OPRE 6366 : Quiz on 11 October 2011

This is an open textbook and open lecture notes exam. You may use a calculator although leaving quantities as fractions, additions or products is perfectly acceptable and preferable. **No cellular communication devices** (laptops, phones, etc) can be used during the exam. **Do not forget** to define any variables you introduce. This quiz has 4 bonus points. Good luck ...

NAME (please print): ______________________________

Question	Out of	Points
A. True-False	44	
B. Short Answer	39	
C. Essay-Type	21	
Total	104	

A. Put **T** before a statement if you think that statement is true. Otherwise put **X**.

1. These questions are removed.

B. Short Answer (1-line) Questions.

1. During the lunch time, a student enters the UTD cafeteria every 10 seconds and each student spends an average of 40 minutes in the cafeteria. How many students do you expect to find in the cafeteria at an arbitrary time during the lunch time?

   ANSWER: Arrival rate is 6 per minute; time in the syetme is 40 minutes. The inventory is 240=6*40 by the Little's Formula.

For the next 6 questions, consider the location/transportation formulation below where $x_{ij} = 1$ if market j is supplied by plant i and 0 otherwise; $y_i = 1$ if plant i is opened and 0 otherwise.

Objective: Minimize $12000y_1 + 10000y_2 + 4800x_{11} + 4800x_{12} + 6000x_{13} + 4000x_{21} + 8400x_{22} + 10800x_{23}$,

Constraint type A: $x_{11} + x_{21} = 1$; $x_{12} + x_{22} = 1$; $x_{13} + x_{23} = 1$;

Constraint type B: $40x_{11} + 60x_{12} + 30x_{13} \leq 120y_1$; $40x_{21} + 60x_{22} + 30x_{23} \leq 100y_2$.

2. How many markets and plants are there in this formulation?

   ANSWER: 3 markets and 2 plants.

3. What are the demands in the markets?

   ANSWER: Demands are 40, 60 and 30 for the first, second and third markets.

4. What are the costs of opening up plants?

   ANSWER: 12000 for the first plant, 10000 for the second plant.

5. What are the capacities of the plants?

   ANSWER: 120 for the first plant, 100 for the second plant.

6. What is transportation cost per unit for shipping from plant 1 to market 2?

   ANSWER: $4800x_{12}$ is the total transshipment cost of shipping all of the demand for market 2 from plant 1. Demand of market 2 is 60. Transportation cost $1 \to 2$ is 80=4800/60.

7. What is transportation cost per unit for shipping from plant 2 to market 1?

   ANSWER: $4000x_{21}$ is the total transshipment cost of shipping all of the demand for market 1 from plant 2. Demand of market 1 is 40. Transportation cost $2 \to 1$ is 100=4000/40.

For the next 2 questions, consider the inventory level I_t and backorder level S_t at the end of period t along with production P_t and demand D_t that happen during period t. Some of these numbers are given as below for a 3 period problem:

Period t	I_{t-1}	S_{t-1}	P_t	D_t
1	u	10	40	x
2	15	y	z	40
3	w	10	15	35

8. What are x, y values in the table above?

   ANSWER: x=15, y=0.

9. What are z, w values in the table above?

   ANSWER: z=15, w=0.

For the next 4 questions, consider a version of the transportation problem below. There are two markets. The demand at the first market is 60 with probability 0.3 and 70 with probability 0.7. Demand at the second market is also random. Thee are two demand scenarios but the supplies are the same under both scenario.

Objective: Minimize $0.3[2x_{11}^1 + 3x_{12}^1 + 4x_{21}^1 + 5x_{22}^1] + 0.7[2x_{11}^2 + 3x_{12}^2 + 4x_{21}^2 + 5x_{22}^2]$,

Constraint type A: $x_{11}^1 + x_{21}^1 = 60$; $\quad x_{12}^1 + x_{22}^1 = 40$; $\quad x_{11}^2 + x_{21}^2 = 70$; $\quad x_{12}^2 + x_{22}^2 = 50$;

Constraint type B: $x_{11}^1 + x_{12}^1 \leq 80$; $\quad 40x_{21}^1 + 60x_{22}^1 \leq 50$; $\quad x_{11}^2 + x_{12}^2 \leq 80$; $\quad 40x_{21}^2 + 60x_{22}^2 \leq 50$;

Constraint type C: $x_{11}^1, x_{12}^1, x_{21}^1, x_{22}^1, x_{11}^2, x_{12}^2, x_{21}^2, x_{22}^2 \geq 0$.

