[S] - gives a direct measure of μwave ckts & relates traveling waves to each other $[V^-] = [S][V^+]$

There are other matrices that are useful and we'll show how they are defined

[Z] matrix relates V and I (total V & I)

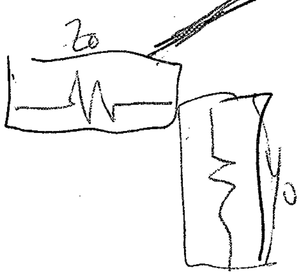
total voltage & current

$[Y]$ - admittance matrix $= \frac{1}{[Z]} = [Z]^{-1}$ inverse of the impedance matrix

[ABCD] matrix - relates 2 ports; total currents & voltages can be cascaded to give a total "system" matrix

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$ represents Z_0

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$ represents Y_0



can be cascaded

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Z matrix

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Y matrix

$[Z] \leftrightarrow [Y] \leftrightarrow [S] \leftrightarrow [ABCD]$; table 4.2 in text p. 187

If given $[S]$ – find $[Z]$ $\Rightarrow$ math intensive

Pozar example 3dB attenuator → using a schematic

We found S_{11}, S_{12}, S_{21}, S_{22}

S_{11} = reflection coefficient Γ if we are matched to the characteristic impedance (at port 1)

S_{21} = transmission coefficient at port 2, due to port 1

$S_{11} = RL = 20\log|\Gamma|$ or return loss

$S_{21} = IL = 20\log|T|$ or insertion loss

We talked about properties.

(a) Linear ckt if V & I are linearly related.

Ex R, C, L are linear
diode – non linear

(b) Symmetric – The same characteristics at each port

Ex. TML

(c) Reciprocal – response at Port 2 from Port 1 is the same as response from Port 1 due to Port 2

Ex: passives are reccprocal
transistors are not [unidirectional S_{21} gain ~ 3dB
$S_{12} \neq$ 3dB

Reciprocity - independent of direction of propagation

$$S_{ij} = S_{ji} \qquad Y_{ij} = Y_{ji}$$

Symmetry - making 2 or more coefficients identical
Ex. $S_{11} = S_{22}$ if The network is a mirror image when looking between ports.

Losses - 3dB attenuator; if Resistor is used ⇒ Lossy network

lossless network - impedance & admittance matrix coefficients are imaginary

$$\text{Re}\{Z_{ij}\} = 0 \qquad \text{Re}\{Y_{ij}\} = 0$$

For S-parameters - satisfy unitary matrix condition

A lossless reciprocal network has a [S] That is unitary

$$[S]^t [S]^* = [I] \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & & 1\,0 \end{bmatrix}$$

t - transpose
* - complex conjugate

Lossless $\sum_{n=1}^{N} |S_{nj}|^2 = 1 \quad \forall j$ and $\sum_{n=1}^{N} S_{in} S_{jn}^* = 0, \quad i \neq j$

$N=3 \quad i=3, \; j=2$

$S_{31}S_{21}^* + S_{32}S_{22}^* + S_{33}S_{23}^* = 0$

Example

$N=3$

$j=2$

3 ports

$$\begin{bmatrix} S_{11} & \boxed{S_{12}} & S_{13} \\ S_{21} & \boxed{S_{22}} & S_{23} \\ S_{31} & \boxed{S_{32}} & S_{33} \end{bmatrix}$$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{32}|^2 = 1$$

Lossy

$$\sum_{n=1}^{N} |S_{nj}|^2 \neq 1, \; < 1 \quad \forall j$$

2 port $\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$ $\quad |S_{11}|^2 + |S_{21}|^2 = 1$

percentage of input power dissipated into heat

$$\frac{P_{diss}}{P_{in,j}} = 1 - \sum_{n=1}^{N} |S_{nj}|^2 \quad \forall j$$

$P_{in,j}$ is incident power in port j

P_{diss} is total power dissipated in the network

Input Match

Load impedance is equal to Z_0

$S_{nn} = 0$ match

Given $[S] = \begin{bmatrix} \overset{S_{11}}{0} & \overset{S_{12}}{0.9} & \overset{S_{13}}{0} \\ \overset{S_{21}}{0} & \overset{S_{22}}{0} & \overset{S_{23}}{0.9} \\ \overset{S_{31}}{0.9} & \overset{S_{32}}{0} & \overset{S_{33}}{0} \end{bmatrix}$

$S_{11} = S_{22} = S_{33} = 0 \Rightarrow$ match
All ports are matched.

