OPRE 6377. DemReMan : Basic Price Optimization

1 Willingness to Pay

Willingness to pay by a population of customers is denoted by $w(x)$ and

$$1 - W(p) = W(\infty) - W(p) = \int_p^\infty w(x)dx$$

is the portion of the customers willing to pay price p or more. Following two equalities are used to relate W and w

$$W(p) = \int_0^p w(x)dx \quad \text{and} \quad w(x) = \frac{d}{dx}W(x).$$

$W(p)$ is the portion of the customers who are willing to pay at most p, that is they are willing to pay p or lower. This portion does not buy the product if it is priced slightly above p.

We call W as the cumulative willingness to pay and w as the exact willingness to pay. Often such a distinction is not necessary as W and w are closely associated with each other throught the formulas above.

The willingness to pay (WTP) of a customer for a product of a company depends on

1. The price history of that product,
2. Prices currently charged for similar products by the company,
3. Prices currently charged for similar products by other companies.

WTP of customers can be managed by explaining the value of a product to customers. Consider the case of drug coated coronary stent developed and sold by Johnson and Johnson (J&J) in 2003. This stent was priced at $3500 per piece while the regular stents were priced about $1000 per piece. IN other words, the new stent was priced 250% more than the regular stent. J&J customers (doctors and patients) criticized this price and called it price gouging. Clearly, they were not willing to pay $3500 for the new stent. To pull WTP up, J&J started to communicate the economic benefits of the new stent to the customers. Two numbers are used in the economic benefit analysis: drug coated stent reduces the chances of a repeated operation from 20% to 5% in a year and the cost of an operation runs about $30,000. Putting these numbers together, the new stent offers a value of 0.15(30,000)= $4500. With this economic analysis, J&J answered the critics and pulled the WTP up. Since Pharmaceutical companies frequently bring new drugs into the marketplace and they would like to set high prices to compensate for R&D costs, these companies can ususally find it difficult to reconcile the price of a product with the WTP of the customers. They can also use the tactic of communicating the economic value to customers, as it is exemplified by J&J.

An important concept is the *reference price*. This is the price that we associate with an aggregate group of products. It is rather a rough estimate, one that is made without knowing the properties of the product. This estimate is certainly shaped by the factors that affect the WTP: price history, price of similar products, price of competitor's products, value of the product.

For a reference price example, consider how much textbooks cost nowadays. Certainly they cost much more than what they should. Even from this we can infer that we have an expectation for the price of a textbook. For example, your reference price can be $150 which could be based on the prices of the latest textbooks that you bought. Then we say that your reference price for a textbook is $150.

A customer's WTP is strongly related to the reference price. To increase WTP, we can increase the reference price. This can be done by pointing out and emphasizing expensive products. For example, your reference price for a textbook can be increased by telling you that a graduate level accounting textbook costs \$240. This consciously or unconsciously will force you to revise your reference price, or at least it will make you uncertain of your reference price. After announcing the accounting textbook's price, we can look at the price of the textbook for OPRE 6377. The list price is \$64 and the current discounted Amazon price (at the time of writing this example) is \$50. Since the accounting textbook costs 380% more than the OPRE 6377 textbook and the high cost of accounting textbooks have already pulled your WTP up, you are willing to pay \$50 to the OPRE 6377 textbook. A bookstore clerk who wants to sell OPRE 6377 textbook should then first tell the customers the prices of the accounting textbooks. This tactic of starting with a higher priced item to pull WTP (or reference price) up is called top-down selling.

Another tactic is giving 100% to a customer but taking a smaller percentage from the customer. But you give 100% of something small and take a smaller percentage of something much bigger. As an example consider free Internet access at premium hotels. The cost of providing free Internet access is much lees than the premium the hotels charge for it. This extra charge may be increasing the hotel bill by \$10, which is 5% of \$200 – average nightly rate at a premium hotel. In this case hotel bills 5% more to the customer but gives 100% free Internet service. Many customers prefer to pay 5% more in their bill in return for 100% free Internet service. This happens because of our habit of focusing on percentages rather than absolute values when we compare prices. Consider the stent and textbook examples above, if you have paid more attention to percentages of 250% and 380% than the absolute differences of \$2500 and \$190, you also have the habit of focusing on the percentages.

In order to highlight our focus on percentages, consider a \$10 saving that you can make by switching where you are shopping for clothes. When you are spending less than \$40 (more than 25% saving) you are more likely to switch the location than when you spend more than \$100 (less than 10% saving). This has been established with customer surveys and experiments. Roughly speaking, we are less concerned about small savings when what we are buying is big or important. For example maintainance costs of a car is not a deal maker or breaker when buying a car. Another example is the cost of the restaurant that you will have your birthday party, the saving that you can have by going to a cheap restaurant does not sound interesting, does it?

