

Experiment 7: Response of Second Order RLC Circuits

1 Objectives

In this experiment the natural and step responses of RLC circuits are examined. Specifically, simple parallel RLC circuits and series RLC circuits are considered.

2 Introduction and Test Circuits

The extension of the concepts presented in Experiment 6 to circuits with two elements capable of storing energy is considered here. Natural responses of RLC circuits are generated by the release of energy by the inductor or capacitor or both as a consequence of an abrupt change in the DC voltage or current in the circuit. Similarly, the step response of RLC circuits is generated when the inductor or capacitor or both acquire energy after a sudden application of DC voltage or current to the circuit. The description of the voltages and currents in this type of circuits is done in terms of differential equations of *second order*. For example, by applying Kirchhoff's current law to the parallel RLC circuit of Fig. 7-1, and after some mathematical manipulation, the following expression is found in terms of the voltage v :

$$\frac{d^2v}{dt^2} + \frac{1}{RC} \frac{dv}{dt} + \frac{v}{LC} = 0 \quad (7-1)$$

Similarly, for the series RLC circuit in Fig. 7-3, application of Kirchhoff's voltage law and some mathematical manipulation yield, in terms of the current i :

$$\frac{d^2i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = 0 \quad (7-2)$$

These are second-order ordinary differential equations whose general solution can be found from the roots of the *characteristic equation*:

$$s^2 + 2\alpha s + \omega_o^2 = 0 \quad (7-3)$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} \quad (7-4)$$

where α is known as the *neper frequency* (in rad/s), given by

$$\alpha = \frac{1}{2RC} \quad (7-5)$$

for the parallel circuit and, for the series circuit

$$\alpha = \frac{R}{2L} \quad (7-6)$$

The third term in (7-3) is the *resonant radian frequency* (in rad/s), given by

$$\omega_o = \frac{1}{\sqrt{LC}} \quad (7-7)$$

The values of α and ω_o^2 determine the form of the natural (or step) response of a second order parallel (or series) RLC circuit. The form of the response can be categorized as:

- *Overdamped.* The response is overdamped when $\alpha^2 > \omega_o^2$. In this case the natural (or step) response approaches its final value without oscillation.
- *Underdamped.* In this case $\alpha^2 < \omega_o^2$, and the response oscillates about its final value.
- *Critically damped.* In this case $\alpha^2 = \omega_o^2$, and the response is on the verge of oscillating about its final value.

Depending on the damping, the solution to the differential equation describing the response of the circuit can be found by applying the appropriate set of equations as is explained next.

2.1 Natural response

Determining the natural response of a second-order circuit consists in finding the type of damping and applying the appropriate set of equations. Notice that in order to determine the coefficient values, a set of two linear equations needs to be solved. Here, $x(0)$ and $dx/dt(0)$ represent the initial values of the variable representing the response (voltage or current) and its derivative evaluated at $t = 0$, respectively.

<u>Damping</u>	<u>Natural response equations</u>	<u>Coefficient equations</u>
Overdamped	$x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$	$x(0) = A_1 + A_2$ $dx/dt(0) = A_1 s_1 + A_2 s_2$
Underdamped	$x(t) = (B_1 \cos \omega_d t + B_2 \sin \omega_d t) e^{-\alpha t}$	$x(0) = B_1$ $dx/dt(0) = -\alpha B_1 + \omega_d B_2$ where $\omega_d = \sqrt{\omega_o^2 - \alpha^2}$
Critically damped	$x(t) = (D_1 t + D_2) e^{-\alpha t}$	$x(0) = D_2$ $dx/dt(0) = D_1 - \alpha D_2$

2.1.1 Natural response of a parallel RLC circuit

The determination of the natural response of a parallel RLC circuit (Fig. 7-1) consists of finding the *voltage* v generated across the parallel branches by the release of energy stored in the inductor or capacitor or both. The initial conditions of voltage across the capacitor and current through the inductor are represented by V_o and I_o respectively.

A test circuit is shown in Fig. 7-2. Here, V_s provides a square signal with a DC offset voltage such that the bottom part of the waveform is aligned with the zero volts level. R_s is the internal resistance of the voltage source. This circuit is analogous to that in Fig. 7-1 when the square wave output by the function generator goes to zero providing that $R = R_c + R_s$. The natural response of the circuit can be generated in a periodic fashion providing that the period T of the square wave is long enough to allow the charge and discharge of the respective elements.