$S_{12} = 0.9 \quad S_{21} = 0$

$S_{13} = 0 \quad S_{31} = 0.9 \quad S_{ij} \neq S_{ji}$

$S_{32} = 0 \quad S_{23} = 0.9$

$\Rightarrow$ Not a reciprocal network

$$\sum_{n=1}^{N=3} |S_{nj}|^2 = 1 \quad \forall j$$

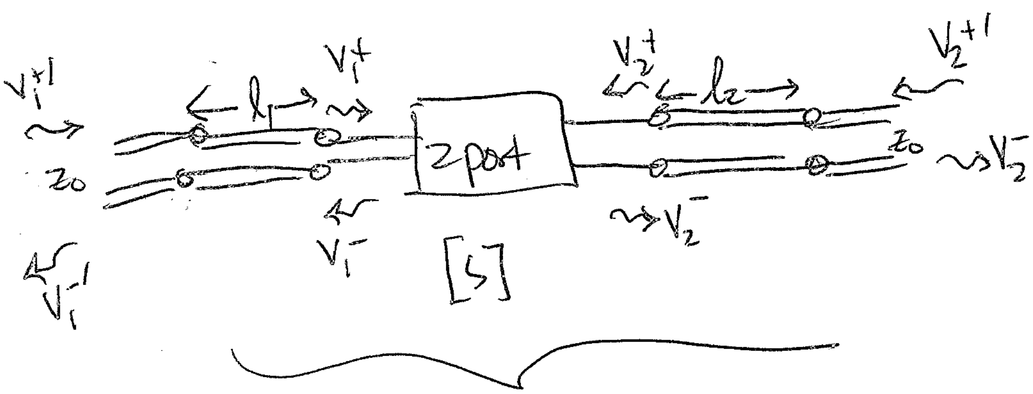
$|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 = 0 + 0 + 0.81 < 1 \Rightarrow$ Lossy

$|S_{12}|^2 + |S_{22}|^2 + |S_{32}|^2 = 0.81$

yes No NO

Unique 3 ports - cannot be <u>matched</u> if it's <u>lossless</u> & <u>reciprocal</u>

<u>Shift in reference planes</u>

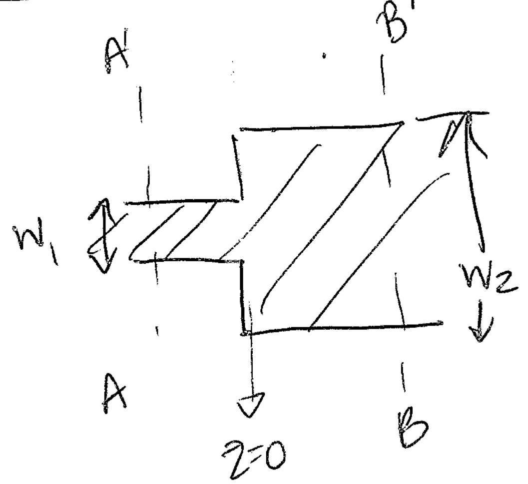


$[S']$

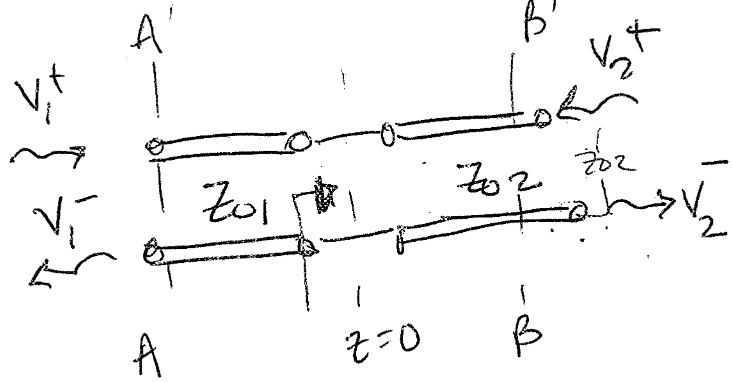
If we operate a single mode TEM type line, reference shifts by phase only $[S'] = \begin{bmatrix} e^{-j\theta_1} & 0 \\ 0 & e^{-j\theta_2} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & 0 \\ 0 & e^{-j\theta_2} \end{bmatrix}$

