

# Price Differentiation

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## Outline

- ◆ **Price Differentiation: Limitations and Tactics**
- ◆ **Volume Discounts**
- ◆ **Pricing with Arbitrage and Cannibalization**
- ◆ **Consumer Welfare**

**Based on Phillips (2005) Chapter 4**

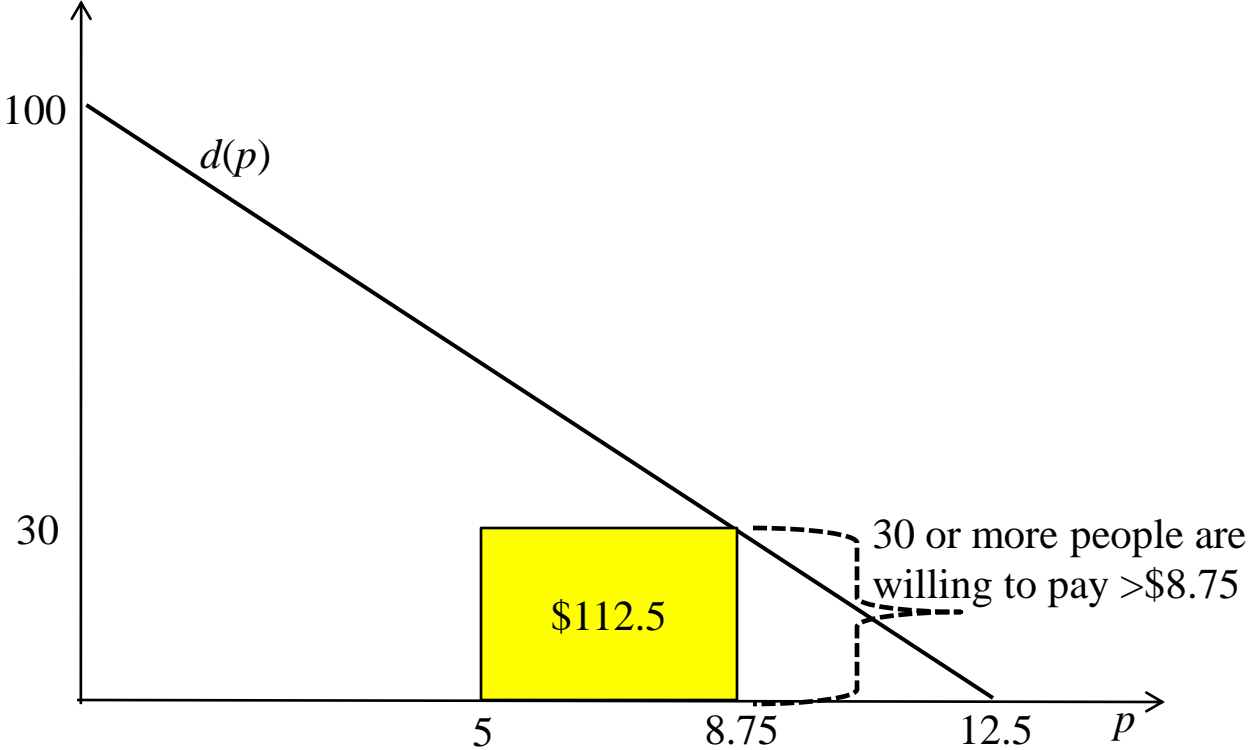
# Price Differentiation

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- ◆ Price differentiation: Charging different customers different prices
  - For exactly the same product
  - For a slightly different product
    - » By customer group – UTD students
    - » By product version – Older version
    - » By geography – US prices
    - » By distribution channel – E-commerce
  
- ◆ Price differentiation requires understanding customer segments
  - What are the segments?
  - How do they differ from each other?
  - Do they remain the same over time?
  - Are there customers switching segments (cannibalization)?
  - Can a product sold to a segment can end up in another segment (arbitrage)?
  - How can we create barriers between segments?

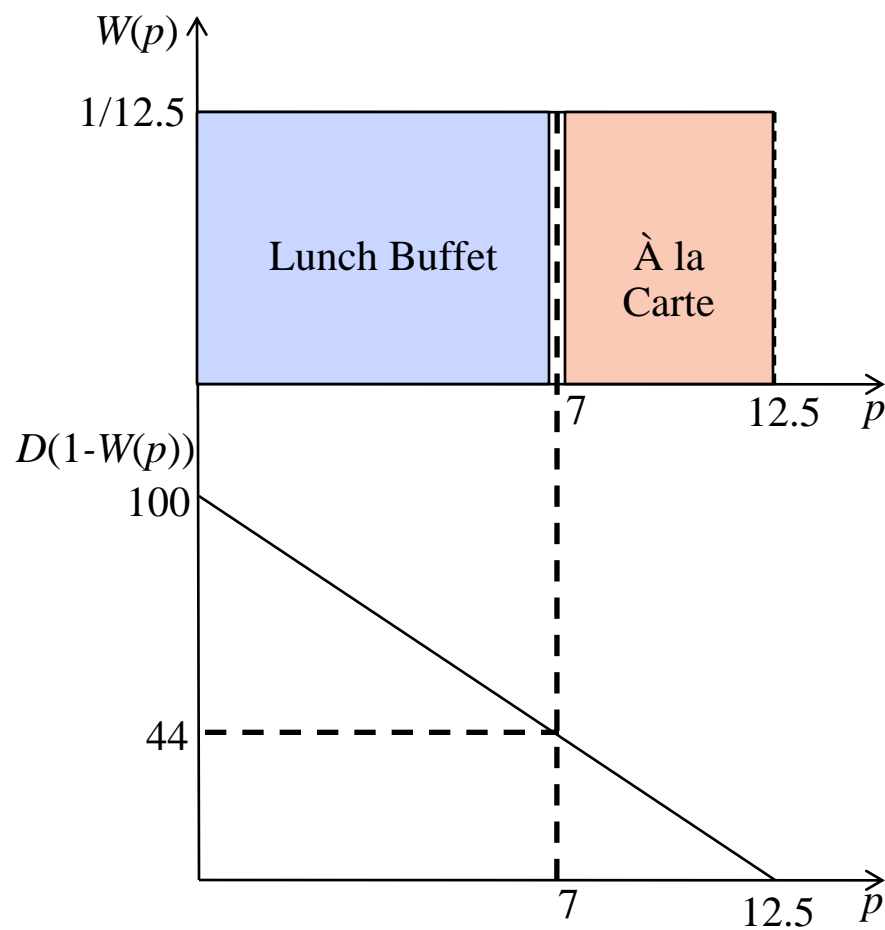
# Price Differentiation: À la Carte

- ◆ Consider Nazar restaurant in Addison and suppose that their demand function for lunch is  $d(p)=100-8p$  and the cost per lunch is  $c=\$5$ .
- ◆ The total contribution (profit)  $[-8p^2+140p-500]$  maximizing price is  $\$8.75=\$140/16$ .
- ◆ The demand at this price is 30 people per lunch time, the profit is  $\$112.5$ .



