Differential Space-Time Coding for Frequency-Selective Channels

S. N. Diggavi N. Al-Dhahir A. Stamoulis A. R. Calderbank
AT&T Shannon Laboratory
180 Park Avenue
Florham Park NJ 07932
{suhas,naofal,as,rc}@research.att.com

Abstract

In this paper we introduce two space-time transmission schemes which allow full-rate and full-diversity non-coherent communications using two transmit antennas over fading frequency–selective channels. The first scheme operates in the frequency domain where it combines differential Alamouti space–time block–coding with OFDM. The second scheme operates in the time domain and employs differential time–reversal space–time block–coding to guarantee blind channel identifiability without the need for temporal oversampling or multiple receive antennas.

1 Introduction

Since their invention, space–time block codes (STBC) [10] have sparked wide interest as they promise to significantly increase transmission rates in wireless communications. With N transmit and M receive antennas, over frequency-selective channels, the total number of channel parameters that needs to be estimated is $NM(\nu+1)$, where ν is an upper bound on the order of the underlying finite–impulse–response (FIR) channels. The overhead of frequent re-training coupled with the increased cost of channel estimation and the degradation of tracking quality in fast time-varying environments make non-coherent schemes attractive alternatives.

Previous related work includes differential STBC schemes with two [9] or more [7] transmit antennas, group differential coding schemes [2, 6] (and references therein) for flat–fading channels and coherent STBC schemes for frequency–selective channels (see e.g. [8]). To the best of our knowledge, there has been no previous work on differential STBC schemes for frequency–selective channels with an arbitrary number of transmit antennas, which is the subject of this paper.

Blind identification techniques also enable the elimination of training symbols in transmission. The identification of FIR channels based only on second–order statistics without multiple receive antennas (or oversampling) is not possible since the phase information is lost [11]. In this paper, we present a differential space-time encoding scheme which makes blind channel identification possible with second–order statistics using only symbol–spaced samples and a single receive antenna. In summary, the main contributions of this paper are

- A differential space–time OFDM–based transmit–diversity scheme over frequency–selective channels with an arbitrary number of transmit antennas.

- A time–domain differential space–time transmit–diversity scheme that guarantees blind channel identifiability from second–order statistics without requiring temporal oversampling or multiple receive antennas.

2 Review of STBC-based Differential codes

With two transmit antennas, Alamouti's STBC [1] groups the input symbols into pairs (x_1, x_2) which are fed to the space-time block encoder

$$\left(\begin{array}{c} x_1 \\ x_2 \end{array}\right) \longrightarrow \left(\begin{array}{cc} x_1 & x_2 \\ -\bar{x}_2 & \bar{x}_1 \end{array}\right) \begin{array}{cl} \rightarrow & \text{time} \\ \downarrow & \text{space} \end{array} , \tag{1}$$

where $(\bar{.})$ denotes the complex–conjugate transpose operation. Assuming a single receive antenna, the received signals (over two consecutive time slots) can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \, , \; \mathbf{H} = \left(\begin{array}{cc} h_1 & h_2 \\ -\bar{h}_2 & \bar{h}_1 \end{array}\right) , \tag{2}$$

where $\mathbf{y} = [y_1, -\bar{y}_2]^T$, $\mathbf{x} = [x_1, -\bar{x}_2]^T$, $\mathbf{n} = [n_1, n_2]^T$, n_1 and n_2 are independent AWGN processes each with variance $N_o/2$ per dimension. Under the assumption of quasi-static channels (constant over the transmission of two symbols) $\mathbf{H}$ is orthogonal[1], *i.e.,* $\bar{\mathbf{H}}\mathbf{H} = ||\mathbf{h}||^2 \mathbf{I}_2$, where $||\mathbf{h}||^2 = (|h_1|^2 + |h_2|^2)$ and h_1, h_2 are the channel coefficients from the first and second transmit antennas, respectively, to the receive antenna. We cast the differential STBC scheme of [9] in a matrix form to facilitate the development of our novel differential STBC schemes for frequency–selective channels in the next section. Manipulating (2), we obtain

$$\left(\begin{array}{cc} y_1 & y_2 \\ -\bar{y}_2 & \bar{y}_1 \end{array}\right) = \left(\begin{array}{cc} h_1 & h_2 \\ -\bar{h}_2 & \bar{h}_1 \end{array}\right) \left(\begin{array}{cc} x_1 & x_2 \\ -\bar{x}_2 & \bar{x}_1 \end{array}\right) + \text{noise} \, . \tag{3}$$