2.1.2 Natural response of a series RLC circuit

The determination of the natural response of the series RLC circuit shown in Fig. 7-3 consists of finding the *current* i generated through the series elements by the release of energy stored in the inductor or capacitor or both. A test circuit is shown in Fig. 7-4. This circuit is analogous to that in Fig. 7-3 during the time when the square wave output by the function generator goes to zero providing that $R = R_c + R_s$. The period T of the square signal must last long enough to allow the charge and discharge of the respective elements.

2.2 Step response

Determining the step response of a second-order circuit consists in finding the type of damping and applying and solving the appropriate set of equations. Here, X_f is the final value of $x(t)$, and $x(0)$ and $dx/dt(0)$ are initial values (at $t = 0$).

<u>Damping</u>	<u>Step response equations</u>	<u>Coefficient equations</u>
Overdamped	$x(t) = X_f + A_1 e^{s_1 t} + A_2 e^{s_2 t}$	$x(0) = X_f + A_1 + A_2$ $dx/dt(0) = A_1 s_1 + A_2 s_2$
Underdamped	$x(t) = (B_1 \cos \omega_d t + B_2 \sin \omega_d t) e^{-\alpha t}$	$x(0) = X_f + B_1$ $dx/dt(0) = -\alpha B_1 + \omega_d B_2$
Critically damped	$x(t) = X_f + (D_1 t + D_2) e^{-\alpha t}$	$x(0) = X_f + D_2$ $dx/dt(0) = D_1 - \alpha D_2$

2.2.1 Step response of a parallel RLC circuit

The determination of the step response of a parallel RLC circuit (Fig. 7-5) consists of finding the *current* i_L that appears through the inductor as a consequence of the sudden application of a DC current source. This current is of particular interest because it does not approach zero as t increases. The initial conditions of voltage across the capacitor and current through the inductor may or may not be zero.

A test circuit is shown in Fig. 7-2. This circuit is analogous to that in Fig. 7-5 when the square wave output by the function generator takes the value of V_s volts with $V_s = I_s R$ and $R = R_c + R_s$. Again, the step response of the circuit can be observed periodically providing that the period T of the square wave is long enough to allow the charge and discharge of the respective elements.

2.2.2 Step response of a series RLC circuit

The determination of the step response of the series RLC circuit shown in Fig. 7-7 consists of finding the *voltage* v_C across the capacitor generated by the sudden application of a DC voltage source. This voltage is of particular interest because it does not approach zero as t increases. A test circuit is shown in Fig. 7-4. This circuit is analogous to that in Fig. 7-7 during the time when the square wave output by the function generator takes the value of V_s volts and providing that $R = R_c + R_s$.

3 Preparation

In preparation for the lab do the following activities:

1. Natural response of a parallel RLC circuit. Consider the circuit shown in Fig. 7-2 and the component values of Table 7-1. Assume that the signal provided by the function generator switches from 10 to 0 volts at $t = 0$. Determine α , ω_o , the type of damping of this circuit and find the expression for the voltage $v(t)$. Generate a computer graph of $v(t)$ versus time in the interval from 0 to 2 ms.
2. Natural response of a series RLC circuit. Consider the circuit shown in Fig. 7-4 and the component values of Table 7-1. Assume that the signal provided by the function generator switches from 10 to 0 volts at $t = 0$. Determine α , ω_o , the type of damping of this circuit and find the expression for the current $i(t)$. Generate a computer graph of $i(t)$ versus time in the interval from 0 to 1 ms.
3. Step response of a parallel RLC circuit. Consider the circuit shown in Fig. 7-2 and the component values of Table 7-2. Assume that the signal provided by the function generator switches from 0 to 10 volts at $t = 0$. Determine α , ω_o , the type of damping of this circuit and find the expression for the current $i_L(t)$ through the inductor. Generate a computer graph of $i_L(t)$ versus time in the interval from 0 to 100 μs .
4. Step response of a series RLC circuit. Consider the circuit shown in Fig. 7-4 and the component values of Table 7-2. Assume that the signal provided by the function generator switches from 0 to 10 volts at $t = 0$. Determine α , ω_o , the type of damping of this circuit and find the expression for the voltage $v_C(t)$ across the capacitor. Generate a computer graph of $v_C(t)$ versus time in the interval from 0 to 200 μs . Determine the value of R'_c to replace R_c , such that the circuit is critically damped. Find the expression for $v'_C(t)$ and generate a computer graph of $v'_C(t)$ versus time in the interval from 0 to 100 μs .