2 Multinomial Model

In the multinomial model, demand of a product is proportional to that product's attractiveness while it decreases with other products' attractiveness. This is a reasonable method to split demands among products. But it requires the computation of *attractiveness* of a product. The multinomial model can be thought as a hierarchical method of assigning demands to products. This hierarchy has two steps: computation of attractiveness of each product and splitting demand among products depending on relative attractiveness.

Attractiveness: We consider only price as a variable although one can append quality, functionality or other aspects of a product to this framework. Suppose that products are indexed by letter i, the price of product i is p_i and its attractiveness is $A_i(p_i)$. Attractiveness is nonngeative, i.e., $A_i(p_i) \geq 0$. Since higher prices make a product less atractive, A_i must be a decreasing function. This is not a very workable construction until we make $A_i(p_i)$ more explicit.

In the multinomial model, we set $A_i(p_i) = \exp(-b_i p_i)$ by using parameter $b_i \geq 0$. This parameter is a measure of drop in attractiveness as the price increases. The drop in attractiveness is proportional to the parameter b_i and the current attractiveness. To make this more concrete suppose that p_i is the current price and it yields the attractiveness $A_i(p_i)$ and further suppose that the price is increased to $p_i' > p_i$. The attractiveness at the higher price p_i' is $A_i(p_i')$. Since A_i is a decreasing function we have $A_i(p_i') < A_i(p_i)$. The quantity of interest is incremental change in the attractiveness $A_i(p_i') - A_i(p_i)$. In the multinomial model, this incremental change is given by

$$A_i(p_i') - A_i(p_i) \approx -b_i A_i(p_i) < 0.$$

In words, the attractiveness drops faster with price increases if the parameter b_i is large or the current attractiveness is large. If we write the above approximate equality in terms of a derivative, we obtain

$$\frac{d}{dp_i} A_i(p_i) = -b_i A_i(p_i).$$

The only atractiveness function that solves this equality is $A_i(p_i) = \exp(-b_i p_i)$, which is the multinomial model.

Relative Attractiveness: Suppose that the market has I products, each product i has its price p_i and attractiveness $A_i(p_i)$ as formulated above. The relative attractiveness f_i for product i depends on the price vector, i.e., all of the prices $p = [p_1, p_2, \ldots, p_i, \ldots, p_I]$. Algebraically, it is given by

$$f_i(p) = f_i([p_1, p_2, \ldots, p_i, \ldots, p_I]) = \frac{A_i(p_i)}{\sum_{i=1}^{I} A_i(p_i)}.$$

Since f_i is relative attractiveness, it is the ratio of product i's attractiveness to the sum of all products' attractiveness. Fuguratively speaking, you can think of product i trying to overcome the attractiveness of all of the other products to be sold to the customer. This ratio automatically gives us the following four properties:

$$f_i(p) \geq 0; \ \ \frac{d}{dp_i} f_i(p) \leq 0; \ \ \frac{d}{dp_j} f_i(p) \geq 0; \ \ \sum_{i=1}^{I} f_i(p) = 1.$$

Attractiveness and relative attractiveness concepts are more general than the multinomial model. Put differently, the multinomial model is a special case of attractiveness and relative attractiveness concepts. These concepts together are called attraction models.

Example of alternative attractiveness: The multinomial model has exponential attractiveness $A_i(p_i) = \exp(-b_i p_i)$. By changing the attractiveness function, we can obtain another model. We can consider $A_i'(p_i) = b_i/p_i$ as an alternative. This is a valid alternative because A_i' is positive and decreasing in price. Then, for three products, we have the relative attractiveness of

$$f_1(p) = \frac{b_1/p_1}{b_1/p_1 + b_2/p_2 + b_3/p_3}; \ \ f_2(p) = \frac{b_2/p_2}{b_1/p_1 + b_2/p_2 + b_3/p_3}; \ \ f_3(p) = \frac{b_3/p_3}{b_1/p_1 + b_2/p_2 + b_3/p_3}.$$