# Price Differentiation: Lunch Buffet and À la Carte Demands

- ◆ Nazar restaurant divides its market into two: those paying below and above \$7. The restaurant offers a lunch buffet for low willingness to pay customers .
- ◆ 44 people are willing to pay more than \$7. Market size for high-payers is 44 and for low-payers is 56.

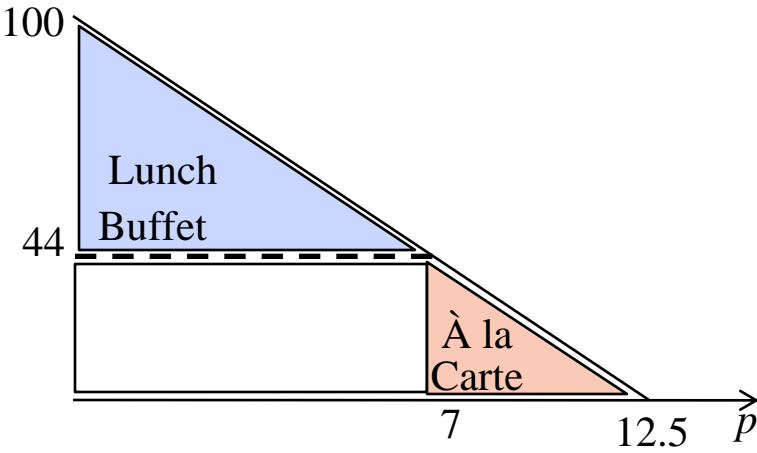
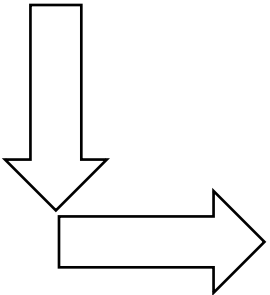
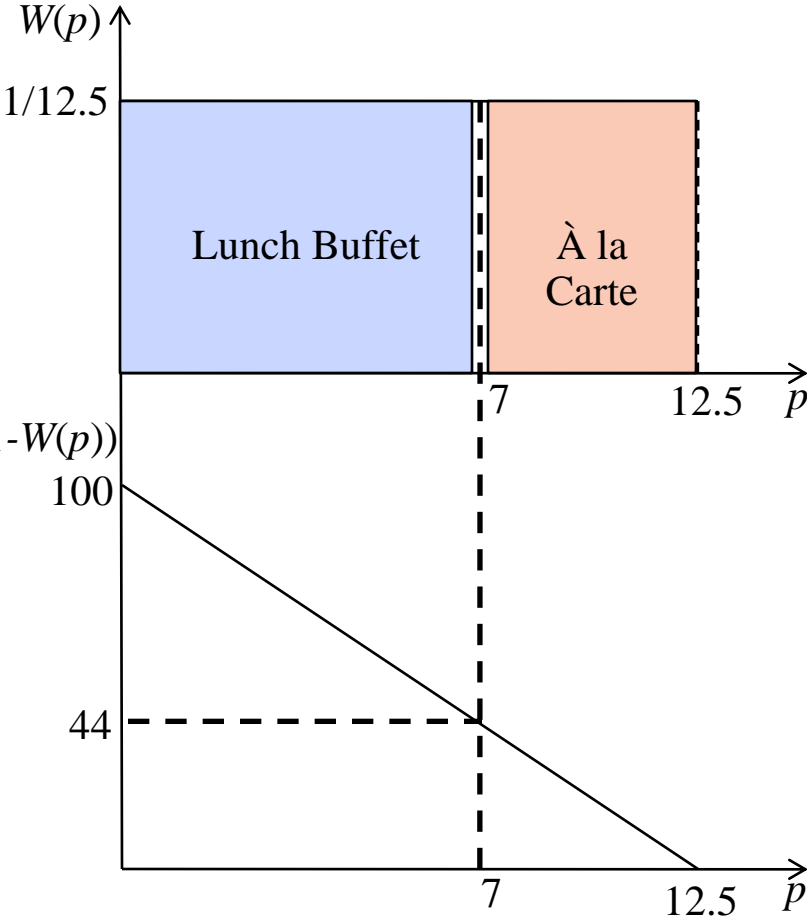


$$\begin{aligned}
 d_1(p) &= D_1(1 - W_1(p)) \\
 &= 44 \left( 1 - \frac{(p-7)^+}{5.5} \right) = 44 - (8p - 56)^+ \\
 &= \min\{44, (100 - 8p)^+\} \text{ for } 0 \leq p \leq 12.5
 \end{aligned}$$
  

$$\begin{aligned}
 d_2(p) &= D_2(1 - W_2(p)) \\
 &= 56 \left( 1 - \frac{p-0}{7} \right)^+ = (56 - 8p)^+ \text{ for } 0 \leq p \leq 12.5
 \end{aligned}$$

To indicate the positive part of number  $x$  we write  $x^+ = \max\{0, x\}$ .

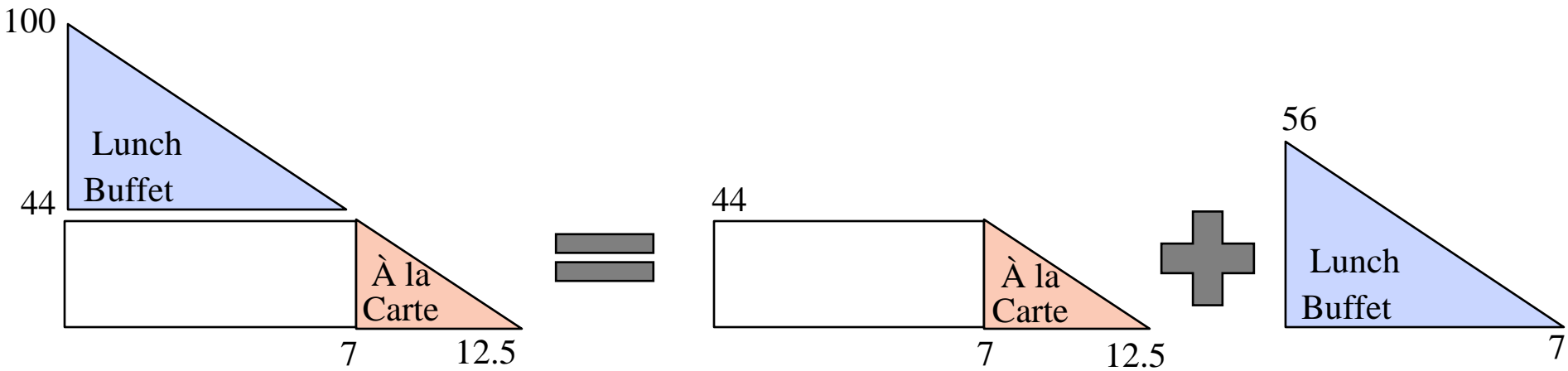
# Lunch Buffet and À la Carte Demands via Pictures