10. What are the possible values of demand at the second market and what are the probabilities associated with those demands?

    ANSWER: The demand at the second market is 40 with probability 0.3 and 50 with probability 0.7.

11. Express variable x_{21}^2 in English.

    ANSWER: x_{21}^2 is the amount shipped from second supplier to the first market under the second scenario.

12. Is variable x_{21}^2 anticipatory or non-anticipatory? No explanation necessary.

    ANSWER: It depends on scenario so it is anticipatory.

13. Are the supplies anticipatory or non-anticipatory? No explanation necessary.

    ANSWER: They are the same under both scenarios so they are non-anticipatory.

C. [*Tailored two product delivery*] A retailer buys products A and B from a supplier. Product A is demanded a lot with respect to product B so the retailer receives a truck of product A delivery every Monday. It is **not** clear whether B should be in every truck (relative frequency $n_B = 1$). When product B is carried on a truck, there is a loading/unloading (handling) cost of \$4000 for the retailer. Moreover, the weekly demand for B is 800 units and the holding cost rate per week per unit is \$1 at the retailer. The fixed cost of handling product B is $s = 4000$. Let us fix time unit to a single week. Then demand is $R = 800$ per week, and the holding cost $H = 1$ per week per unit.

[4*pts*] a) Using the relative order frequency n_B of product B, express the total B handling and inventory holding cost over n_B weeks.

$$TC(n_B; \text{over } n_B \text{ weeks}) =$$

ANSWER: With a relative order frequency of n_B, product B is ordered very other n_B weeks. Thus, the order size must suffice for n_B weeks. The order size then is $n_B R$. Since the retailer inventory at maximum is $n_B R$ and drops to zero at the uniform rate of R, the average inventory is $n_B R/2$.

$$\begin{aligned} TC(n_B; \text{over } n_B \text{ weeks}) &= \underbrace{s}_{\text{Fixed ordering cost}} + \underbrace{\underbrace{n_B R/2}_{\text{Average inventory}} \underbrace{n_B}_{\text{kept } n_B \text{ weeks}} \underbrace{hC}_{\text{at the holding cost rate } hC}}_{\text{Inventory holding cost over } n_B \text{ weeks}} \\ &= 4000 + \frac{1}{2} 800(1) n_B^2, \end{aligned}$$

where $H = hC = 1$ is holding cost per item per week.

[3*pts*] b) Use $TC(n_B; \text{over } n_B \text{ weeks})$ above to express the total cost (handling and holding) over 1 week. This is the average cost incurred per week.

$$TC(n_B; \text{over 1 week}) =$$

ANSWER:

$$TC(n_B; \text{over 1 week}) = \frac{TC(n_B; \text{over } n_B \text{ weeks})}{n_B} = \frac{s}{n_B} + \frac{1}{2} R hC n_B = \frac{4000}{n_B} + \frac{1}{2} 800(1) n_B.$$

[4*pts*] c) When finding the relative frequency n_B^0, decide whether we should minimize $TC(n_B; \text{over } n_B \text{ weeks})$ or $TC(n_B; \text{over 1 week})$. In a sentence, explain the rationale behind your decision. By taking the derivative of the appropriate objective function and setting equal to zero, compute n_B^0.

ANSWER: When finding the relative frequency, we cannot use an objective that depends on the relative frequency; That is circular logic. In management, our objectives are often computed over fixed and independent (from the decisions) time intervals. Such a time interval is 1 week. If you wish you can also use 2 weeks or 5 weeks as your time intervals, in these cases you must multiply the objective below by 2 and 5 respectively and recall that multiplying an objective by a positive number does not affect the outcome of the optimization.

