OPRE 6377. DemReMan : Estimation

1 Inferring Demand from Price Elasticity

In this section, we suppose that the relationship between demand and price is linear: $d(p) = D - mp$. This relationship has two parameters D, m to estimate. This theoretically simple estimation can be challenging in practice due to lack of data. Here is what might happen in the case of 40-foot container shipment price in the Hong Kong/Los Angeles lane.

Executive summary of Rate-Capacity Nexus Report by ComPair Data (www.compairdata.com/Reports/rcn3.asp) mentions price elasticities below in August 2011:

> *On the bellwether lanes of Hong Kong/New York and Hong Kong/Los Angeles-Long Beach, capacity rose in the second quarter, which affected rates. At the end of June, Hong Kong/Los Angeles-Long Beach capacity was 25 percent higher than 12 months earlier while rates were 33.3 percent lower. Hong Kong-to-New York rates fell 18 percent in the second quarter as capacity rose 14 percent in the same period. Its hard to deny the impact capacity has on rates on key U.S. inbound lanes.*

Unfortunately this quote is about capacity but not demand. However, we assume that capacity and demand growths are parallel to each other, i.e., they grow (or drop) by the same percentage. Consequently, we read the quote as demand (in addition to capacity) was 25 percent higher than 12 months earlier in the Hong Kong/Los Angeles lane.

Suppose that price and demand of last year is denoted by (p_0, d_0). Then this year's price and demand is $(2p_0/3, 5d_0/4)$. Using these and demand-price relationship, we obtain

$$\begin{aligned} d_0 &= D - mp_0 && \text{Last year's demand price relationship,} \\ 5d_0/4 &= D - m2p_0/3 && \text{This year's demand price relationship.} \end{aligned}$$

These two equations do not help much because they include unknowns $(p_0, d_0), D, m$. We have 4 unknowns and 2 equations. That is as far as we can go with the quote above.

We can find last year's price of shipping a 40-foot container on Hong Kong/Los Angeles lane from the Internet. Suppose that the average price is p_0=\$2700, which is a realistic estimate. With this additional price information,

$$\begin{aligned} d_0 &= D - 2700m && \text{Last year's demand price relationship,} \\ 5d_0/4 &= D - 1800m && \text{This year's demand price relationship.} \end{aligned}$$

Now we have 3 unknowns and 2 equations so we still cannot solve for d_0, D, m.

Surprisingly, without solving for all the parameters, we can find the effect of price on the demand in terms of d_0 if we assume that neither D nor m changes in the next year. Suppose that the price will increase to \$2000 next year, what will the demand be in terms of d_0?

It is easy to represent next year's demand in terms of D, m, although these are unknown parameters: $d(p = 2000) = D - 2000m$. What remains is to use $d_0 = D - 2700m$ and $5d_0/4 = D - 1800m$ to compute $D - 2000m$ in terms of d_0. Basically we need to weigh $d_0 = D - 2700m$ by, say, α and $5d_0/4 = D - 1800m$ by, say, β so that

$$D - 2000m = (D - 2700m)\alpha + (D - 1800m)\beta.$$

Note that we are not requiring α or β to be nonnegative. This equality holds if $1 = \alpha + \beta$ and $2000 = 2700\alpha + 1800\beta$. That is $\alpha = 2/9$ and $\beta = 7/9$. Inserting these α, β, and $d_0 = D - 2700m$ and $5d_0/4 = D - 1800m$ the equation above,

$$D - 2000m = (D - 2700m)\alpha + (D - 1800m)\beta = (2/9)d_0 + (7/9)5d_0/4 = 43d_0/36.$$

This result can be stated as follows. Price of \$2000 will increase the demand by 19.4% (=7/36) in the forthcoming year.

An increase of 19.4% in demand may not be sufficient. The shipping company may ask what price will lead to 20% increase in demand. Then we need to search for p such that $D - 6d_0/5 = mp$. What remains is to use $D - d_0 = 2700m$ and $D - 5d_0/4 = 1800m$ to compute $D - 6d_0/5$ in terms of m. Basically we need to weigh $D - d_0 = 2700m$ by, say, γ and $D - 5d_0/4 = 1800m$ by, say, δ so that

$$D - 6d_0/5 = (D - d_0)\gamma + (D - 5d_0/4)\delta.$$

This equality holds if $1 = \gamma + \delta$ and $6/5 = \gamma + 4\delta/4$. That is $\gamma = 1/5$ and $\delta = 4/5$. Inserting these γ, δ, and $D - d_0 = 2700m$ and $D - 5d_0/4 = 1800m$ in the equation above,

$$D - 6d_0/5 = (D - d_0)\gamma + (D - 5d_0/4)\delta = (1/5)2700m + (4/5)1800m = 1980m.$$

This result can be stated as follows. To increase the demand by 20% in the forthcoming year, price must be \$1980.

2 Homework Questions

1. Consider the quote: "*At the end of June, Hong Kong/Los Angeles-Long Beach capacity was 25 percent higher than 12 months earlier while rates were 33.3 percent lower*. With regard to the demand at $p_0 = 2700$, we have obtained that price of \$2000 increases the demand by 19.4% and price of \$1980 increases the demand by 20%. The elasticity of price change from \$2700 to \$2000 can be computed. The price drops by 700/2700 and demand rises by 7/36, so the elasticity is (7/36)/(700/2700)= 0.75.
   a) Compute the elasticity for the price change from \$2700 to \$1980.
   b) Compare the elasticity of 0.75 with what you computed in a). Comment on how the elasticities should differ or should not differ at all.

2. Consider the quote: "*Hong Kong-to-New York rates fell 18 percent in the second quarter as capacity rose 14 percent in the same period.*" Suppose that capacity and demand grow (drop) by the same percentage and demand is linear in the price. With (p_0, d_0) as the last year's price and linear demand assumption, first check that we have

$$\begin{aligned} d_0 &= D - mp_0 && \text{Last year's demand price relationship,} \\ 114d_0/100 &= D - m82p_0/100 && \text{This year's demand price relationship.} \end{aligned}$$

   Suppose that p_0=\$3000 for a 40-foot container. Suppose that the price will drop to \$2000 next year, what will the demand be in terms of d_0?