# Lunch Buffet and À la Carte Demands

## Segment Demands via Pictures

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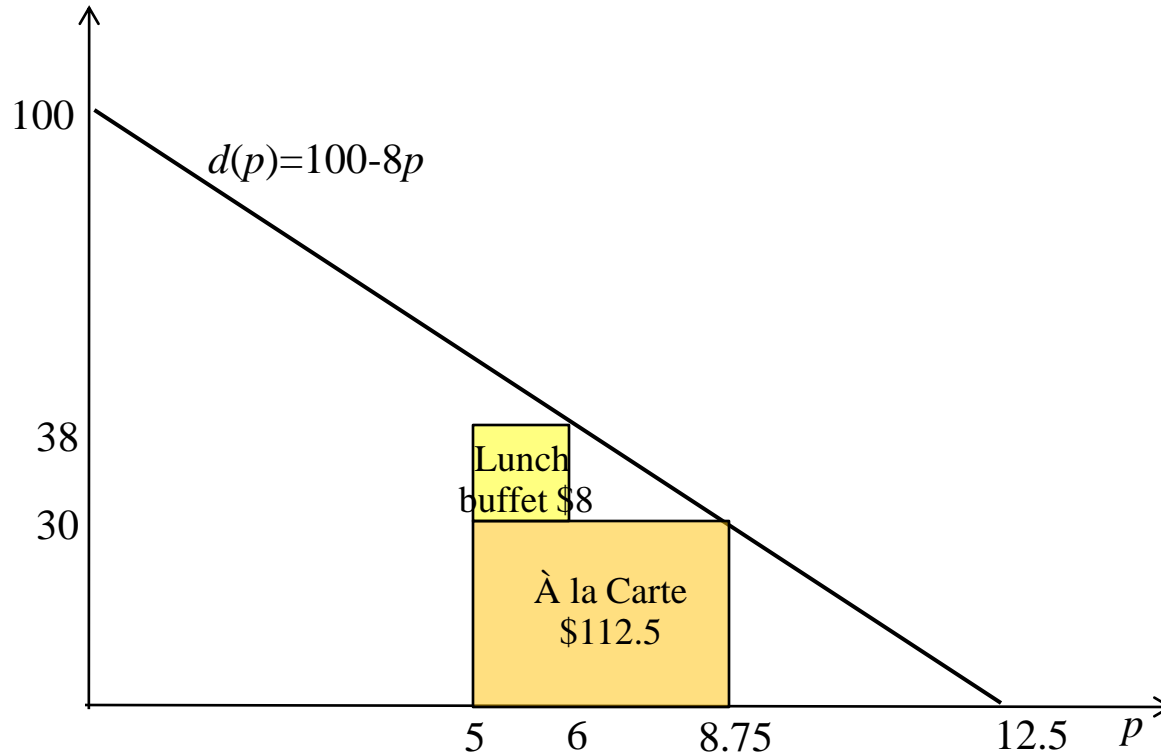
Demand functions for high and low willingness to pay customers from the pictures above:

$$d_1(p) = \min\{44, (100 - 8p)^+\}$$

$$d_2(p) = (56 - 8p)^+$$

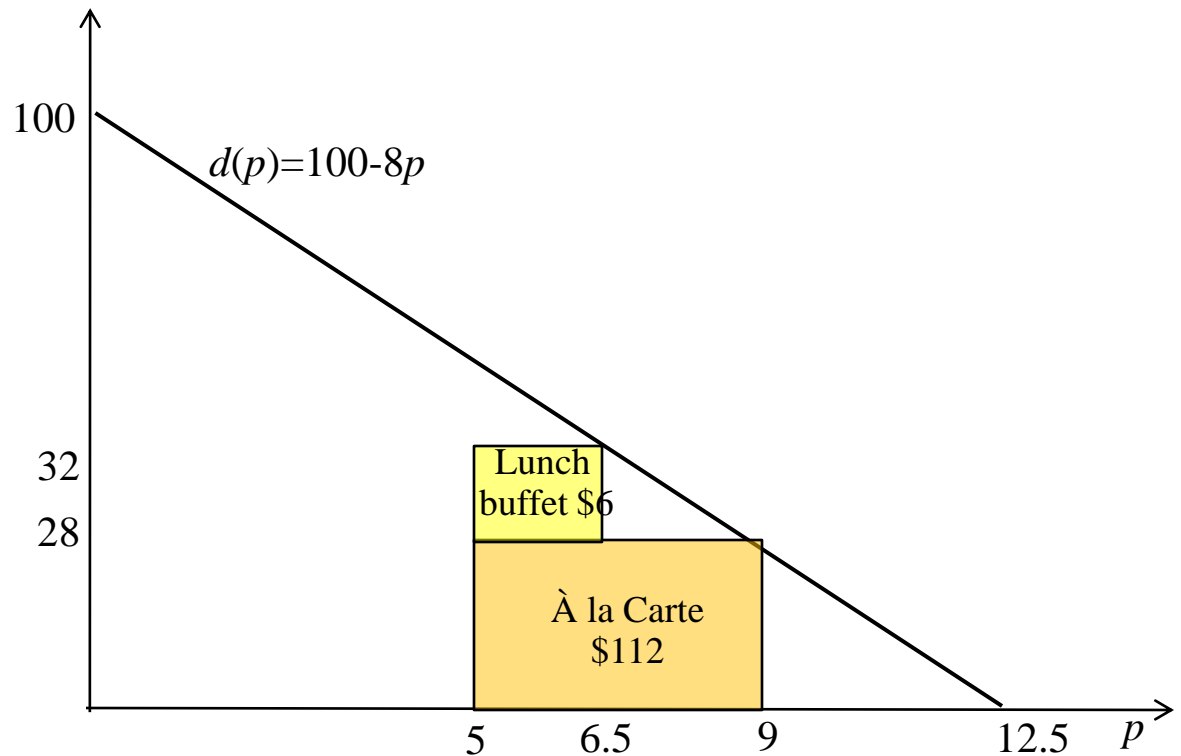
# Price Differentiation: Lunch Buffet at \$6 and À la Carte at \$8.75

- ◆ Nazar restaurant continues to charge an average of \$8.75 for à la cart service.
- ◆ Still  $d_1(p=8.75)=30$  people are willing to pay more than \$8.75 and bring a profit of  $112.5=30(8.75-5)$  per lunch time.
- ◆ Suppose that the restaurant prices the lunch buffet at \$6. Then the buffet demand turns out to be  $d_2(p=6)=8$  people. This brings in an additional profit of  $8=8(6-5)$  per lunch time.
- ◆ The profit increases from \$112.5 to \$120.5 per lunch time. This difference can draw the fine line between success and bankruptcy in the restaurant business.



