

OPRE 6377. DemReMan : Price Differentiation

1. **[Segmentation of in a Mortgage Market]** LowCost loan company is in the business of sourcing house loans (mortgage) from financial markets and then selling the servicing of the loan to big banks with loan servicing infrastructure: Chase Manhattan Bank, Wells Fargo, Citibank. The loan customers (borrowers) pay for the cost loan of origination, documentation fees, recording fees, appraisal fee, survey fee to the loan company (lender). We call these as loan costs. Borrowers also pay a title company for the services such as title insurance and settlement of a loan. We call these as title costs. Suppose that each of the loan costs and title costs is about \$1000, so the total loan closing cost is \$2000. LowCost company has funded a borrower survey which has concluded that a borrower is willing to pay between \$2000 and \$4000 for closing a loan. That indicates  $W(p) = p/2000 - 1$  for  $2000 \leq p \leq 4000$ .
  - a) Suppose that LowCost has a monthly potential demand of at most 100 borrowers, express the monthly demand  $d(p)$  if LowCost charges  $p \geq 2000$  for closing costs.
  - b) Suppose that LowCost has an affiliated title company so it and the title company together make a profit of  $p - 2000$  when a borrower is charged  $p$  for closing cost. What is optimal  $p$  that maximizes the profit of LowCost and its affiliated title company?
  - c) If LowCost needs, it can waive/reduce some closing costs. For example, if a borrower submits a relatively recent copy of the house survey (a depiction of house, fences, yards), survey fee of \$400 can be waived. If the ratio of loan amount to house value is less than 50%, a full appraisal can be turned into a drive-by appraisal, which reduces house appraisal cost by about \$200. Explain how these cost reductions/eliminations can be used to segment the borrower market. Also comment on arbitrage and cannibalization possibility in a segmented borrower market.
  
2. **[Optimal Prices in a Segmented Mortgage Market]** LowCost loan company, introduced above, classifies any borrower with wtp above \$3000 as a high-paying borrower. The others are low-paying customers. LowCost can waive survey fee, reduce appraisal fee and loan origination fee for low-paying customers.
  - a) Let  $d_1, d_2$  denote the demand of high-paying and low-paying customers, obtain the demand functions  $d_1(p)$  and  $d_2(p)$  for  $3000 \leq p \leq 4000$ .
  - b) Find the optimal prices  $p_1^*$  and  $p_2^*$  for the markets specified in part b).
  
3. **[Segmentation of in a Lunch Market]** Nazar restaurant makes a market analysis to find out that the minimum willingness to pay (wtp) for the lunch is \$2.5 and maximum wtp is \$12.5. Suppose that the wtp is uniformly distributed between these limits, that there are 100 lunch customers considering to go to the Nazar restaurant and that the cost of the lunch is \$5.
  - a) The restaurant classifies anybody with wtp more than \$8.5 as a high-paying customer. The rest are considered low-paying customers. The restaurant will offer à la cart service to high-paying customers and a lunch buffet to low-paying customers. Let  $d_1, d_2$  denote the demand of high-paying and low-paying customers, obtain the demand functions  $d_1(p)$  and  $d_2(p)$  for  $2.5 \leq p \leq 12.5$ .
  - b) If you are told that the demand functions are
 
$$d_1(p) = \min\{50, 125 - 10p\},$$

$$d_2(p) = (75 - 10p)^+,$$
 at what level of wtp is the market split in to high- and low-paying customers?
    - c) Find the optimal prices  $p_1^*$  and  $p_2^*$  for the markets specified in part b).
    - d) What is the total profit when Nazar charges  $p_1 = 9$  and  $p_2 = 6$  in the markets specified in part b).

4. **[Avoiding Cannibalization at Nazar Restaurant]** When the high- and low-paying customer demands are given by

$$\begin{aligned}d_1(p) &= \min\{50, 125 - 10p\}, \\d_2(p) &= (75 - 10p)^+, \end{aligned}$$

find the optimal prices and see if they satisfy the constraint  $|p_1 - p_2| \leq 2$ . If they satisfy the constraint, less than \$2 difference between prices will be effective in eliminating the cannibalization. That is high-paying customers will buy à la cart service while low-paying customers will stick to the lunch buffet. If prices do not satisfy the constraint, find optimal prices by satisfying the constraint  $|p_1 - p_2| \leq 2$ .

5. **[Incorporating Cannibalization at Nazar Restaurant]** When there is no cannibalization, high- and low-paying customer demands are given by

$$\begin{aligned}d_1(p) &= \min\{50, 125 - 10p\}, \\d_2(p) &= (75 - 10p)^+. \end{aligned}$$

However, Nazar has surveyed high-paying customers to find out that  $\alpha$  portion of them will buy a lunch buffet, irrespective of the price difference between the buffet and the à la cart service. Nazar modifies the demands in light of this cannibalization information and denotes them as  $\bar{d}_1$  and  $\bar{d}_2$ .

- Obtain expressions for  $\bar{d}_1(p)$  and  $\bar{d}_2(p)$ .
  - Prove that cannibalization cannot decrease the total demand irrespective of  $\alpha$ :  $\bar{d}_1(p) + \bar{d}_2(p) \geq d_1(p) + d_2(p)$ .
  - Find the optimal prices with demands  $\bar{d}_1(p)$  and  $\bar{d}_2(p)$  that incorporate cannibalization. Your prices can depend on  $\alpha$ .
  - Since the cannibalization cannot decrease the total demand, can it decrease the total profit? Explain.
6. **[Arbitrage vs. Cannibalization]** When a seller offers a volume discount to a buyer, the seller should eliminate arbitrage and cannibalization.
- Consider the CPA classes at the bank. Explain if/how arbitrage happens. Explain if/how cannibalization happens.
  - Consider a software licensed to multiple users at a single company and the licensing fee per user has a volume discount. Explain if/how arbitrage happens. Explain if/how cannibalization happens.
  - Consider a commercial building contractor who spends millions of dollars on buying galvanized pipe. The contractor buys the pipes with a volume discount. Explain if/how arbitrage happens. Explain if/how cannibalization happens.
  - Consider the three contexts above. In which context arbitrage is more likely to happen? In which context cannibalization is more likely to happen?