OPRE 6302 Quiz [1]

I understand that

- This is a closed books/notes exam but I can use a calculator.
- A cheat sheet including complex formulas is provided so I will NOT bring my own.
- I will choose the most appropriate answer for the questions. I will NOT get any credit by marking multiple choices for a single question. When changing my answers, I will erase the paper properly.
- I will NOT forget to define any variables I introduce.
- I will NOT use laptops, cellular phones, any cellular communication device, and I will turn off all these devices before starting my exam.
- My conduct during the exam will reside entirely within the limits of the UTD regulations governing scholastic honesty -detailed in the handbook of operating procedures Title V Chapter 49.

NAME (please print): ______________________________

Question	Out of	Points
1		
2		
3		
Total	100	

Q1. Put **T** before a statement if you think that statement is true. Otherwise put **X**.

1. (   ) Economic order quantity cannot decrease with lower set up costs. X
2. (   ) Production mix at a facility can change the bottleneck. T

[1] This is a sample as such it only indicates the type of the questions but not the number of questions

Q2: Choose the most appropriate answer and mark your answer on this paper.

1. Heuristic rules are usually applied in practice when a problem

   (a) Deals with high costs or revenues
   (b) Has a small number of alternative solutions
   (c) Is important for the upper management
   (d) Has a large number of alternative solutions
   (e) Both a and c

   D

2. A production line is to be designed for a job with three tasks. Task times are 0.5, 0.7 and 0.9 minutes. The maximum and minimum possible cycle times are:

   (a) 0.9, 0.5
   (b) 2.7, 1.5
   (c) 2.1, 0.9
   (d) 2.1, 1.5
   (e) 1.6, 0.5

   C

3. In an assembly operation at a furniture factory, seven employees assemble an average of 350 standard dining chairs per 5-day week. What is the labor productivity of this operation?

   (a) 50 chairs/(worker x week)
   (b) 70 chairs/week
   (c) 50 chairs/week
   (d) 10 chairs/day
   (e) 70 chairs/worker

   A

4. Determine which of these are true.
   I: Labor content of the final good (product/service) is higher in service industries than in manufacturing.
   II: Measuring productivity is easier in service than in manufacturing.
   III: Standardization can be achieved easily in manufacturing than in service.

   (a) I, II and III are all true
   (b) Only I and II are true
   (c) Only III is true
   (d) Only I and III are true
   (e) None are true

   D

5. Which of the following is typically dealt by operations managers?

   (a) Facility location
   (b) Set of products to be offered
   (c) Motivation of workers
   (d) Scheduling
   (e) All of the above

   E

6. Models are **not** built to
   (a) Improve communication
   (b) Make concepts more abstract
   (c) Allow experimentation
   (d) Standardize the situation at hand for analysis
   (e) Improve understanding

   B

7. In queeing, pooling refers to
   (a) Putting different customer classes into a single class
   (b) Replacing a server with a faster server
   (c) Sending overflowing customers from one server to another
   (d) Allowing for only one queue for multiple servers
   (e) Aggregating customer service times to end up with an average service time

   D

8. Erlang's thruput loss formula gives losses due to
   (a) Server breakdowns
   (b) Impatient customers leaving the queue after waiting for a certain amount of time
   (c) Customers not entering the queue due to busy servers and unavailable buffer space
   (d) Customers not entering the queue due to busy servers and full buffers
   (e) Customers not entering the queue due to long queue

   C

Essay type questions of the following form.

[Spring 11] TexEx (Texan Express) is a credit card company that collects check payments from three major regions in the U.S.: South (S), West (W) and East (E). The average daily value of payments mailed to TexEx are $120 K from customers in the South, $80 K from customers in the West and $100 K from customers in the East. TexEx must decide where these payments should be sent to. Three potential locations are San Angelo (s), Texas for the southern customers; Walla Walla (w), Washington for the western customers; Edison (e), New York for the eastern customers. Annual cost of operating a check-receiving office is $200 K in San Angelo, $120 K in Walla Walla and $250 K in Edison. If all offices are operated, TexEx will assign customer regions to offices as follows: S-s, W-w, E-e. However, given the cost of operating the offices, operating offices in all three locations may not be profitable.