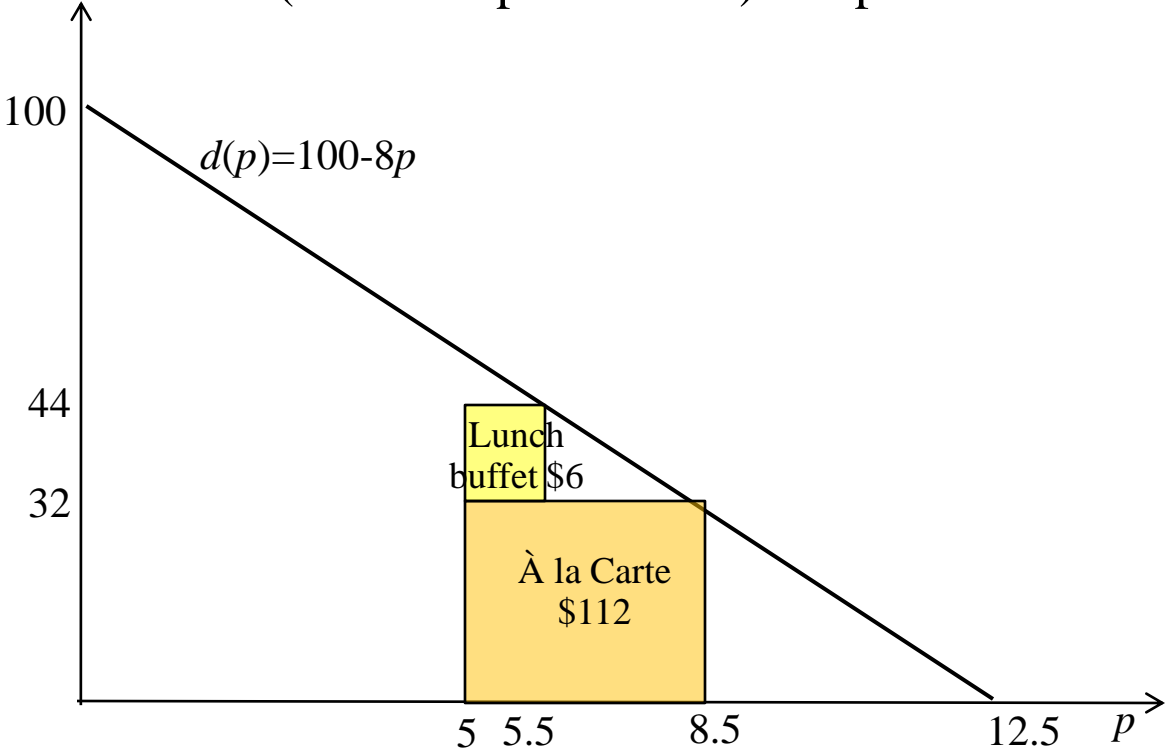
# Price Differentiation: Lunch Buffet at \$6.5 and À la Carte at \$9

- ◆ With \$9 for à la cart service, demand is  $d_1(p=9)=28$  which brings a profit of  $112=28(9-5)$  per lunch time.
- ◆ With \$6.5 for lunch buffet, demand is  $d_2(p=6.5)=4$  which brings a profit of  $6=4(6.5-5)$  per lunch time.
- ◆ This prices increase the profit from 112.5 to 118. Note that lower prices on the previous page yielded higher total profit.



# Price Differentiation: Lunch Buffet at \$5.5 and À la Carte at \$8.5

- ◆ With \$8.5 for à la cart service, demand is  $d_1(p=8.5)=32$  which brings a profit of  $112=32(8.5-5)$  per lunch time.
- ◆ With \$5.5 for lunch buffet, demand is  $d_2(p=5.5)=44$  which brings a profit of  $6=12(5.5-5)$  per lunch time.
- ◆ This prices increase the profit from 112.5 to 118 while a profit of 120.5 is possible with prices of \$6 and \$8.75.
- ◆ Price differentiation alone (without optimization) is a profitable strategy.



# Perfect Price Differentiation is an Utopia

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- ◆ There are limitations to achieve perfect price differentiation
  - Imperfect segmentation: Are you willing to pay high or low? This is hard to answer on your own. Your willingness to pay can change depending on various factors, some are objective (such as income) and other are subjective (such as mood).
    - » Tactics to identify and segment the customers are very important.
  - Cannibalization: Customers with high willingness to pay may discover the low cost alternative and purchase that alternative. All of the À la Carte customers at Nazar restaurant may buy the lunch buffet. Then the lunch buffet demand  $d_1(p=6)=44$  due to high paying customers and the lunch buffet demand  $d_2(p=6)=8$  due to low paying customers. This gives a profit of only  $52=52(6-5)$  which is far below the original profit of 112.5 without price differentiation.
    - » Tactics to identify and segment the customers are very important.
  - Arbitrage: Third party arbitrageurs buy the product at low price and sell it to high willingness to pay customers.
    - » Tactics to identify and segment the customers are very important.
  
- ◆ When segmentation fails due to customer effort, it is cannibalization. When it fails due to third party effort, it is arbitrage.

# Tactics to Segment Customers

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## ◆ Group pricing: Student discounts

- Need
  - Unambiguous indicator of group membership: UTD-ID card
    - Experience Dallas Program of Comet Center
  - High correlation between group membership and willingness to pay
  - Difficult to trade of product/services among customers to avoid cannibalization and arbitrage. Services are difficult to trade.
  - Legally and culturally accepted grouping: Lower health care premiums for nonsmokers??

## ◆ Channel pricing

- Low cost distribution in some channels like Internet
  - Tablet users are impulse buyers who buy in large quantities.
    - Tablet users' conversion rate (= # of purchases/# of visits) = 4-5%. Traditional PC users' conversion rate=3%. Tablet users place 10-20% larger orders than PC users.
    - iPad uses apps but not Flash software when browsing.
 

Source: Tablets: Ultimate Buying Machines by D. Mattioli in WSJ, Sep 28, 2011.
- Customer's channel choice indicates willingness to pay: Outlet stores.

## ◆ Regional pricing

- Costs are higher in affluent areas so are the prices.
- Willingness to pay are higher in affluent areas so are the prices.