$S'_{nn} = e^{-j2\theta_n} S_{nn} \qquad \theta_n = \beta_n l_n$

Example



terminal plane



$$Z_{in} = Z_{02}$$

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_{2+}=0,\text{ or matched}} \qquad \Gamma = \frac{Z_{02}-Z_{01}}{Z_{02}+Z_{01}} = S_{11}$$

$W_2 = 2W_1$

$f = 2\,GHz$

$W_1 = 1.27\,mm$

$\epsilon_r = 10.8$

$h = 0.635\,mm$

Find [S]

$$S_{22} = \frac{V_2^-}{V_2^+} = \frac{Z_{01}-Z_{02}}{Z_{01}+Z_{02}}$$

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_{2+}=0}$$

$$V_2 = \cancel{V_2^+} + V_2^- = V_2^-$$

$$V_1^- = V_1^+ S_{11}$$

$$V_1 = V_1^+ + V_1^- = V_1^+(1+S_{11})$$

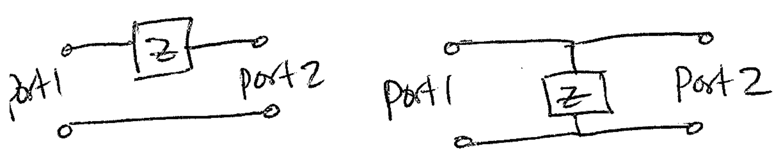
$$\frac{V_2^-}{V_1^+} = \frac{V_2}{V_1^+} = \frac{V_1}{V_1^+} = \frac{\cancel{V_1^+}(1+S_{11})}{\cancel{V_1^+}} = 1+S_{11}$$

$$1+S_{11} = 1+\frac{Z_{02}-Z_{01}}{Z_{02}+Z_{01}}$$

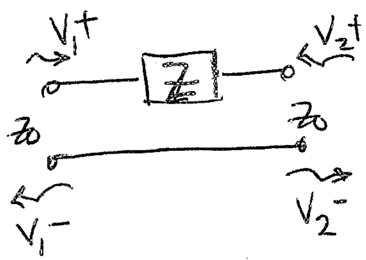
$$= \frac{2Z_{02}}{Z_{02}+Z_{01}} = \left.\frac{V_2^-}{V_1^+}\right|_{V_{2+}=0}$$

$$[S] = \begin{bmatrix} \dfrac{Z_{02}-Z_{01}}{Z_{02}+Z_{01}} & \dfrac{2Z_{01}}{Z_{02}+Z_{01}} \\ \dfrac{2Z_{02}}{Z_{02}+Z_{01}} & \dfrac{Z_{01}-Z_{02}}{Z_{01}+Z_{02}} \end{bmatrix}$$

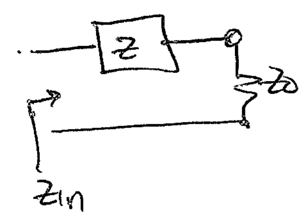
<u>Example</u> Find [S] for series & shunt loads. Assume Z_o characteristic impedance.