Example of multinomial with power-functions for attractiveness: A valid question is what happens if we set a slightly different attractiveness $A_i'(p_i) = 3^{-b_i' p_i}$ rather than $A_i(p_i) = \exp(-b_i p_i) = e^{-b_i p_i}$. $3^{-b_i p_i}$ and $e^{-b_i p_i}$ are quite alike but can they give us the same attractivenss? The answer is yes beacuse

$$A_i'(p_i) = 3^{-b_i' p_i} = e^{-b_i p_i} = A_i(p_i) \ \text{ provided that } \ b_i' \ln 3 = b_i,$$

where "ln" denotes natural logarithm. Consequently, we obtain the same multinomial demand models wheteher we set $A_i'(p_i) = 3^{-b_i' p_i}$ or $A_i(p_i) = e^{-b_i p_i}$ when $b_i' \ln 3 = b_i$. Parameters are estimated in practice, if you work with $A_i'(p_i) = 3^{-b_i' p_i}$, you will estimate $\hat{b}_i'$ from the historical data. Alternatively, if you work with $A_i(p_i) = e^{-b_i p_i}$, you will estimate $\hat{b}_i$ from the historical data. Except for small estimation errors, you will obtain $\hat{b}_i' \ln 3 = \hat{b}_i$. In other words, you gain or lose nothing by considering the alternative attractiveness formulation $A_i'(p_i) = 3^{-b_i' p_i}$ over the original formulation $A_i(p_i) = e^{-b_i p_i}$. Above argument can be generalized for any alternative attractiveness formulation with power functions: $A_i'(p_i) = c^{-b_i' p_i}$, where c is a given constant.

3 Homework Questions

1. Top-down selling tactic is initially offering high-priced items to a customer to pull WTP (reference price) up regardless of whether the customer is interested in or is able to afford these high-priced items. Suppose that you are
   a) a real estate agent selling houses,
   b) a mail catalog designer for a blouse company,
   c) a Wal-Mart store employee stocking pasta shelves,
   how would you implement the top-down selling tactic?

2. Suppose that a product has a reference price of $\hat{p}$ and willingness to pay is uniform over the interval $[\hat{p} - \Delta, \hat{p} + \Delta]$ for a known $\Delta > 0$:

$$w(x;\hat{p}) = \left\{ \begin{array}{ll} 0 & \text{if } x < \hat{p} - \Delta, \\ \dfrac{1}{2\Delta} & \text{if } \hat{p} - \Delta \le x \le \hat{p} + \Delta, \\ 0 & \text{if } \hat{p} + \Delta < x \end{array} \right\}.$$

   The width of the WTP interval is 2Δ. a) For each of the three possible cases of price p, $p < \hat{p} - \Delta$, $\hat{p} - \Delta \le p \le \hat{p} + \Delta$ and $\hat{p} + \Delta < p$, express the percentage of the customers who are willing to pay at most p in terms of $\hat{p}$, Δ and numbers.
   b) For each of the three posiible cases of price p, $p < \hat{p} - \Delta$, $\hat{p} - \Delta \le p \le \hat{p} + \Delta$ and $\hat{p} + \Delta < p$, express the percentage of the customers who are willing to pay at least p in terms of $\hat{p}$, Δ and numbers.
   c) Suppose that a top-down selling tactic increases the reference price from $\hat{p}$ to $\hat{p} + 1$ but it does not change the uniformity of WTP or the width of the WTP interval, express the new WTP $w(x;\hat{p}+1)$ in terms of $\hat{p}$, Δ and numbers. How much does this tactic increase the percentage of the population who buy the product at price p for $\hat{p} - \Delta + 1 \le p \le \hat{p} + \Delta$? Express the increase in terms of $\hat{p}$, Δ and numbers.

3. [Less rent or free month(s)] When the apartment rental market cools down, the apartment owners think of reducing the rental prices either directly or indirectly to stimulate the demand. Direct reduction is simply reducing the annual rent. One of the indirect reduction strategies is keeping the rent constant but not charging any the rent for the first month of a 12-month lease (also called free first-month). Among these two strategies, which one keeps the reference price higher? Explain. Is it in the interest of the apartment owners to keep the reference price higher? In your own apartment renting experience, have you seen more direct rent reductions or free first-months?

4. [Inattention bias] It is empirically observed that the price of a car decreases with its mileage. However, this decrease is not continuous at every mileage. For example, the decrease in price with 1 extra mile is significant if the car already has 19,999 miles. This decrease is small if the car has 19,998 miles. In general, price drops by a significant amount when it reaches milage of 20 K, 30 K, 40 K, 50 K, and so on. These drops are attributed to inattention bias of the buyers. In other words, buyers typically pay attention only to the first digit of the mileage for the cars below 100 K. Thus, cars with 60100 miles and with 69900 miles are perceived to closer in terms of milage than cars with 69900 miles and with 70100 miles. The inattention bias can be strengthen in a sales transaction by increasing the amount of information the buyer receives.
   a) If you are a car salesperson and want to sell a car with 50213 miles on it, would you plan to strengthen the inattention bias or weaken it?
   b) How would your responde to a) change if the car has 49787 miles?
   c) What are the specific ways to strengthen the inattention bias that a car salesperson can implement?