# Tactics to Segment Customers

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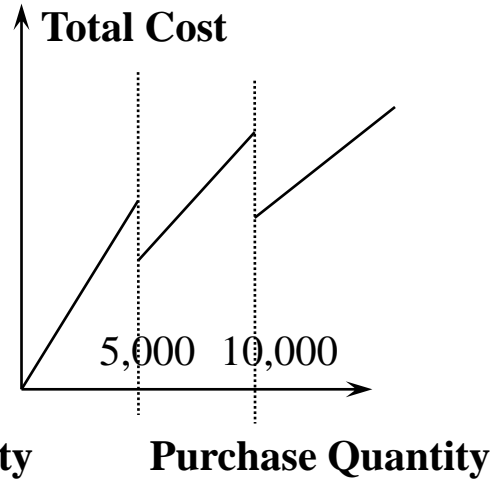
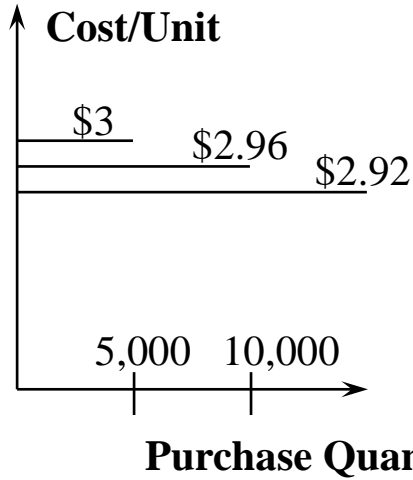
- ◆ **Coupons and Self-Selection**
  - Those willing to make extra effort to get the discount are price sensitive
    - Manufacturer issued mail-in discount/rebate coupons
    - Not everybody mails the coupon
    - Sometimes no rebates are given even after coupon is mailed in – make the effort
- ◆ **Versioning**
  - Using minor differences between version of products to exploit price sensitivity
  - Adding minor features to create a superior good from an inferior one
    - Proctor -Silex iron ready button
  - Deleting minor features to create an inferior good from a superior one
    - Disabled math processor of Intel 486DX processor
  - Creating product lines
    - Software with basic-pro-premier-enterprise editions
- ◆ **Time-based differentiation**
  - Faster delivery costs more

# Volume Discounts

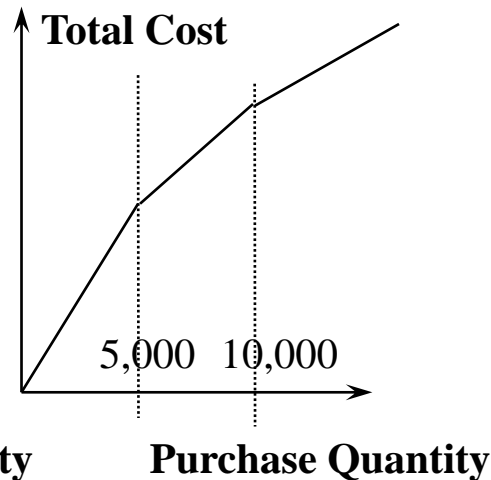
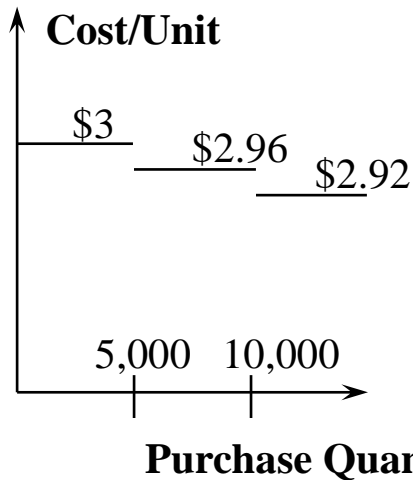
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- ◆ Per unit price of an item decreases when many more items are purchased.
- ◆ Why to discount?
  - Cost based pricing: Cheaper to produce/transport items in big quantities.
    - Recall Millie's cider shop and allocation of rent to cider bottles. If Millie sells more ciders, she allocates less of her rent to a single cider bottle.
  - Value based pricing: Marginal utility decreases for the customer.
    - First glass of the water is the most valuable when you are thirsty.
  - Indicator of price sensitive customers: These customers buy in big quantities.
    - Costco customers are more price sensitive than Target customers.

# Two Discounting Schemes



- ◆ All-unit quantity discount
  - Discount is applied to **every** unit



- ◆ Marginal-unit quantity discount, or
- ◆ Incremental quantity discount
  - Discount is applied to **additional** units

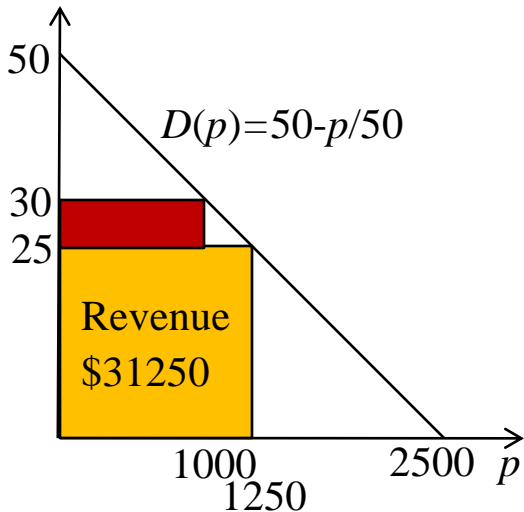
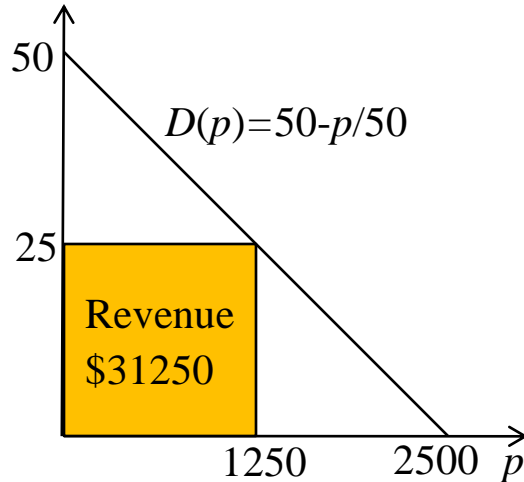
Other uses of discounting schemes are in  
OPRE 6366 Supply Chain Management.

# Decreasing Marginal Value of Number of CPA Certified Employees

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- ◆ CPA is a common accounting certification. A community college offers CPA classes for companies at the premises of the companies. This is convenient for the personnel of the companies.
- ◆ A certain bank has 50 employees at the level of associate, manager and senior manager, assistant director who consider taking the CPA classes if they are offered at the bank's premises. The accounting personnel at the level of director and above already have a CPA.
- ◆ The utility of the CPA class for employees are not the same.
  - An assistant director requires a CPA to be promoted to be a director so the utility of the CPA class is high for him/her.
  - Associates can be promoted all the way up to an assistant director over time without a CPA so the utility of the CPA class is low for them.
  - The community college estimates the utility of the class to range uniformly over \$0-\$2500 for the employees.
- ◆ Using the number of employees interested in CPA and its utility estimate, the community college forecasts the demand to be  $D(p)=50(1-p/2500)=50-p/50$ .
- ◆ Revenue  $R(p)=50p-p^2/50$  so the revenue maximizing price is found from solving  $50-p/25=0$  and it is \$1250.