[2*pts*] a) A check sent from anywhere in the South to San Angelo arrives in two days. What is the total value of checks at any time in the mail sent from the South to San Angelo?

ANSWER: The question gives thruput and time in the (postal) system and asks for inventory. The inventory is 2*120 K=240 K.

[2*pts*] b) This is independent of a). TexEx assesses the value of checks in the mail sent from the South to San Angelo. That value is $360 K but this amount is in the mail and can not be invested by TexEx. TexEx can earn 20% annually by investing the payments. What is the annual opportunity cost of not immediately investing the payments sent from the South to San Angelo?

ANSWER: TexEx has an inventory of $360 K in the mail and the annual cost of that is 0.2*360 K=72 K.

[4*pts*] c) This is independent of a) and b). TexEx can earn 10% annually by investing the payments. The annual opportunity costs of not immediately investing the payments sent from a region (S, W, E) to offices (s, w, e) are given by the following table.

K $	s	w	e
S	36	60	48
W	40	24	48
E	40	60	30

In this table, the numbers in the first row 36, 60, 48 are obtained by assuming in turn that southern customers are assigned to s, or w, or e. Similar comments apply to the remaining two rows. Customers in a region are all assigned to an office. If the office in San Angelo is not operated and southern customers are assigned to Edison, these customers know only Edison and cannot send payments to any other office. Use the above table to deduce the number of days it takes for a mail to arrive from a region to a city, and fill in the table below:

Days	s	w	e
S			
W			
E			

ANSWER:

Days	s	w	e
S	3=36/(0.1*120)	5=60/(0.1*120)	4=48/(0.1*120)
W	5=40/(0.1*80)	3=24/(0.1*80)	6=48/(0.1*80)
E	4=40/(0.1*100)	6=60/(0.1*100)	3=30/(0.1*100)

Each entry worths 4/9 points.

[4pts] d) The rest of this question depends on c). To assign regions to offices (x) and to determine operating offices (y), TexEx needs a formulation. Define the decision variables (x, y) with appropriate subscripts/superscripts for this formulation and decide if they are fractional (continuous), integer or binary.
ANSWER: To determine operating offices, we use binary variables y_s, y_w, y_e. If $y_s = 1$, open the San Angelo office; otherwise do not. Other y variables are defined similarly. To determine region to office assignment, we use binary variables $x_{Ss}, x_{Sw}, x_{Se}, x_{Ws}, x_{Ww}, x_{We}, x_{Es}, x_{Ew}, x_{Ee}$.
The assignment of S-s, W-w, E-e is captured by $x_{Ss} = 1$, $x_{Ww} = 1$, $x_{Ee} = 1$ and all other x variables are zero.
The assignment of S-w, W-w, E-e is captured by $x_{Sw} = 1$, $x_{Ww} = 1$, $x_{Ee} = 1$ and all other x variables are zero.
The assignment of S-w, W-w, E-w is captured by $x_{Sw} = 1$, $x_{Ww} = 1$, $x_{Ew} = 1$ and all other x variables are zero.
In general, an x variable is indexed by a region and an office and it takes the value of 1 only when that region is assigned to that office.

[2pts] e) TexEx wants to minimize opportunity cost of not investing immediately plus the cost of operating offices. Express this objective using the decision variables defined in d).
ANSWER: Min $200y_s + 120y_w + 250y_e + 36x_{Ss} + 60x_{Sw} + 48x_{Se} + 40x_{Ws} + 24x_{Ww} + 48x_{We} + 40x_{Es} + 60x_{Ew} + 30x_{Ee}$.

[2pts] f) Write constraint(s) so that every region is assigned to a single office.
ANSWER:
$x_{Ss} + x_{Sw} + x_{Se} = 1$ for southern region;
$x_{Ws} + x_{Ww} + x_{We} = 1$ for western region;
$x_{Es} + x_{Ew} + x_{Ee} = 1$ for eastern region.

