OPRE 6377: Quiz B on November 14, 2011

This is an open written-material exam. You may use a calculator although leaving quantities as fractions, additions or products is perfectly acceptable. **Do not forget** to define any variables you introduce. Good luck ...

Questn	1	2	3	4	**Total**	**Please Print Your Name Below**
Out of	5.0	5.0	7.0	4.0	21.0	
Points						

1. Put **T** before a statement if you think that statement is true. Otherwise put **X**.

1. (   ) When a low-fare class is not available, some customers planing to buy a low-fare ticket can buy a high-fare ticket. Presence of this possibility should increase the protection level for the high-fare class compared to the case where this possibility is absent. T
2. (   ) Bid price for a flight increases when the capacity on the flight increases. X
3. (   ) Setting booking limits by allotments is a method of finding nested booking limits. X
4. (   ) In a case of 10 fare classes (1 is the highest), nested booking limit b_7 limits the bookings for classes 7, 8, 9 and 10. T
5. (   ) The fee obtained by renting a GPS (global positioning satellite) device along with cars is an example of ancillary fee for a car rental company. T

2. **[Capacity Allocation]** An airline has fare classes from higher to lower as 1, 2 and 3 for an aircraft with 100 coach seats. The ticket prices for the classes are p_1=\$400, p_2=\$300, p_3=\$200. The uniformly distributed demands for these fare classes are denoted by D_1, D_2 and D_3: $D_1 \in [20, 60]$, $D_2 \in [10, 40]$, $D_3 \in [20, 60]$. The airline applies EMSR-a heuristic to find the protection levels for classes.

[1P] a) Find $y_{1,3}$, the number of seats protected for the first class while booking for the third class.
ANSWER: $\mathrm{P(D_1 > y_{1,3}) = p_3/p_1} = 2/4$ or $\mathrm{P(D_1 \le y_{1,3})} = 1/2$, so $y_{1,3} = 20 + (1/2)(60 - 20) = 40$.

[1P] b) Find $y_{1,2}$, the number of seats protected for the first class while booking for the second class.
ANSWER: $\mathrm{P(D_1 > y_{1,2}) = p_2/p_1} = 3/4$ or $\mathrm{P(D_1 \le y_{1,2})} = 1/4$, so $y_{1,2} = 20 + (1/4)(60 - 20) = 30$.

[1P] c) What should y_2 be?
ANSWER: $\mathrm{P(D_2 > y_{2,3}) = p_3/p_2} = 2/3$, or $\mathrm{P(D_2 \le y_{2,3})} = 1/3$, so $y_{2,3} = 10 + (1/3)(40 - 10) = 20$. $y_2 = y_{1,3} + y_{2,3} = 40 + 20 = 60$.

[1P] d) Suppose that $y_2 = 200/3$, what is the booking limit b_3 for the third class?
ANSWER: Since $b_3 + y_2 = 100$, $b_3 = 100 - 200/3 = 100/3$.

[1P] e) How does protection level y_1 change when the second class demand increases to $D_2 \in [20, 50]$ from $D_2 \in [10, 40]$?
ANSWER: $y_1 = y_{1,2}$ and depends on D_1 so it does not change with demand D_2.

3. **[Booking with EMSR-b]** Regular rooms at a hotel have 5 fare classes with Normally distributed demands with the following means and standard deviations:

$$\mu_1 = 10, \delta_1 = \sqrt{4}; \quad \mu_2 = 20, \delta_2 = \sqrt{12}; \quad \mu_3 = 30, \delta_3 = \sqrt{20}; \quad \mu_4 = 40, \delta_4 = \sqrt{x}; \quad \mu_5 = y, \delta_5 = \sqrt{36},$$

for unknowns x, y. Nightly room rates are $p_1 = 120;\ \ p_2 = 100;\ \ p_3 = 80;\ \ p_4 = 60;\ \ p_5 = 40$. You can use normdist and norinv in your answers.

[1P] a) What are the mean and standard deviation of aggregate demand class composed of classes 1, 2 and 3, i.e., $\mu_{1,2,3}, \delta_{1,2,3}$?

ANSWER: $\mu_{1,2,3} = 10 + 20 + 30 = 60$, $\delta^2_{1,2,3} = 4 + 12 + 20 = 36$, so $\delta_{1,2,3} = 6$.

[1P] b) What is the price $p_{1,2,3}$ for the aggregate demand class composed of classes 1, 2 and 3?

ANSWER: $p_{1,2,3} = (10/60)120 + (20/60)100 + (30/60)80 = 20 + 33.3 + 40 = 93.3$.

[2P] c) According to EMSR-b heuristic what is the protection level y_2 for classes 1 and 2?

ANSWER: $\mu_{1,2} = 10 + 20 = 30$, $\delta^2_{1,2} = 4 + 12 = 16$, so $\delta_{1,2} = 4$. $p_{1,2} = (10/30)120 + (20/30)100 = 40 + 66.6 = 106.6$. $\mathrm{P}(\mathrm{D}_{1,2} > \mathrm{y}_2) = \mathrm{p}_3/\mathrm{p}_{1,2} = 80/106.6$ or $\mathrm{P}(\mathrm{D}_{1,2} \le \mathrm{y}_2) = 26.6/106.6$, so $y_2 = norminv(26.6/106.6, 30, 4)$.

