EE6351 Homework 2: SOLUTION OF MATRIX EQUATIONS

Assignment objective:

- Understand and program Gaussian elimination
- Examine the effect of
  - pivoting
  - scaling
  - ill-conditioned matrices
- Compare the results with linpack or other canned matrix solver
- Programming skills –
  - matrix manipulation
  - use of Matlab integration functions

1. Attempt to solve the following matrix equations by hand using Gaussian elimination WITHOUT pivoting or scaling. Use only 2 decimal places of accuracy. Show all intermediate matrices. (Note: Do not use the matrix solution from your calculator, as it will work much like Matlab. Do these problems by hand on paper, dropping the accuracy of your calculations to 2 decimal places.)

   a) Well conditioned matrix which does not require scaling or pivoting.

   $$\begin{aligned} x_1 &+ x_2 &+ x_3 &= 6 \\ 3x_1 &+ 0.5x_2 &+ 5x_3 &= 17 \\ 2x_1 &+ x_2 &+ 4x_3 &= 14 \end{aligned}$$

   (Answer: $x_1 = x_2 = x_3 = 2$)

   b) Well conditioned matrix which requires pivoting.

   $$\begin{aligned} -x_1 &+ 5x_2 &+ x_3 &= 12 \\ &- x_2 &+ 5x_3 &= 13 \\ 5x_1 &- x_2 & &= 3 \end{aligned}$$

   (Answer: $x_1 = 1$, $x_2 = 2$, $x_3 = 3$)

   c) Well-conditioned matrix which requires scaling.
   (Poorly scaled version of part a. Do not divide by 10^{15}, just attempt to solve as is.)

   $$\begin{aligned} 10^{15} x_1 &+ 10^{15} x_2 &+10^{15} x_3 &= 6\text{x}10^{15} \\ 3x_1 &+ 0.5x_2 &+ 5x_3 &= 17 \\ 2x_1 &+ x_2 &+ 4x_3 &= 14 \end{aligned}$$

   (Answer: $x_1 = x_2 = x_3 = 2$)

   d) Ill-conditioned matrix.

   $$\begin{aligned} x_1 &+ x_2 &+ x_3 &= 6 \\ 2x_1 &+ 2.000000001x_2 &+ 1.999999998\ x_3 &= 12 \\ 2x_1 &+ x_2 &+ 4x_3 &= 14 \end{aligned}$$

   (Answer: matrix is ill-conditioned, rows 1 and 2 are nearly coincident.)

   e) Calculate the determinant of the matrix in part d

2. Write a program to solve a matrix equation using Gaussian Elimination. Use your program to solve the sets of equations in (a) through (d) above. Explain any differences between answers obtained by your program and your initial calculations.

3. Use Matlab to solve the matrix equations in (a) through (d) from part 1. Use this software to find the condition number or determinant of your matrices. Matlab functions (read their help files to see what they are doing) include trap, quad, quad8.

4. Summarize your results.
   Explain when and why pivoting and scaling may be important.
   Explain the effect of ill-conditioning. Can you improve conditioning by going to double precision? If you had infinite precision, could you solve any ill-conditioned system?

Turn in:

[ ] Hand calculations for part 1.

[ ] Program(s) for solving matrix equations.

- [ ] WELL commented
- [ ] Proof that they work (you decide how to verify your code)

[ ] Summary in part 4.