.

We will illustrate the conversion procedure with an example.

A company manufactures a product in two cities, which are Dallas and Houston. The daily production capacities at Dallas and Houston are 160 and 200, respectively. Products are shipped by air to customers in San Francisco and New York. The customers in each city require 140 units of the product per day. Because of the deregulation of air fares, the company believes that it may be cheaper to first fly some products to Chicago or Los Angeles and then fly the products to their final destinations. The costs of flying one unit of the product between these cities are shown in the table below.

	TO					
FROM	Dallas	Houston	Chicago	L. A.	S. F.	N. Y.
Dallas	\$0	–	\$9	\$14	\$26	\$29
Houston	–	\$0	\$16	\$13	\$27	\$26
Chicago	–	–	\$0	\$7	\$17	\$18
L. A.	–	–	\$7	\$0	\$15	\$17
S. F.	–	–	–	–	\$0	–
N. Y.	–	–	–	–	–	\$0

The company wants to minimize the total cost of daily shipments of the required products to its customers.

We shall first define our terminology more carefully. We will define a *source* as a city that can send products to another city but cannot receive any product from any other city. Similarly, a *sink* is defined as a city that can receive products from other cities but cannot send products to any other city. Finally, a *transshipment point* is defined as a city that can both receive products from other cities and send products to other cities.

According to these definitions, Dallas and Houston are sources, with (daily) supplies of 160 and 200 units respectively. Chicago and Los Angeles are transshipment points. San Francisco and New York are sinks, each with a (daily) demand of 140 units.

Observe that the total supply equals 360 and the total demand equals 280. Therefore, we should create a dummy sink, with a demand of 80, to balance the two. With this revision, we have a problem with 2 sources, 3 sinks, and 2 transshipment points.

Since each transshipment point can both receive and send out products, it plays the dual roles of being a sink and a source. This naturally suggests that we could attempt a reformulation in which each transshipment point is "split" into a corresponding sink and a corresponding source. A little bit of reflection, however, leads us to the realization that while the demand and the supply at such a pair of sink and source should be set at the same level (since there is no gain or loss in units), it is not clear what that level should be. This is a consequence of the fact that we do not know a priori how many units will be sent into and hence shipped out of a transshipment point. Fortunately, upon further reflection, it turns out that this difficulty can actually be circumvented by assigning a "sufficiently-high" value as the demand and the supply for such a sink-source pair and by allowing *fictitious* shipments from a given transshipment point back to itself at zero cost.

More specifically, suppose the common value of the demand and the supply at the corresponding sink and source of a given transshipment point is set to c; and suppose x units of "real" shipments are sent into and shipped out of that transshipment point. Then, under the assumption that c is no less than x, we can interpret this as saying: (i) a total of c units of the product are being sent into the corresponding sink, of which x units are sent from other points (or cities) and $c - x$ units are sent (fictitiously) from the transshipment point to itself; and (ii) a total of c units of the product are being shipped out of the corresponding source, of which x units are shipped to other points (or cities) and $c - x$ units are shipped (fictitiously) back to the transshipment point itself.

Notice that since a shipment from a transshipment point back to itself is assumed to incur no cost, the proposed reformulation preserves the original objective function.

The only remaining question now is: What specific value should be assigned to c? The default answer to this question is to let c equal to the total supply in the original problem. Such a choice is clearly sufficient because no shipment can exceed the total available supply. It follows that we have indeed resolved the difficulty alluded to earlier.

In our problem, there are two transshipment points. Therefore, we should replace these by two new sources and two new sinks (all of which will retain their original city names). Furthermore, the new sources and sinks should have a common supply or demand of 360. Our reformulation, therefore, yields an equivalent transportation problem with 4 sources and 5 sinks. This equivalent transportation problem is given in the tableau below.

	S. F.	N. Y.	Chicago	L. A.	Dummy	
Dallas	26	29	9	14	0	160
Houston	27	26	16	13	0	200
Chicago	17	18	0	7	0	360
L. A.	15	17	7	0	0	360
	140	140	360	360	80	

Since the solution method for transportation problems has been explained in detail, we will not attempt to solve this problem.

An interesting variation of the above example is to allow shipments between Dallas and Houston, say, at a cost of \$5 per unit either way. This would make Dallas and Houston transshipment points. It follows that we should introduce into the above tableau two new sinks to represent Dallas and Houston, respectively. Both of these two new sinks should have a demand of 360. Correspondingly, it will also be necessary to increase the supply from Dallas by 360, to 520, and the supply from Houston by 360, to 560. These revisions result in the formulation below.

	S. F.	N. Y.	Chicago	L. A.	Dallas	Houston	Dummy	
Dallas	26	29	9	14	0	5	0	520
Houston	27	26	16	13	5	0	0	560
Chicago	17	18	0	7	M	M	0	360
L. A.	15	17	7	0	M	M	0	360
	140	140	360	360	360	360	80	

Notice that we have introduced 4 "big" M's as the transportation costs in four cells. These are intended to insure that shipments into Dallas and Houston from other cities do not appear in the optimal solution.

It should be clear that other variations can also be handled similarly.