For block k, denote the source symbols as $\mathbf{u}(k) = [u_1(k), u_2(k)]^T$, the transmitted matrix as $\mathbf{X}(k)$, and the received matrix as $\mathbf{Y}(k)$. Then, in the absence of noise, (3) is written as $\mathbf{Y}(k) = \mathbf{H}\mathbf{X}(k)$. Assuming that the channel is quasi-static over two consecutive blocks, it follows that

$$\bar{\mathbf{Y}}(k-1)\,\mathbf{Y}(k) = ||\mathbf{h}||^2\, \bar{\mathbf{X}}(k-1)\,\mathbf{X}(k) \, . \tag{4}$$

Since we would like to estimate the source symbols contained in $\mathbf{U}(k) \overset{def}{=} \left(\begin{array}{cc} u_1(k) & u_2(k) \\ -\bar{u}_2(k) & \bar{u}_1(k) \end{array}\right)$, we define the differential transmission rule

$$\mathbf{X}(k) = \bar{\mathbf{X}}^{-1}(k-1)\,\mathbf{U}(k) \, , \tag{5}$$

which forms the basis for the differential STBC scheme for flat-fading channels [9]. Next we discuss how this differential transmission rule can be extended to frequency–selective channels.

3 Differential STBC for Frequency-Selective Channels

We consider STBC schemes with two transmit and one receive antennas. These techniques trivially extend to multiple receive antennas. We have also extended these ideas to more than two antennas at a rate loss, see [4] for more details. The frequency–selective channels from the transmit antennas to the receive antenna are modeled as FIR filters with memory ν, denoted by length–$(\nu+1)$ vectors $\{h_1^{(k)}(n)\}$ and $\{h_2^{(k)}(n)\}$, respectively. An alternative useful representation is in terms of their D-transforms defined by $H_i^{(k)}(D) \overset{def}{=} \sum_{n=0}^{\nu} h_i^{(k)}(n) D^n$.

Next we present two schemes that extend the differential transmission scheme in (5) to frequency-selective channels. In both schemes, each transmission block is divided into two sub-blocks, each of length P, separated by ν guard symbols to avoid inter-sub-block interference (see Figure 1).

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[1] In this paper for a matrix (or vector) $\mathbf{A}$ we denote by $\mathbf{A}^T$ its transpose and by $\bar{\mathbf{A}}$ its Hermitian transpose.

3.1 Frequency-Domain Scheme

For channels which are constant over a transmission block, orthogonal frequency division multiplexing (OFDM) provides a set of parallel flat-fading channels. Consequently, the differential transmission techniques of the previous section can be readily applied in the frequency domain. We consider two symbols $X_1(m)$ and $X_2(m)$ drawn from a PSK constellation which, following the Alamouti encoding scheme, are mapped as

$$\mathbf{X}^{(1)}(m) = [X_1(m), X_2(m)]^T, \; \mathbf{X}^{(2)}(m) = [-\bar{X}_2(m), \bar{X}_1(m)]^T, \tag{6}$$