<u>Soln</u>



$S_{11} = \dfrac{V_1^-}{V_1^+}\Big|_{V_{2+}=0} \Rightarrow$



$Z_{in} = Z + Z_o$

$S_{ii} = \Gamma$ seen looking into port i when all other ports are terminated in matched loads

$S_{ij} = T$ from port j to i when all other ports are terminated in matched loads

$$\therefore S_{11} = \frac{V_1^-}{V_1^+}\Big|_{V_2^+=0} = \Gamma^{(1)}\Big|_{V_2^+=0} = \frac{Z_{in}^{(1)} - Z_o}{Z_{in}^{(1)} + Z_o}\Bigg|_{Z_o \text{ on port 2}} = \frac{Z+Z_o-Z_o}{Z+Z_o+Z_o} = \frac{Z}{Z+2Z_o}$$

Because of symmetry $S_{22} = S_{11} = \dfrac{Z}{Z+2Z_o}$

$S_{21} = \dfrac{V_2^-}{V_1^+}\Big|_{V_2^+=0}$ Applying an incident wave at port 1, V_1^+ and measuring the outcome at port 2, V_2^-. We can use voltage division.

$$V_1 = V_1^+(1+\Gamma) = V_1^+\left(1 + \frac{Z}{Z+2Z_o}\right); \quad V_2 = \cancel{V_2^+}^{\,0} + V_2^- \Rightarrow V_2 = V_2^-$$

$$V_2 = \frac{Z_0}{Z_0 + Z} V_1$$

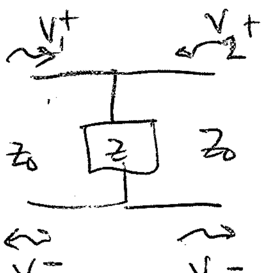
$$V_2 = \cancel{V_2^+} + V_2^- = V_2^-$$

$$V_1 = V_1^+ + V_1^- = V_1^+(1+\Gamma) = V_1^+\left(1 + \frac{Z}{Z+2Z_0}\right) = V_1^+\left(\frac{Z+2Z_0+Z}{Z+2Z_0}\right) = \frac{V_1^+(2Z+2Z_0)}{Z+2Z_0}$$

$$\frac{V_2^-}{V_1^+} = \frac{Z_0}{\cancel{Z_0+Z}} \cdot \frac{\cancel{(2Z+2Z_0)}}{Z+2Z_0} = \frac{2Z_0}{Z+2Z_0}$$

$$\therefore \frac{V_2^-}{V_1^+} = S_{21} = \frac{2Z_0}{Z+2Z_0} \text{ ; by symmetry } S_{12} = S_{21}$$

$$[S] = \begin{bmatrix} \frac{Z}{Z+2Z_0} & \frac{2Z_0}{Z+2Z_0} \\ \frac{2Z_0}{Z+2Z_0} & \frac{Z}{Z+2Z_0} \end{bmatrix}$$



$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0}$$

$$\Gamma = S_{11} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \quad \text{where } Z_{in} = Z_0 // Z = \frac{Z \cdot Z_0}{Z + Z_0}$$

$$\Gamma = \frac{\frac{Z \cdot Z_0}{Z_0 + Z} - Z_0}{\frac{Z_0 Z}{Z_0 + Z} + Z_0} = \frac{\cancel{ZZ_0} - Z_0^2 - \cancel{Z_0 Z}}{ZZ_0 + Z_0^2 + Z_0 Z} = \frac{-Z_0^{\cancel{2}}}{\cancel{Z_0}(2Z + Z_0)} = \frac{-Z_0}{2Z+Z_0} \text{ ; by symmetry } S_{11} = S_{22}$$

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0} \qquad V_1 = V_1^+(1+\Gamma) = V_1^+\left(1 - \frac{Z_0}{2Z+Z_0}\right)$$

$V_2 = \cancel{V_2^+} + V_2^-$; The Voltage at port 1 is the same as Voltage at port 2

$$[S] = \begin{bmatrix} \frac{-Z_0}{2Z+Z_0} & \frac{2Z}{2Z+Z_0} \\ \frac{2Z}{2Z+Z_0} & \frac{-Z_0}{2Z+Z_0} \end{bmatrix}$$

$$\therefore V_2^- = V_1^+\left(1 - \frac{Z_0}{2Z+Z_0}\right) \Rightarrow \frac{V_2^-}{V_1^+} = \frac{2Z + \cancel{Z_0} - \cancel{Z_0}}{2Z+Z_0} = \frac{2Z}{2Z+Z_0}$$