4 Procedure

This part of the experiment requires assembling the circuits discussed in the previous section and measuring data from all of them.

Equipment settings

- Function generator. *Type of waveform:* Square wave. *Amplitude*=5 V. *Offset*=2.5 V. *Frequency*=250 Hz . Notice that the actual amplitude of the signal is 10 V.
- Oscilloscope. *External trigger:* from the *SYNC out* terminal of the function generator. *Channel 2:* disconnected (disconnect the probe from channel 2). Attenuation factor of channel 1 10X.
- Computer. Enter the computer network password at start-up. Load *channell.vi* (a shortcut is located in the computer desktop). A floppy disk is required to collect data.

The procedure necessary to determine the circuit responses experimentally is described next. Measure all the components and report their actual values in the corresponding entries of Tables 7-1 and 7-2.

1. Natural response of a parallel RLC circuit.

- Using the component values of Table 7-1, assemble the circuit shown in Fig. 7-2.
- Connect the oscilloscope probe of channel 1 across the capacitor. Press **Autoscale** . Press **Main/Delayed** and select **Lft** under **Time Ref** from the soft menu in the oscilloscope screen. Press **Slope/Coupling** and select the second slope in the soft menu (going from high to low). Adjust the **Time/Div** knob to 100 μs /. At this point the natural response of the parallel RLC circuit should be displayed in a form that is comparable with the plot created in the Preparation section.
- In the front panel of the virtual instrument *channell.vi*, enter a suitable data filename, i.e., A:\RL.txt. Use always drive A to store the datafiles. Move the switch to ON to allow the recording of the data. Run the virtual instrument by pressing **Ctrl-R** in the computer keyboard. Print a hard copy of the computer screen by pressing **Ctrl-P**.

2. Natural response of a series RLC circuit.

Repeat the above procedure to obtain the natural response of the series RLC circuit with the following modifications. Use the circuit of Fig. 7-4 and the component values of Table 7-1. To avoid ground conflicts with the equipment, connect the components in an order such that the resistor is connected to the negative terminal of the function generator (it is a series circuit, so the order of connection is irrelevant). Connect the oscilloscope probe across the resistor to observe a *scaled* version of $i(t)$. Use a different filename to store the data.

3. Step response of a parallel RLC circuit.

Repeat the procedure of (1) to obtain the step response of a parallel RLC circuit with the following modifications. Use the circuit of Fig. 7-6 and the component values of Table 7-2. Notice that there is a small resistor connected in series with the inductor. This is because the oscilloscope can measure voltage only, therefore, it is necessary to measure a scaled version of $i_L(t)$. Connect the oscilloscope probe across the 5.1 Ω resistor. Select the first slope (going from low to high). Set the **Time/Div** knob to 10 μs . Use a different filename to store the data.

4. Step response of a series RLC circuit.

(a) Repeat the procedure of (1) to obtain the step response of a series RLC circuit with the following modifications. Use the circuit of Fig. 7-4 and the component values of Table 7-2. To avoid ground conflicts with the equipment, connect the components in an order such that the capacitor is connected to the negative terminal of the function generator. Connect the oscilloscope probe across the capacitor. Select the first slope (going from low to high). Set the **Time/Div** knob to 20 μs . Use a different filename to store the data.

(b) Repeat part (a) replacing the value of R_c by R'_c , the value that produces a critically damped response found in the Preparation section.

5 Analysis

This section is intended for the analysis and comparison of the experimental and theoretical results. Answer all the questions.

1. Complete the entries corresponding to the error percentage in Tables 7-1 and 7-2.
2. Compare the theoretical and experimental graphs of all the responses. Explain any differences between them.
3. Use PSpice to simulate the responses of all the circuits in the Procedure section. Use the actual component values that were measured during the lab and compare the simulated responses with the experimental graphs.

Parameter	Unit	Theor	Exper	%Error
R_c	Ω	1000		
L	mH	1.0		
C	μF	0.1		

Table 7-1: RLC circuit values to determine the natural response.