where $\mathbf{X}^{(1)}$ represents the information-bearing vector for the first transmit antenna and $\mathbf{X}^{(2)}$ corresponds to the second transmit antenna[2]. Let P denote the fast Fourier transform (FFT) size, then $\mathbf{X}^{(1)}$ and $\mathbf{X}^{(2)}$ are length–$2P$ vectors holding the symbols to be transmitted by the two transmit antennas. More explicitly, for the first transmit antenna, the first symbol $X_1(m)$ is sent with the first OFDM sub-block on its mth subcarrier, and the second symbol $X_2(m)$ is sent with the second OFDM sub-block on its mth subcarrier. A similar construction of the OFDM sub-blocks is done on the second transmit antenna, with symbols $-\bar{X}_2(m), \bar{X}_1(m)$. Consequently, after taking the FFT at the receiver, Equation (3) becomes (at subcarrier m)

$$\begin{pmatrix} Y_1(m) & Y_2(m) \\ -\bar{Y}_2(m) & \bar{Y}_1(m) \end{pmatrix} = \begin{pmatrix} H_1(m) & H_2(m) \\ -\bar{H}_2(m) & \bar{H}_1(m) \end{pmatrix} \begin{pmatrix} X_1(m) & X_2(m) \\ -\bar{X}_2(m) & \bar{X}_1(m) \end{pmatrix} + \text{noise}\,, \tag{7}$$

where $H_1(m)$ and $H_2(m)$ are the frequency responses of channels $h_1(n)$, $h_2(n)$ at subcarrier m. It is clear from (7) that the differential space–time Alamouti scheme for flat–fading channels can be readily applied at each subcarrier using (5). A simulation example showing that this differential OFDM scheme achieves a BER performance which is 3dB away from coherent reception (with perfect channel knowledge) at high SNR is given in [4].

3.2 Time-Domain Scheme

Being a multi–carrier scheme, the differential OFDM–STBC scheme of the previous section suffers from a high peak–to–average ratio and is sensitive to frequency synchronization errors. In this section, we present a single–carrier differential STBC scheme that overcomes these drawbacks and where all processing is performed in the spatial and temporal domains. We transmit information by encoding over two transmission blocks over which the channel is assumed to be quasi-static. Therefore, in the D-transform notation, the received sequences for the first and second sub-blocks of the kth block are given by

$$\begin{aligned} Y_1^{(k)}(D) &= H_1^{(k)}(D)X_{1,1}^{(k)}(D) + H_2^{(k)}(D)X_{2,1}^{(k)}(D) + Z_1^{(k)}(D) \\ Y_2^{(k)}(D) &= H_1^{(k)}(D)X_{1,2}^{(k)}(D) + H_2^{(k)}(D)X_{2,2}^{(k)}(D) + Z_2^{(k)}(D), \end{aligned} \tag{8}$$

where $X_{i,l}(D)$ denote the transmitted sequences from the ith transmit antenna for the lth sub-block, $i = 1, 2$ and $l = 1, 2$. For two information sequences $\{\mathbf{x}_1^{(k)}(n)\}, \{\mathbf{x}_2^{(k)}(n)\}$, we transmit the sequences and the time-reversed conjugated versions $(\{\bar{\mathbf{x}}_1^{(k)}(-n)\}, \{\bar{\mathbf{x}}_2^{(k)}(-n)\})$ over the sub-blocks as shown in Figure 2 (similar to the technique used in [8]). For the kth transmission block, we can write the D–transform of the received sequence as

$$\mathbf{Y}^{(k)}(D) = \begin{bmatrix} Y_1^{(k)}(D) & Y_2^{(k)}(D) \end{bmatrix} = \begin{bmatrix} H_1^{(k)}(D) & H_2^{(k)}(D) \end{bmatrix} \begin{bmatrix} X_1^{(k)}(D) & X_2^{(k)}(D) \\ -\bar{X}_2^{(k)}(\bar{D}^{-1}) & \bar{X}_1^{(k)}(\bar{D}^{-1}) \end{bmatrix} + \begin{bmatrix} Z_1^{(k)}(D) & Z_2^{(k)}(D) \end{bmatrix} \tag{9}$$

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[2] Intuitively, each OFDM subcarrier can be thought of as a flat–fading channel and the Alamouti code is applied to each of the OFDM subcarriers. As a result, the Alamouti code yields diversity gains at every subcarrier.