# Cost of CPA Class by an Marginal-unit Discounting

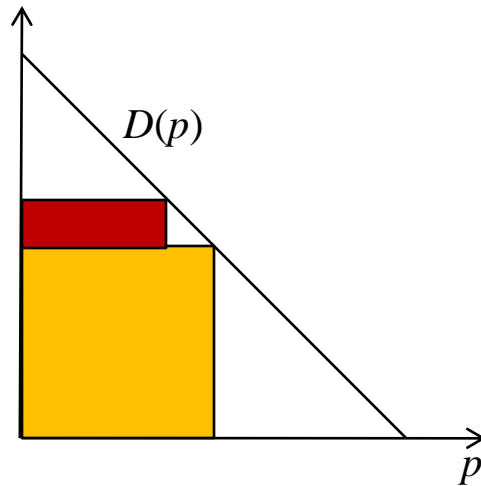


- ◆ With the price of \$1250 per employee, 25 bank employees take the CPA class.
- ◆ Community college makes a revenue of \$31,250 by offering classes at the premises of the bank.
- ◆ For the community college, the cost of offering class to 25 or to 30 people is identical. The college computes that if the price were \$1000, 30 people would take the course. This gives a revenue of only \$30,000.
- ◆ To obtain more than the revenue of \$31250 and to enroll 30 people, the college offers a volume discounting scheme:
  - Each of the first 25 people pay \$1250
  - Each additional person pays \$1000.
- ◆ This discounting scheme provides the bank with an additional revenue of \$5000, represented by the red box on the left.

# Market Segmentation vs. Volume Discounting

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- ◆ Pictures that have boxes representing revenue/profit underneath the demand curve are common in market segmentation and volume discounting. Each box indicates some more revenue.



- ◆ If all we are doing with both market segmentation and volume discounting is inserting boxes under the demand curve, can we apply both in the same buyer-seller context?
  - Either you say “yes”,
  - Or you say “no” and explain the difference(s) between the contexts.

# Calculating Differentiated Prices by Eliminating Arbitrage

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- Arbitrage happens when a lower priced product  $i$  is
  - transported to be sold as a higher priced product  $j$ 
    - transportation cost  $a_{ij}$
  - upgraded to be sold as a higher priced product  $j$ 
    - upgrade cost  $a_{ij}$
  - inventoried to be sold as a higher priced product  $j$ 
    - inventory holding cost  $a_{ij}$
- In all the cases above, we eliminate the arbitrage opportunity with the constraint

$$p_j \leq p_i + a_{ij}$$

on the price decision variables.

- The constraint makes the cost of using arbitrage  $p_i + a_{ij}$  higher than the price  $p_i$ .
- The constraint can be called as “arbitrage elimination constraint”.

# Motivating Example of Chips

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- Intel chips are sold both in US (country 1) and in Brazil (country 2). The demand functions are given by

$$d_1(p) = a_1 - b_1 p = 508000 - 100000p \quad \text{and} \quad d_2(p) = a_2 - b_2 p = 48600 - 10000p$$

- The transportation cost per chip is \$0.08 between these countries.
- With the constant slope demand curves (lines), the revenue maximizing prices are given as

$$p_1^0 = \frac{a_1}{2b_1} = 2.54 \quad \text{and} \quad p_2^0 = \frac{a_2}{2b_2} = 2.43 \quad \text{but} \quad p_1^0 = 2.54 > 2.51 = p_2^0 + a_{21}$$

Since the constraint is not satisfied, an arbitrageur can buy chips in Brazil and transport them to US to sell them in US.

- We need to jointly optimize prices in US and Brazil to eliminate arbitrage rather than separately as we have done above.

# Quadratic Program Formulation to Eliminate Arbitrage

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- When we have linear demands  $d_i(p_i)$ , the profit  $(p_i - c_i)d_i(p_i)$  is a quadratic function (it is a polynomial of degree 2). Summing these profits over  $N$  regions, we still have a quadratic objective for the joint optimization problem:

$$\max_{p_1 \dots p_N} \sum_{i=1}^N (p_i - c_i) d_i(p_i)$$

subject to

$$p_j \leq p_i + a_{ij} \quad \text{for } 1 \leq i \neq j \leq N$$

$$p_i \geq 0 \quad \text{for } 1 \leq i \leq N$$

- This is a quadratic programming problem and it can be solved efficiently with algorithms similar to those used to solve linear programming problems.

# Solving Quadratic Program for $N=2$

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- When there are two regions (US and Brazil),  $N=2$  and we can solve the quadratic program without a software.

$$\max_{p_1, p_2} p_1(a_1 - b_1 p_1) + p_2(a_2 - b_2 p_2)$$

subject to

$$p_1 \leq p_2 + a_{21} \quad \text{and} \quad p_2 \leq p_1 + a_{12}$$

$$p_1, p_2 \geq 0$$

- If the separately found prices satisfy the arbitrage eliminating constraints, they are optimal and we stop.
- Else let the region with higher price be 1 and the other be 2, we have

$$p_1^0 > p_2^0 + a_{21} \quad \text{and} \quad p_2^0 \leq p_1^0 + a_{12}$$

- In the optimal solution, the violated constraint must be satisfied as an equality, so we can let

$$p_1 = p + a_{21} \quad \text{and} \quad p_2 = p \quad \text{for some } p \text{ that we shall find next}$$

- This reparameterization reduces the number of decision variables to one and hence simplifies the solution procedure.

# Solving Quadratic Program for $N=2$

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- After price reparameterization, the constraints will hold and we focus on the objective

$\max_p (p + a_{21})(a_1 - b_1(p + a_{21})) + p(a_2 - b_2p)$  whose derivative is

$a_1 - 2b_1(p + a_{21}) + a_2 - 2b_2p = 0$  which yields

$p = \left( \frac{a_1 + a_2}{2} - b_1 a_{21} \right) \frac{1}{b_1 + b_2}$  this can be shown to be nonnegative

by using the violated constraint  $a_{21} < p_1^0 - p_2^0$ .

This argument also leads to  $p_2 = p > p_2^0$ .

