Microwave and RF Design, A Systems Approach

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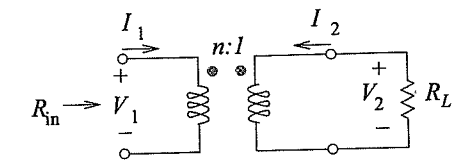


Figure 8-6 A transformer as a matching network.

8.2.4 Microstrip Resonator

Figure 8-4 shows a gap-coupled microstrip resonator comprising a half-wavelength section and coupling to the resonator controlled by the gap. The half-wavelength resonator will have a standing wave established at the resonant frequency. Energy is stored on the line segment and energy is lost to resistive dissipation and radiation. The Q of the half-wavelength section is Q_U. The gap provides a small capacitance that couples energy into the line segment so that the measured or loaded Q is Q_L. The smaller the gap capacitance, the closer Q_L is to Q_U as the loading becomes smaller.

8.3 Impedance Transforming Networks

8.3.1 The Ideal Transformer

The ideal transformer shown in Figure 8-6 can be used to match a load and a source if the source and load impedances are real. This will be shown by starting with the constitutive relations of the transformer:

$$V_1 = nV_2 \text{ and } I_1 = -\frac{1}{n}I_2. \tag{8.11}$$

Here n is the transformer ratio. For a wire-wound transformer, n is the ratio of the number of windings on the primary side, Port 1, to the number of windings on the secondary side, Port 2. So the input resistance, R_{in}, is related to the load resistance, R_L, by

$$R_{in} = \frac{V_1}{I_1} = -n^2\frac{V_2}{I_2} = n^2 R_L. \tag{8.12}$$

The matching problem with purely resistive load and source impedances is solved by choosing the appropriate winding ratio, n. Unfortunately resistive-only problems at RF are rare. However at a few hundred megahertz and below, transformer-based matching sometimes works quite well, as parasitics are relatively small so that resistive load and source impedance are more closely approximated. Also, the parasitics of a wire-wound transformer (such as interwinding capacitance) can then be small.

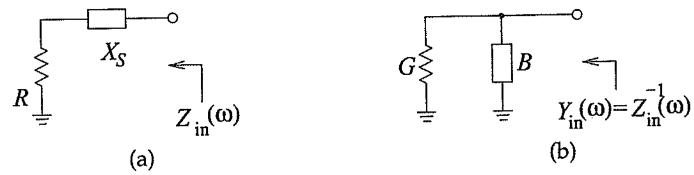


Figure 8-7 Matching using a series reactive element: (a) the series reactive element; and (b) the equivalent transformed shunt circuit.

8.3.2 A Series Reactive Element

Matching using lumped elements is based on the impedance and admittance transforming properties of series and shunt reactive elements. Even a single reactive element can achieve limited impedance matching. Consider the series reactive element shown in Figure 8-7. In Figure 8-7(a), the reactive element, with reactance X_S, is in series with a resistance R. The shunt equivalent of this network is shown in Figure 8-7(b) with a shunt susceptance of B. In this transformation the resistance R has been converted to a resistance $R_P = 1/G$. The mathematics describing this transformation is as follows. The input admittance of the series connection (Figure 8-6(a)) is

$$Y_{\text{in}}(\omega) = \frac{1}{Z_{\text{in}}(\omega)} = \frac{1}{R + jX_S} = \frac{R}{R^2 + X_S^2} - j\frac{X_S}{R^2 + X_S^2}. \tag{8.13}$$

Thus the elements of the equivalent shunt network are

$$G = \frac{R}{R^2 + X_S^2} \quad \text{and} \quad B = -\frac{X_S}{R^2 + X_S^2}. \tag{8.14}$$

The "resistance" of the network, R, has been transformed to a new value,

$$R_P = G^{-1} = \frac{R^2 + X_S^2}{R} > R. \tag{8.15}$$

This is an important start to matching, as X_S can be chosen to convert R (a load, for example) to any desired resistance value (such as the resistance of a source). However, there is still a residual reactance. Before moving on to the solution of this problem consider the following example.

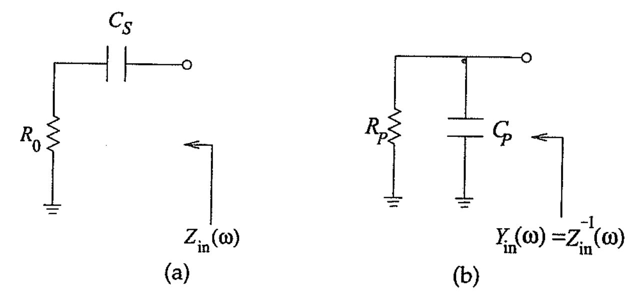


Figure 8-8 Impedance transformation using a series resistive element: (a) a resistor with a series capacitor; and (b) its equivalent shunt circuit.