where $\bar{x}(-n) \Longleftrightarrow \bar{X}(\bar{D}^{-1})$ indicates conjugated time-reversed sequences. We can write[3]

$$\begin{aligned} \mathbf{Y}_Q^{(k)}(D) &= \mathbf{H}_Q^{(k)}(D)\mathbf{X}_Q^{(k)}(D) + \mathbf{Z}_Q^{(k)}(D) \\ \bar{\mathbf{Y}}_Q^{(k)}(\bar{D}^{-1}) &= \bar{\mathbf{X}}_Q^{(k)}(\bar{D}^{-1})\bar{\mathbf{H}}_Q^{(k)}(\bar{D}^{-1}) + \bar{\mathbf{Z}}_Q^{(k)}(\bar{D}^{-1}) \ , \end{aligned} \tag{10}$$

where we have defined

$$\begin{aligned} \mathbf{X}_Q^{(k)}(D) &\overset{def}{=} \begin{bmatrix} X_1^{(k)}(D) & X_2^{(k)}(D) \\ -\bar{X}_2^{(k)}(\bar{D}^{-1}) & \bar{X}_1^{(k)}(\bar{D}^{-1}) \end{bmatrix}, \\ \bar{\mathbf{X}}_Q^{(k)}(\bar{D}^{-1}) &\overset{def}{=} \begin{bmatrix} \bar{X}_1^{(k)}(\bar{D}^{-1}) & -X_2^{(k)}(D) \\ \bar{X}_2^{(k)}(\bar{D}^{-1}) & X_1^{(k)}(D) \end{bmatrix}, \end{aligned} \tag{11}$$

and similar notation for $\mathbf{Y}_Q^{(k)}(D), \bar{\mathbf{Y}}_Q^{(k)}(\bar{D}^{-1}), \mathbf{Z}_Q^{(k)}(D), \bar{\mathbf{Z}}_Q^{(k)}(\bar{D}^{-1})$. The matrices $\mathbf{H}_Q^{(k)}(D)$ and $\mathbf{Z}_Q^{(k)}(D)$ are defined in an analogous way. Note that we have

$$\bar{\mathbf{X}}_Q^{(k)}(\bar{D}^{-1})\mathbf{X}_Q^{(k)}(D) \overset{def}{=} X_{eq}(D)\mathbf{I}, \tag{12}$$

where we have defined

$$X_{eq}(D) = X_1^{(k)}(D)\bar{X}_1^{(k)}(\bar{D}^{-1}) + X_2^{(k)}(D)\bar{X}_2^{(k)}(\bar{D}^{-1}), \tag{13}$$

and where $\mathbf{I}$ is the identity matrix. This shows that the 2×2 polynomial matrix $\mathbf{X}_Q^{(k)}(D)$ has a simple representation for its inverse. Also note that the 2×2 polynomial matrices defined in (11) form a multiplicative group, *i.e.* if we denote by $\mathcal{Q}(D)$ the set of 2×2 polynomial matrices which have the structure given in (11) they have the following properties:

$$\begin{aligned} &\text{For } \ \mathbf{V}_1(D), \mathbf{V}_2(D) \in \mathcal{Q}(D), \mathbf{V}_1(D)\mathbf{V}_2(D) \in \mathcal{Q}(D). \\ &[\mathbf{V}(D)]^{-1} = \frac{1}{V_{eq}(D)}\bar{\mathbf{V}}(\bar{D}^{-1}) \in \mathcal{Q}(D). \end{aligned} \tag{14}$$