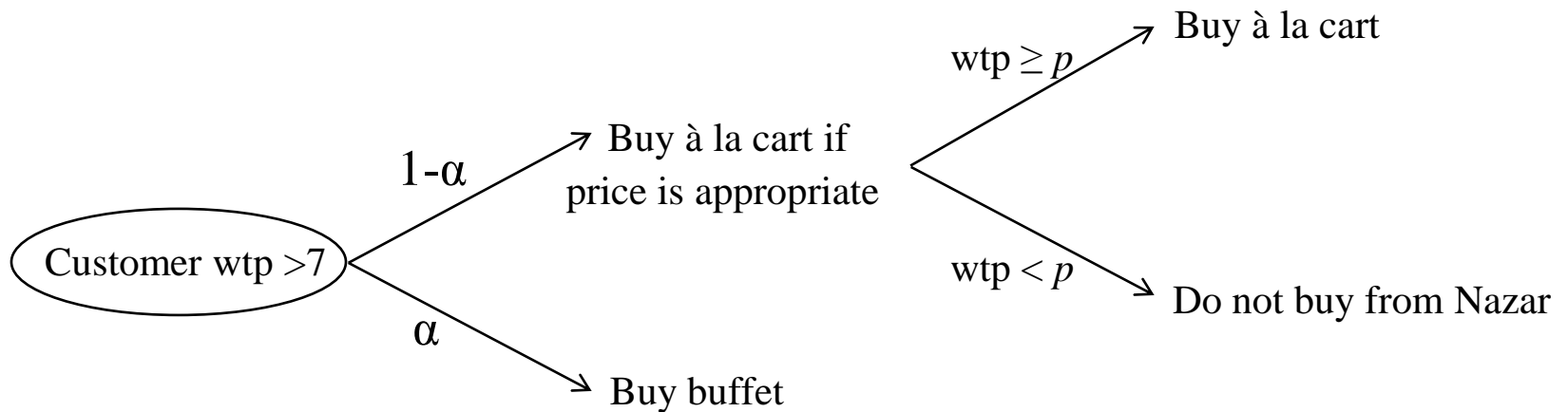
- Plugging in the parameters of US-Brazil pricing problem, we obtain

$$\begin{aligned}
 p &= \left( \frac{a_1 + a_2}{2} - b_1 a_{21} \right) \frac{1}{b_1 + b_2} = \\
 &= \left( \frac{508000 + 48600}{2} - 10000(0.08) \right) \frac{1}{100000 + 10000} = 2.457
 \end{aligned}$$

$$p_1 = 2.457 + 0.08 = 2.537 \text{ and } p_2 = 2.457$$

# Calculating Differentiated Prices by Incorporating Cannibalization

- ❑ Market segmentation often assumes that markets are perfectly segmented in that higher wtp (willingness to pay) customers do not buy at lower price.
- ❑ Recall Nazar restaurant, some à la cart customers can buy lunch buffet so cannibalization can happen.
- ❑ Suppose that  $0 \leq \alpha \leq 1$  percentage of high wtp buy at lower price.
- ❑ Cannibalization alters the demands at Nazar restaurant from



$$\begin{aligned}
 d_1(p) &= \min\{44, (100 - 8p)^+\} & \text{to} & & d_1(p) &= (1 - \alpha) \min\{44, (100 - 8p)^+\} \\
 d_2(p) &= (56 - 8p)^+ & & & d_2(p) &= (56 - 8p)^+ + 44\alpha
 \end{aligned}$$

# Calculating Differentiated Prices by Incorporating Cannibalization

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- The optimal price for à la cart service is found by maximizing

$$(p_1 - 5)(1 - \alpha) \min\{44, (100 - 8p_1)^+\} = (p_1 - 5)(1 - \alpha)(100 - 8p_1) \quad \text{for } 7 \leq p_1 \leq 12.5$$

The optimal price satisfies  $(1 - \alpha)(140 - 16p_1) = 0$ , so  $p_1 = 8.75$ .
  
- The optimal price for lunch buffet is found by maximizing

$$(p_2 - 5)((56 - 8p_2)^+ + 44\alpha) = (p_2 - 5)(56 + 44\alpha - 8p_2) \quad \text{for } 0 \leq p_2 \leq 7$$

The optimal price satisfies  $(96 + 44\alpha - 16p_2) = 0$ , so  $p_2 = \min\{7, 6 + 2.75\alpha\}$ .

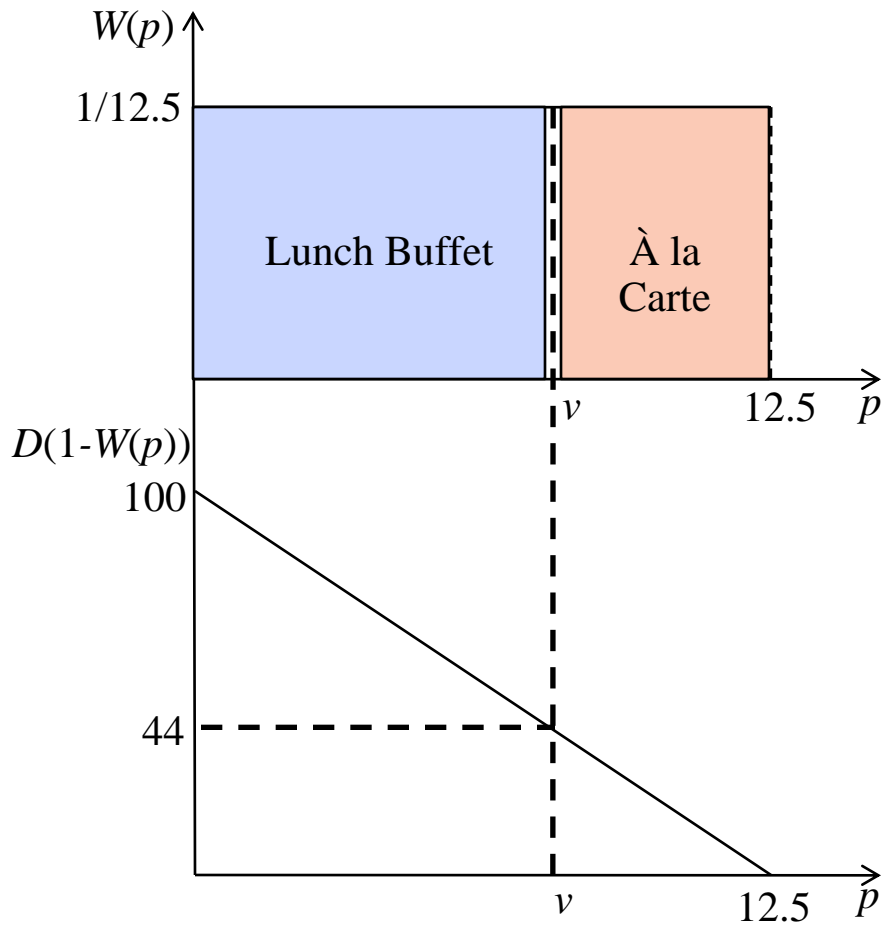
# Effect of Cannibalization on Total Profit

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- Suppose  $\alpha=0.1$ , profit from à la cart service is  $(8.75-5)(1-0.1)(100-8(8.75))=101.25$
- Then the profit from the lunch buffet is  $(6.275-5)(60.4-8(6.275))=13.005$ .
- Profit with optimal segment prices under cannibalization  $\alpha=0.1$  is 114.255
- The same profit without cannibalization is 120.5 and without segmentation is 112.5.
  
- Suppose  $\alpha=0.2$ , profit from à la cart service is  $(8.75-5)(1-0.2)(100-8(8.75))=90$
- Then the profit from the lunch buffet is  $(6.55-5)(64.8-8(6.55))=19.22$
- Profit with optimal segment prices under cannibalization  $\alpha=0.2$  is 109.22.
- Profit of 109.22 is less than 112.5 obtained without segmentation.
  