[1P] d) Suppose that y_4 is given as $norminv(1 - p_5/p_{1,2,3,4}, 100, 8)$, what is $\delta_4 = \sqrt{x}$?

ANSWER: We must have $8=\delta_{1,2,3,4}$ or $64=\delta^2_{1,2,3,4} = 4 + 12 + 20 + x$. This gives $x = 28$.

[1P] e) Suppose that price $p_{1,2,3,4,5} = 50$ for the aggregate class composed of classes 1, 2, 3, 4 and 5, what is y?

ANSWER: We need to solve for y from

$$50 = \frac{10}{100+y}120 + \frac{20}{100+y}100 + \frac{30}{100+y}80 + \frac{40}{100+y}60 + \frac{y}{100+y}40.$$

We obtain $8000 + 40y = 5000 + 50y$, so $y = 300$.

[1P] f) Protection level for class 1 is found from $y_1 = norminv(1 - p_2/p_1, \mu_1 = 10, \delta_1 = 2)$. How does y_1 change when μ_1 increases to 15? Encircle the star * that has the most accurate and correct answer. No explanation needed.
*) Decreases by an unknown amount; *) Increases by an unknown amount; *) Does not change;
*) Decreases by 15; *) Increases by 15; *) Decreases by 10; *) Increases by 10;
*) Decreases by 2; *) Increases by 2; *) Decreases by 5; *) Increases by 5.

ANSWER: When the mean increases by 5, demand increases by 5. We need to protect 5 more rooms for this demand. The last choice is correct.

4. **[Priceline single- vs. multi-dimensional bidding]** Priceline has a name-your-own-price (NYOP) option where the customers can bid for opaque airline/hotel products. If the customer's bid is high enough (e.g., higher than the airline's/hotel's bid price), the bid is accepted and customer gets the reservation.
[1P] a) Consider American Airlines flight 0402 that departs DFW airport at 1:05 pm and arrives at CLT airport, Charlotte, North Carolina at 4:25 pm on Nov 15, 2011. Give an example of how this flight can be made opaque for presenting/selling to NYOP customers.
ANSWER: It can be presented as flight from DFW to CLT by a major US airline on Nov 14. Similar answers are ok as well.

[1P] b) When buying opaque hotel stays, customers currently make a single bid at once. A street-smart customer can start his bid at a low price and increase it in increments of $1 until it becomes equal to the hotels' bid price. This is not a desirable situation for Priceline because it wants to sell tickets at prices above the hotels' bid prices. How can Priceline prevent the bids increasing in small increments to discover the hotels' bid price?
ANSWER: Priceline can limit the number of bids placed by a computer in a day. This is possible through cookies placed on computers. Another method is limiting the number of bids placed via a credit card in a day. Answers similar to these are ok.

[1P] c) Currently, Priceline allows a customer to make single-dimensional bids only. For example, if you want to bid for a 3-star hotel for Nov 15 stay in downtown San Diego, you can place a bid only for this product at once. An alternative is multi-dimensional bidding where you may place multiple bids at once, say one for a 3-star hotel and another for a 4-star hotel. Such a bid will look like:

[For 3-star hotel $80;   For 4-star hotel $100].

Make an argument based on convenience to explain why customers may like this multi-dimensional bidding feature.
ANSWER: The alternative is making sequential bids of [For 3-star hotel $80] and [For 4-star hotel $100]. Sequential bids take more time and more typing, so they are convenient.

[1P] d) Make an argument why receiving multi-dimensial bid [For 3-star hotel $80; For 4-star hotel $100] is better for Priceline than receiving a sequence of single-bids, such as first the bid [For 3-star hotel $80] and then the bid [For 4-star hotel $100], or first the bid [For 4-star hotel $100] and then the bid [For 3-star hotel $80].
ANSWER: Suppose that bid prices for 3-star and 4-star hotels are b_3 and b_4. With the multi-dimensional bidding, Priceline earns

$$\max\{80 - b_3, 100 - b_4, 0\}$$

Now consider first the bid [For 3-star hotel $80] and then the bid [For 4-star hotel $100]. If the first bid is accepted, i.e., $80 > b_3$, Priceline earns $80 - b_3$; otherwise it earns $\max\{100 - b_4, 0\}$. But we know that

$$\max\{80 - b_3, 100 - b_4, 0\} \geq \left\{ \begin{array}{ll} 80 - b_3 & \text{if } 80 > b_3 \\ \max\{100 - b_4, 0\} & \text{if } 80 \leq b_3 \end{array} \right\}.$$

The inequality above can be strict for example when $b_3 = 79$ and $b_4 = 80$.

The argument above is perhaps more mathematical than you like. To build intuition, consider yourself as a seller and you are receiving bids. Do not you want to receive all bids before deciding? Deciding after all bids (equivalent to postponing the bid-choice decision) is advantageous because it is done with more information. Multi-dimensional bid allows Priceline to decide after receiving multiple bids.

When you look at answers to c) and d) above, we may soon see multi-dimensional bidding becoming an option on Priceline.