We assume that for consecutive transmission blocks k and $k+1$, the channel remains constant, *i.e.* $H_i^{(k+1)}(D) = H_i^{(k)}(D)$. Therefore, using (10) and (12) we can write

$$\bar{\mathbf{Y}}_Q^{(k)}(\bar{D}^{-1})\mathbf{Y}_Q^{(k+1)}(D) \overset{def}{=} H_{eq}^{(k)}(D)\mathbf{U}_Q^{(k+1)}(D) + \tilde{\mathbf{Z}}_Q^{(k+1)}(D) \ . \tag{15}$$

where

$$\tilde{\mathbf{Z}}_Q^{(k+1)}(D) = \bar{\mathbf{Z}}_Q^{(k)}(\bar{D}^{-1})\mathbf{H}_Q^{(k)}(D)\mathbf{X}_Q^{(k+1)}(D) + \bar{\mathbf{X}}_Q^{(k)}(\bar{D}^{-1})\bar{\mathbf{H}}_Q^{(k)}(\bar{D}^{-1})\mathbf{Z}_Q^{(k+1)}(D) + \bar{\mathbf{Z}}_Q^{(k)}(\bar{D}^{-1})\mathbf{Z}_Q^{(k+1)}(D). \tag{16}$$

We have defined the information symbol matrix as

$$\mathbf{U}_Q^{(k+1)}(D) \overset{def}{=} \bar{\mathbf{X}}_Q^{(k)}(\bar{D}^{-1})\mathbf{X}_Q^{(k+1)}(D) \ , \tag{17}$$

yielding a differential space–time encoding/decoding scheme. The equivalent channel polynomial $H_{eq}^{(k)}(D)$ is equal to $H_1^{(k)}(D)\bar{H}_1^{(k)}(\bar{D}^{-1}) + H_2^{(k)}(D)\bar{H}_2^{(k)}(\bar{D}^{-1})$. Note that the equivalent noise sequence $\tilde{\mathbf{Z}}_Q^{(k+1)}(D)$ has approximately twice the power of the original noise process yielding a 3dB penalty. From (17), we obtain the differential encoding scheme

$$\mathbf{X}_Q^{(k+1)}(D) = \frac{\mathbf{X}_Q^{(k)}(D)}{X_{eq}(D)}\mathbf{U}_Q^{(k+1)}(D) \ . \tag{18}$$

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[3] The subscript Q is used to emphasize the quaternionic structure [3].

Equating the first row of both sides of (18), we get

$$\begin{aligned} X_1^{(k+1)}(D) &= \frac{X_1^{(k)}(D)U_1^{(k+1)}(D) - X_2^{(k)}(D)\bar{U}_2^{(k+1)}(\bar{D}^{-1})}{X_1^{(k)}(D)\bar{X}_1^{(k)}(\bar{D}^{-1}) + X_2^{(k)}(D)\bar{X}_2^{(k)}(\bar{D}^{-1})} \\ X_2^{(k+1)}(D) &= \frac{X_1^{(k)}(D)U_2^{(k+1)}(D) + X_2^{(k)}(D)\bar{U}_1^{(k+1)}(\bar{D}^{-1})}{X_1^{(k)}(D)\bar{X}_1^{(k)}(\bar{D}^{-1}) + X_2^{(k)}(D)\bar{X}_2^{(k)}(\bar{D}^{-1})} . \end{aligned} \tag{19}$$

Remarks :

1. To estimate the information symbols from (15), we need to estimate the equivalent channel $H_{eq}^{(k)}(D)$. This channel identification problem is easy because $H_{eq}^{(k)}(D)$ is a correlation sequence, therefore, it enjoys complex–conjugate symmetry about its middle sample which causes its zeros to occur in conjugate–reciprocal pairs. Thus, we only need to identify the minimum–phase zeros of $H_{eq}^{(k)}(D)$ using a standard spectral factorization procedure.

2. The differential encoding scheme shown in (19) can be implemented using a feedback filtering structure with proper transmit power normalization. For example, we can write

$$X_1^{(k+1)}(D)X_{eq}^{(k)}(D) = X_1^{(k)}(D)U_1^{(k+1)}(D) - X_2^{(k)}(D)\bar{U}_2^{(k+1)}(\bar{D}^{-1}), \tag{20}$$

   where as before $X_{eq}^{(k)}(D) = X_1^{(k)}(D)\bar{X}_1^{(k)}(\bar{D}^{-1}) + X_2^{(k)}(D)\bar{X}_2^{(k)}(\bar{D}^{-1})$. If we convert this into time-domain, the feedback structure for $x_1^{(k+1)}(n)$ becomes quite apparent[4]. Since this is done at the transmitter where the previous block is completely known, this would not have error propagation problems.