EXAMPLE 8. 1 Capacitive Impedance Transforming

Consider the impedance transforming properties of a capacitive series element as shown in Figure 8-8. Show that the capacitor can be adjusted to obtain any positive shunt resistance value.

Solution:

The concept here is that the series resistor and capacitor network has an equivalent shunt circuit that includes a capacitor and a resistor, but now the resistor value can be tuned by the series capacitor. The equivalent resistance of the parallel equivalent network is, from Equation (8.15),

$$R_P = G^{-1} = \frac{R_0^2 + (1/\omega^2 C_S^2)}{R_0} = \frac{1 + \omega^2 C_S^2 R_0^2}{\omega^2 C_S^2 R_0} \tag{8.16}$$

and the susceptance is

$$B = \frac{(1/\omega C_S)}{R_0^2 + 1/\omega^2 C_S^2} = \omega \frac{C_S}{1 + \omega^2 C_S^2 R_0^2}. \tag{8.17}$$

Matching, by our definition, is achieved when R_P is equal to the resistance of the source, that is, matching is achieved when $R_P = R_0$. Thus to achieve matching

$$C_P = \frac{B}{\omega} = \frac{C_S}{1 + \omega^2 C_S^2 R_0^2}. \tag{8.18}$$

To match R_0 to a resistive load R_P ($> R_0$) at a given frequency ω_d, then, from Equation (8.16),

$$\omega_d C_S = \frac{1}{\sqrt{R_0 R_P - R_0^2}}, \tag{8.19}$$

and this is the design equation for C_S. To complete the design, place a shunt inductor, as shown in Figure 8-9, where

$$\omega_d C_P = \frac{1}{\omega_d L}, \tag{8.20}$$

the design equation for L. The equivalent impedance in Figure 8-9 is a resistor of value R_P, and the value of this resistor can be adjusted by adjusting C_S.

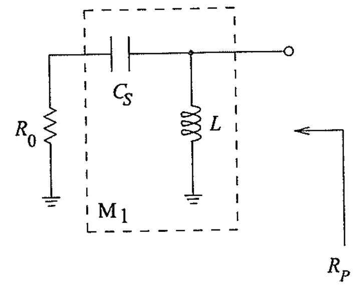


Figure 8-9 An *LC* matching network.

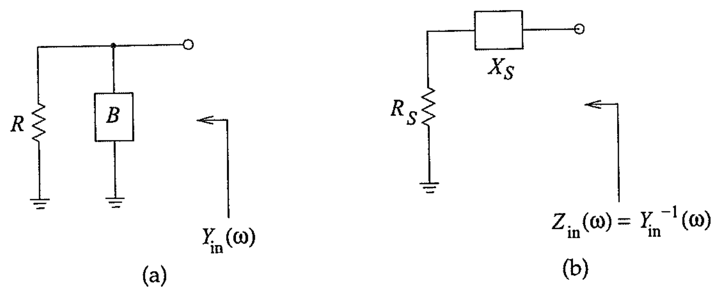


Figure 8-10 A resistor with (a) a shunt parallel reactive element, and (b) its equivalent series circuit.

8.3.3 A Parallel Reactive Element

The dual of the series capacitor matching procedure described in the previous example is obtained with a parallel reactive element, as shown in Figure 8-10(a). In general there are two two-element lumped-element circuits that will enable a particular load resistor to be matched to a source resistor. The input admittance of the shunt circuit is

$$Y_{\text{in}} = \frac{1}{R} + \jmath B. \tag{8.21}$$

This can be converted to a series circuit representation by calculating the impedance, $Z_{\text{in}} = 1/Y_{\text{in}}$:

$$Z_{\text{in}} = \frac{R}{1 + \jmath BR} = \frac{R}{1 + B^2R^2} - \jmath\frac{BR^2}{1 + B^2R^2}. \tag{8.22}$$

So

$$R_S = \frac{R}{1 + B^2R^2} \text{ and } X_S = \frac{-BR^2}{1 + B^2R^2}. \tag{8.23}$$

Notice that $R_S < R$.