Parameter	Unit	Theor	Exper	%Error
R_c	Ω	27		
L	mH	1.0		
C	μF	0.01		

Table 7-2: RLC circuit values to determine the step response.

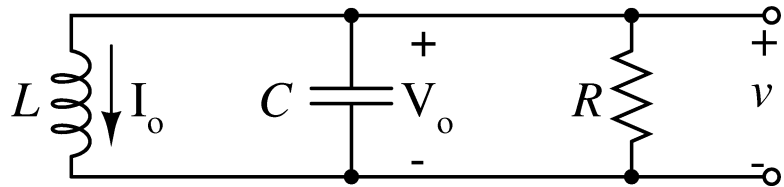


Figure 7-1: Parallel RLC circuit to illustrate the natural response.

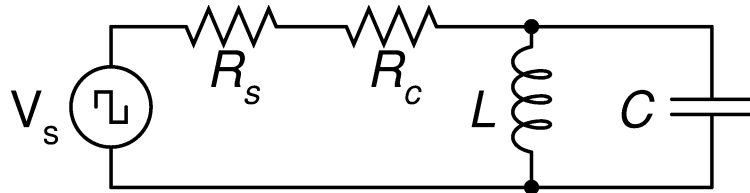


Figure 7-2: Parallel RLC test circuit.

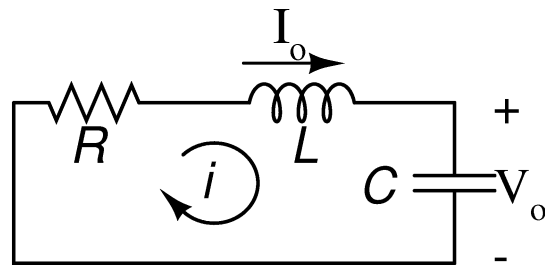


Figure 7-3: Series RLC circuit to illustrate the natural response.

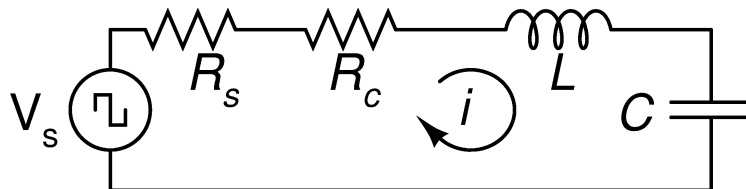


Figure 7-4: Series RLC test circuit.

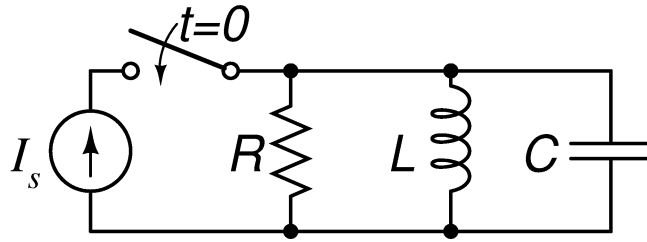


Figure 7-5: Parallel RLC circuit to illustrate the step response.

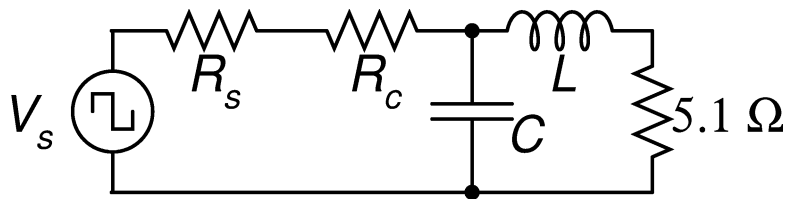


Figure 7-6: Parallel RLC test circuit to observe the step response.

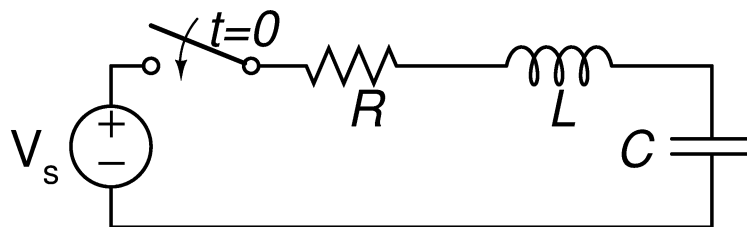


Figure 7-7: Series RLC circuit to illustrate the step response.