3. By virtue of the special spatio–temporal structure imposed by the encoding rule, we have constructed a blind channel identification scheme based on second-order statistics that works in the absence of multiple receive antennas or temporal oversampling.

4. After identification of $H_{eq}^{(k)}(D)$, the detection scheme can be one of many well–known equalization techniques. For example, we can use a whitened–matched filter [5] to convert $H_{eq}^{(k)}(D)$ to its minimum–phase equivalent followed by a Viterbi equalizer. Alternatively a decision-feedback equalizer could also be used.

5. To improve the blind estimation quality of $H_{eq}^{(k)}(D)$ at low SNR, the variance of the noise sequence $\tilde{\mathbf{Z}}_Q^{(k+1)}(D)$ in (15) can be estimated using standard eigen subspace techniques [11].

We conclude this section by summarizing the pros and cons of the frequency–domain and time–domain schemes. The frequency–domain scheme is based on OFDM, hence it inherits its advantages of low computational complexity (due to use of FFT) and superior multi–rate capabilities together with its disadvantages of high peak–to–average ratio and increased sensitivity to frequency synchronization errors. The time–domain scheme is single–carrier hence it avoids these two OFDM drawbacks at the expense of increased computational complexity since it requires blind identification of the channel and noise characteristics. Due to the special encoding rule, this blind identification is possible using second–order statistics only of the received signals without requiring temporal oversampling or multiple receive antennas.

4 Conclusions

We presented two differential full–rate space–time block coding schemes for two transmit antennas, one operates in the frequency domain and the other in the time domain, that achieve spatial and multipath diversity gains for frequency–selective channels. For the time–domain scheme, the special spatio–temporal code structure guarantees blind channel

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[4] Note that this transmission can be made in a finite block length P, as accurately as needed by increasing the transmission block size.

identifiability from second–order statistics with a single receive antenna and no temporal oversampling. Extensions to arbitrary number of transmit antennas are discussed in [4].

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List of Figures

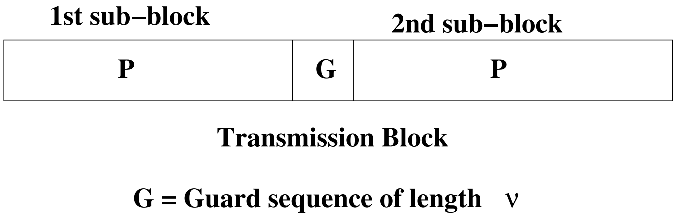

Figure 1: Block transmission format for both frequency-domain and time-domain approaches

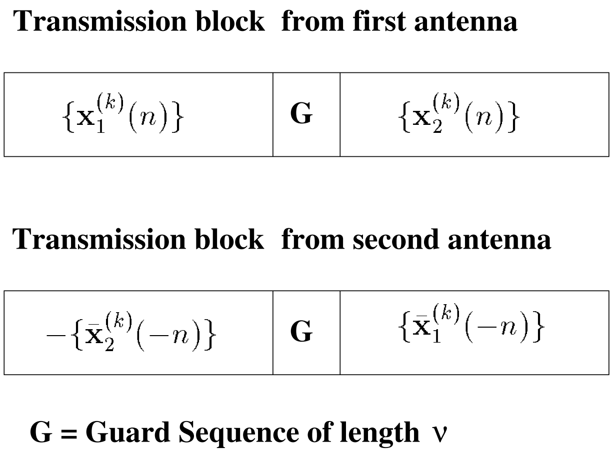

Figure 2: Transmission format for time-domain approach.