We minimize $TC(n_B; \text{over 1 week})$ to find n_B:

$$\frac{d}{dn_b} TC(n_B; \text{over 1 week}) = -\frac{4000}{n_B^2} + \frac{1}{2} 800(1) = 0.$$

So,

$$n_B = \sqrt{\frac{4000}{400}} = \sqrt{10}$$

[4*pts*] d) The frequency n_B^0 computed in c) may not be an integer. Suppose that it is $n_B^0 = 5/3$. Intuitively this may call for three relative frequencies $n_B^1 = 1$, $n_B^2 = 1$, $n_B^3 = 3$ and trucking cycles with period of 5 weeks:

Cargo of a truck on each week														
5-week cycle 1					5-week cycle 2					5-week cycle 3				
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
B	B	B			B	B	B			B	B	B		

Above table indicates that in the 1st, 2nd and the 3rd weeks of the 5-week cycles A and B are received, in the 4th and 5th week only A is received. Express and compute the total cost (handling and holding) B over 5 weeks that will result from the schedule above. Also compute the "average" total cost incurred per week. In particular, $TC(\text{Schedule in the table; over 5 weeks}) = ?$ $TC(\text{Schedule in the table; over 1 week}) = ?$

ANSWER:

$$\begin{aligned} TC(\text{Schedule in the table; over 5 weeks}) &= \overbrace{4000 + \underbrace{\frac{1}{2}800(1)1^2}_{n_B^1=1}}^{Week\ 1} + \overbrace{4000 + \underbrace{\frac{1}{2}800(1)1^2}_{n_B^2=1}}^{Week\ 1} + \overbrace{4000 + \underbrace{\frac{1}{2}800(1)3^2}_{n_B^3=3}}^{Weeks\ 3\text{–}4\text{–}5} \\ &= 12000 + 4400 = \$16,400 \end{aligned}$$

$$TC(\text{Schedule in the table; over 1 week}) = \frac{TC(\text{Schedule in the table; over 5 weeks})}{5} = \$3280$$

[3*pts*] e) Compute $TC(n_B = 5/3; \text{over 1 week})$ by using your expression in b).

ANSWER:

$$TC(n_B; \text{over 1 week}) = \frac{4000}{5/3} + \frac{1}{2}(800)(1)5/3 = 2400 + 2000/3 \approx \$3066.$$

[3*pts*] f) Compare $TC(\text{Schedule in the table; over 1 week})$ in d) and $TC(n_B = 5/3; \text{over 1 week})$ in e). Explain if they need to be the same.

ANSWER: The costs are not the same. Although the average of $n_B^1 = 1$, $n_B^2 = 1$, $n_B^3 = 3$ is $n_B^0 = 5/3$, the contribution of longer cycles to the holding cost is not proportional to the length of the cycle. Indeed, this contribution is proportional to the square of the cycle length. A longer B-cycle like $n_B^3 = 3$ causes disproportionately more costs than their lengths. Since the schedule in the table has such a cycle, we expect $TC(\text{Schedule in the table; over 1 week}) > TC(n_B = 5/3; \text{over 1 week})$. Note that $n_B^0 = 5/3$ is fractional and cannot be implemented, $TC(\text{Schedule in the table; over 1 week}) - TC(n_B = 5/3; \text{over 1 week}) > 0$ can be called the implementability cost. To understand this cost and its magnitude more, consider another solution whose average is still $n_B^0 = 5/3$, for example compute the weekly cost for $n_B^1 = 1$, $n_B^2 = 2$, $n_B^3 = 2$ illustrated below.

Cargo of a truck on each week														
5-week cycle 1					5-week cycle 2					5-week cycle 3				
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
B	B		B		B	B		B		B	B		B	

Is the schedule above different from the one below? (Hint: consider starting a cycle on week 3.)

Cargo of a truck on each week														
5-week cycle 1					5-week cycle 2					5-week cycle 3				
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
B		B	B		B		B	B		B		B	B	