- Cannibalization can **quickly wash away the benefits** of segmentation.
  
- Should the cannibalization depend on the price differential  $p_1-p_2$ ?

# Best Segmentation

- ◆ Nazar restaurant divided its market into two: those paying below and above \$7. Is this value of  $v=7$  the best price to segment the market?
- ◆ We need to generalize our previous analysis.



$$\begin{aligned}
 d_1(p; v) &= D_1(1 - W_1(p)) \\
 &= 100 \frac{12.5 - v}{12.5} \left( 1 - \frac{(p - v)^+}{12.5 - v} \right) \\
 &= 8(12.5 - p) \quad \text{for } v \leq p \leq 12.5
 \end{aligned}$$

$$\begin{aligned}
 d_2(p; v) &= D_2(1 - W_2(p)) \\
 &= 100 \frac{v}{12.5} \left( 1 - \frac{p - 0}{v} \right)^+ \\
 &= 8(v - p) \quad \text{for } 0 \leq p \leq v
 \end{aligned}$$

# Best Segmentation

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- ◆ Formulation of the segmentation problem

$$\max_v \left\{ \max_{p_1} (p_1 - 5)(100 - 8p_1) + \max_{p_2} (p_2 - 5)(8v - 8p_2) \right\} \quad \text{for } v \leq p_1 \leq 12.5 \text{ and } 0 \leq p_2 \leq v$$

- ◆ For a fixed  $v$ , the optimal prices are

$$p_1 = \max\{v, 8.75\} \text{ and } p_2 = \min\{v, (v+5)/2\}$$

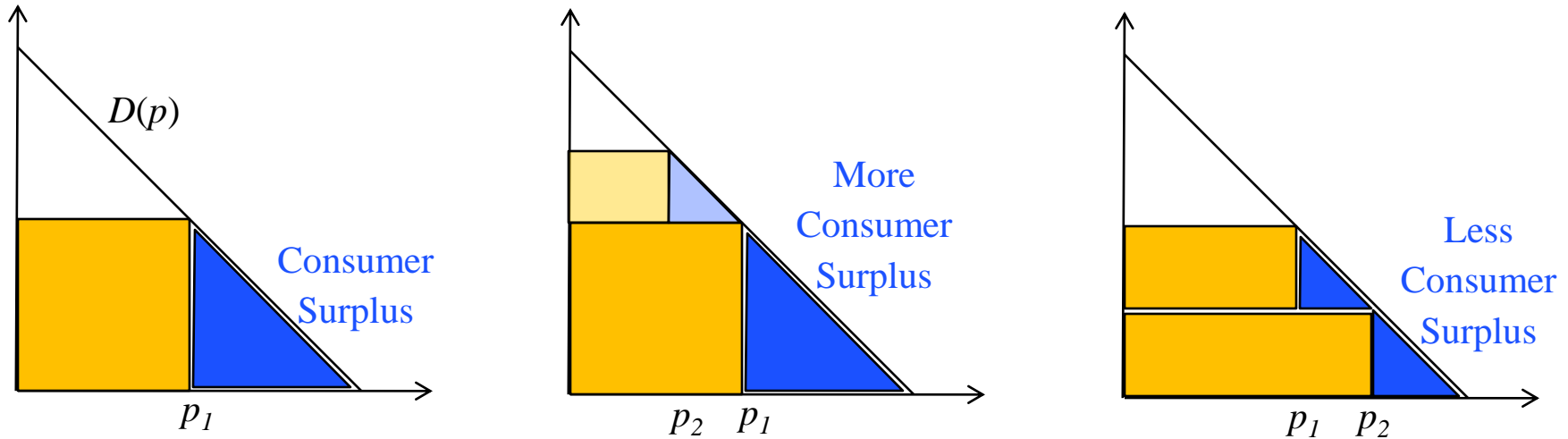
- ◆ Since  $v > 5$ ,  $p_2 = (v+5)/2$ . The objective then is

$$\max_v \left\{ (\max\{v, 8.75\} - 5)(100 - 8 \max\{v, 8.75\}) + (\min\{v, (v+5)/2\} - 5)(8v - 8 \min\{v, (v+5)/2\}) \right\}.$$

- ◆ This objective can be evaluated in Excel for different values of  $v$  to find optimal  $v$ .
- ◆ See `best_segmentation.xlsx` on the course web site.
- ◆ The optimal segmentation happens with  $v = \$10$ .

# Consumer Surplus

- ◆ For each consumer, the positive part of the difference between wtp and the product price is the surplus:  $(wtp-p)^+$ .
- ◆ Summing this up for all consumers, we find total consumer surplus. Graphically,



- ◆ Consumer surplus is positive only for the consumer buying the product.
- ◆ It can be increased by selling to more customers with a lower price; see the middle figure above.
- ◆ It can be decreased by selling to the same number of customers with a higher price; see the figure on the right above.

# Price Differentiation

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## Summary

- ◆ **Price Differentiation: Limitations and Tactics**
- ◆ **Volume Discounts**
- ◆ **Optimal Pricing with**
  - Arbitrage
  - Cannibalization
  - Market segmentation
- ◆ **Consumer Welfare**

# Legally Eliminate Arbitrage

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- ❑ Michigan State offers 10 cents for each used beverage container. The other states do not offer any cash or less such as New York States 5 cents.
- ❑ It is tempting to collect containers in other states, to bring them to Michigan and to deposit them for 10 cents in grocery stores such as Save Plus Superstore in Pontiac, The Larosa Market In Sylvan Lake and Value Foods in Ypsilanti, The Farmer John, Savemart Food Center and the Americana foods, all three in Detroit.
- ❑ This is illegal as it frauds Michigan Bottle Deposit Fund, used to pay for environmental clean up.
- ❑ A pop-can smuggling ring has been arraigned in Michigan. 15 man ring face charge that include maintaining a criminal enterprise (20 year felony) and fraud (5 year felony).
  - ❑ Suspects Arraigned In Pop Can Smuggling Ring. Click on Detroit, Sep 26, 2007.  
<http://www.clickondetroit.com/news/14214576/detail.html>.
- ❑ On a lighter side, watch a 1996 episode of Seinfeld: “**The Michigan Deposit Bottle Scam**” from You Tube <http://www.youtube.com/watch?v=x1blsZxXDCU>
- ❑ Gist of the scam: Newman, who has spent days trying to calculate a profit to the deposit scheme, realizes that there will be a surge of mail the week before Mother’s Day (the "mother of all mail days") to be sorted in Saginaw, Michigan. He tells Kramer that he signed up for a mail truck that would carry spillover mail from the other four main trucks, leaving plenty of space left over in theirs for bottles and cans to refund in Michigan. Kramer realizes that by avoiding truck rental fees, Newman has found a loophole and they set off collecting cans and bottles around the city.
  - ❑ For the info, thanks to Seinfeld fan Osman Kazan, DemReMan 2011 Teaching Assistant.