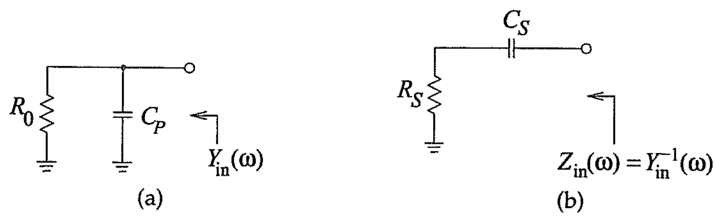


Figure 8-11 A parallel-to-series transformation: (a) a resistor with added shunt resistor; and (b) equivalent series circuit.

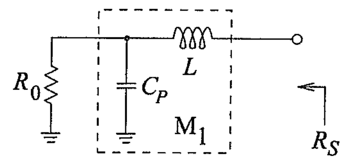


Figure 8-12 Transforming circuit with a series inductor added to the circuit of Figure 8-11(b).

EXAMPLE 8. 2 Parallel Tuning

As an example of the use of a parallel reactive element to tune a resistance value, consider the circuit in Figure 8-11(a) where a capacitor tunes the effective resistance value so that the series equivalent circuit (Figure 8-11(b)) has elements

$$R_S = \frac{R_0}{1 + \omega^2 C_p^2 R_0^2} \tag{8.24}$$

$$X_S = -\frac{\omega C_p R_0^2}{1 + \omega^2 C_p^2 R_0^2} = -\frac{1}{\omega C_S} \tag{8.25}$$

and

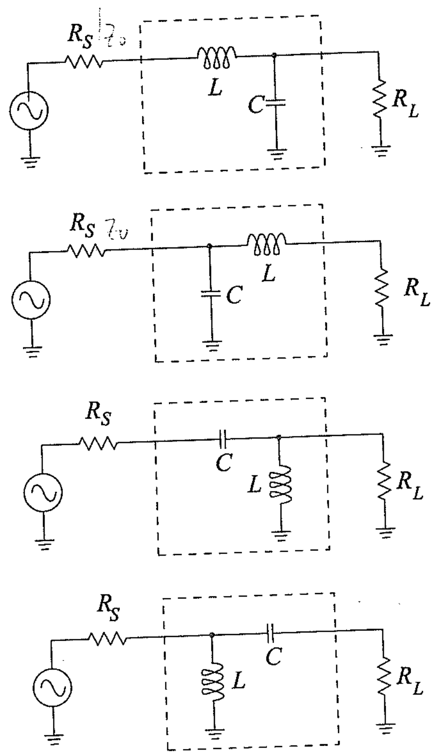
$$C_S = \frac{1 + \omega^2 C_p^2 R_0^2}{\omega^2 C_p R_0^2}. \tag{8.26}$$

Now consider matching R_0 to a resistive load R_S, which is less than R_0 at a given frequency ω_d. To complete the design, place a series inductor to remove the reactive effect of the capacitor, as shown in Figure 8-12. The value of the inductor required is

$$\omega_d L = \frac{1}{\omega_d C_S}. \tag{8.27}$$

This leads to the design equation for L:

$$L = \frac{1}{\omega_d^2 C_S}. \tag{8.28}$$



(a) Lowpass $R_S < R_L$
$Q = \sqrt{R_L/R_S - 1}$
$Q = |X_L|/R_S = R_L/|X_C|$

(b) Lowpass $R_S > R_L$
$Q = \sqrt{R_S/R_L - 1}$
$Q = |X_L|/R_L = R_S/|X_C|$

(c) Highpass $R_S < R_L$
$Q = \sqrt{R_L/R_S - 1}$
$Q = |X_C|/R_S = R_L/|X_L|$

(d) Highpass $R_S > R_L$
$Q = \sqrt{R_S/R_L - 1}$
$Q = |X_C|/R_L = R_S/|X_L|$

Figure 8-13 L matching networks consisting of one shunt reactive element and one series reactive element.

8.4 The L Matching Network

The examples in the previous two sections suggest the basic concept behind lossless matching of two different resistance levels using an L network:

Step 1: Use a series (shunt) reactive element to transform a smaller (larger) resistance up (down) to a larger (smaller) value with a real part equal to the desired resistance value.

Step 2: Use a shunt (series) reactive element to resonate with (or cancel) the imaginary part of the impedance that results from Step 1.

So a resistance can be transformed to any resistive value by using an LC transforming circuit.

Formalizing the matching approach described above, we note that there are four possible two-element L matching networks (see Figure 8-13). The two possible cases, $R_S < R_L$ and $R_L < R_S$, will be considered in the following subsections.