

Objectives:

- ◆ Understand and program the FDTD equations in 1D
- ◆ Observe CW and Pulsed time domain data
- ◆ Observe numerical dispersion
- ◆ Understand and program the Mur 1st order absorbing boundary conditions
- ◆ Understand the relationship between time domain and frequency domain data and use this to calculate reflection coefficient

1. Differential Equations (3D to 1D)

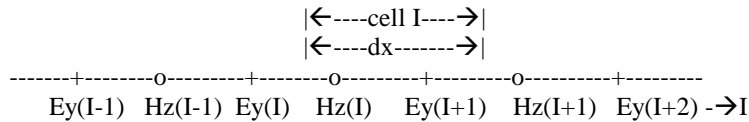
Starting with Maxwell's equations in the time domain (Ampere's and Faraday's laws), differential form, write the 6 coupled differential equations. (Take the cross products and equate vector components.)

Convert these equations to the 1-dimensional TE-to-z case by setting $d/dy = d/dz = 0$ and $E_z = 0$. This represents a plane wave propagating in the x-direction. You should end up with equations for E_y and H_z . (The TM-to-z case would have similar equations for E_z and H_y .)

2. FDTD Equations (1D TE-to-z case)

Convert the 1D TE differential equations above to their FDTD difference form. (Use the central difference formula to approximate the derivatives, and solve for $E_y^{(n+1)}$ and $H_z^{(n+1/2)}$.)

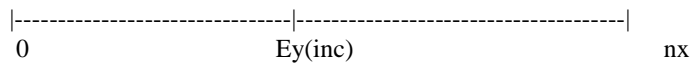
Use the 1D FDTD lattice shown below:



Let the E fields be defined at times $n, n+1, n-1$, etc.
 Let the H fields be defined at times $n-1/2, n+1/2$, etc.

3. Program the FDTD Equations (1D TE case)

Program the equations in (2) for the geometry shown below. Use a forced CW source on E_y : $E_y(I=inc) = \sin(\omega t)$. Note: although all the test cases here are in air, write your code for arbitrary materials.



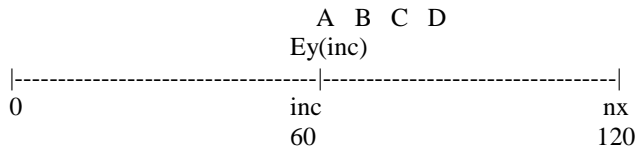
4. Test the FDTD Equations and observe CW Time Domain Data

Use $F = 2\text{GHz}$, $dx = \text{wavelength}/20$, $dt = dx/(2c)$, $nx=120$, $inc=60$.

Plot the E_y and H_z fields at points A,B,C,D as a function of time for 100 time steps. (Note that this stops the simulation just before the wave touches the lack of boundary conditions at the ends.) Give one plot of the four E fields, and another of the four H fields. Store the E fields at point C for use in problem 7.

Using matlab, make a movie of the E fields at all points as a function of time.

Plot the Ey field at point D against the analytical value: $E_y(x) = \sin(\omega t - \beta x)$, where x is the distance from the source.



- A is located at I=60, at source
- B is located at I=63, 3 cells from source
- C is located at I=67, 7 cells from source
- D is located at I=90, 30 cells from source

5. Observe Pulsed Time Domain Data

Change the source to a raised cosine pulse:

$$E_y(\text{inc}) = \begin{cases} 1 - \cos(\omega t) & 0 < t < 1/F_{\text{max}} \\ 0 & t > 1/F_{\text{max}} \end{cases}$$

Use $F_{\text{max}} = 2\text{GHz}$, $dx = \text{wavelength}/20$, $dt = dx/(2c)$, $nx = 120$, $inc = 60$.

Plot the Ey fields at points A,B,C,D as a function of time for 100 time steps. (Notes: If you run more than 120 time steps you will see the waves reflect off the ends of the FDTD mesh.)

Using Matlab, make a movie of the Ey fields as a function of time along the mesh.

6. Observe Numerical Dispersion

Use the raised cosine pulsed source, $F_{\text{max}} = 2\text{GHz}$, $dt = dx/(2c)$, $nx = 220$, $inc = 110$. Run for 200 time steps.

Plot the Ey fields as a function of time 30 cells from the source for $dx = \text{wavelength}/60$, $\text{wavelength}/20$, $\text{wavelength}/10$, and $\text{wavelength}/5$.

Plot the Ey fields 30 cells from the source for the CW source using $dx = \text{wavelength}/5$ and compared to the values observed at point D in part 4.

7. Mur 1st order boundary conditions

- a) Make sure you understand the derivation from class: What approximation is made to make the boundary conditions “first order”?
- b) Write the difference form of the 1st order boundary conditions for Ey.
- c) Program the 1st order boundary conditions for Ey on both boundaries.
- d) Test the boundary conditions:

Incident Fields: Use the Ey fields you stored 7 cells from the source in problem 4 as the “incident” fields. There is no reflection in these fields, because the waves have not yet hit the boundary.

Using the method in part 7, find and plot the incident and reflected fields at these two points as a function of time. Verify that the sum of the incident and reflected fields gives the total fields observed above.

11. Use your program to compute the reflection coefficient from a quarter-wave dielectric transformer as a function of frequency. A quarter-wave dielectric transformer is a layer of material that can be used to "match" two different materials (such as air and fluid). The transformer has a characteristic impedance of $\eta_t = \sqrt{\eta_1 \eta_2}$. The transformer is a quarter of a wavelength (in the transformer material) long. Design a transformer to match air ($\epsilon_r = 1.0$) and water ($\epsilon_r = 40.0$) at 1 MHz. The reflection coefficient should be zero at 1 MHz. Evaluate the reflection coefficient from 0.5 to 2 MHz.

TURN IN:

Hard copy of your code

Derivations of all formulas used in your code (FDTD, boundary conditions, etc.)

Plots

Answers

Summarize and comment on your results

Examples of FDTD code by David Ward: fdtd.davidward.org

Homework adapted from Cynthia Furse, <http://www.ece.utah.edu/~ece6340/>