[2pts] g) Write constraint(s) so that regions are assigned to only operating offices.
ANSWER:
$x_{Ss} + x_{Ws} + x_{Es} \leq 3y_s$ can assign to San Angelo if it is operating;
$x_{Sw} + x_{Ww} + x_{Ew} \leq 3y_w$ can assign to Walla Walla if it is operating;
$x_{Se} + x_{We} + x_{Ee} \leq 3y_e$ can assign to Edison if it is operating.

[Spring 11] PlaIn insurance company of Plano has three claim adjusters in its main office. The office works 5 days per week. People with claims arrive at the office on an average rate of 20 per 8-hour day. An adjuster spends on average 40 minutes with a claimant. Assume that both service and interarrival times are exponentially distributed.
[6*pts*] a) What is the utilization of adjusters? How many hours per week an adjuster spends with a claimant? What is the number of available adjusters on average?
ANSWER: $a = 8 * 60/20 = 24$ mins, $p = 40$ mins, $m = 3$, $u = p/(am) = 40/(72) = 5/9 = 0.55$. An adjuster spends 40*0.55=22.2 hours per week with a claimant.
3*(1-5/9)=12/9=4/3 adjusters are available on average.

[4*pts*] c) How much time does a claimant expect to spend in the main office?
ANSWER:

$$\begin{aligned} T_q &= \frac{40}{3}\frac{(5/9)^{\sqrt{2(3+1)}-1}}{1-5/9}\frac{1^2+1^2}{2} \\ &= \frac{40}{3}\frac{9}{4}(5/9)^{\sqrt{8}-1} \\ &= 30(0.34) \\ &= 10.2 \text{ minutes of waiting for an available adjuster.} \end{aligned}$$

The service time is 40 minutes, adding this to the waiting time, a claimant spends about 50 minutes in the PlaIn office.

[2*pts*] e) Write an expression for the probability that a claimant will find all adjusters busy. Do not attempt to compute the probability.
ANSWER: $r = p/a = 40/24 = 5/3$. That probability with $m = 3$ is

$$\frac{(5/3)^3/3!}{\sum_{i=0}^{3}(5/3)^i/i!}.$$

[Spring 10] UTD has a safe-walk campus escort service (www.utdallas.edu/enroll/visit/safety.php) available 24 hours a day, 365 days a year. To request an escort call ext. 2331. Give your name and the nearest exit door to your location. A safe-walk escort will meet you at that building door and walk with you to any area on campus. Requests for escorts are received on average every 5 minutes, with a coefficient of variation of 1. After receiving a request, the dispatcher answering number 2331 contacts an available escort by a mobile phone. The available escort immediately proceeds to pick up the student and walk him/er to his/er destination. This in total takes about 25 minutes, with a coefficient of variation of 1. Currently there are 7 escorts at UTD. If they are all unavailable, the dispatcher puts the requests in a queue until an escort becomes available.
[3*pts*] a) What is the utilization of escorts?
ANSWER: $a = 5$, $p = 25$, $m = 7$, $u = p/(am) = 25/35 = 5/7 = 0.714$.

[3*pts*] b) What is the number of available escorts on average?
ANSWER: 7*(1-5/7)=2 escorts are available on average.

[5*pts*] c) What is the waiting time in the queue and how long does it take for a student to arrive his/er destination after placing a call to 2331?
ANSWER:

$$\begin{aligned} T_q &= \frac{25}{7} \frac{(5/7)^{\sqrt{2(7+1)}-1}}{1-5/7} \frac{1^2+1^2}{2} \\ &= \frac{25}{7} \frac{(5/7)^3}{2/7} \\ &= \frac{25}{2} \left(\frac{5}{7}\right)^3 \\ &= 4.55 \text{ minutes of waiting for an available escort.} \end{aligned}$$

The service time (pick up by escort and delivery to destination) is 25 minutes. The total time it takes to arrive the destination is 29.55 minutes.

[5*pts*] d) A recently established UTD security committee has decided that the maximum acceptable waiting time in the queue is 2 minutes. What is the minimum number of escorts to achieve this acceptable level?
ANSWER: Set $m = 8$, $u = 25/40 = 5/8$.

$$\begin{aligned} T_q &= \frac{25}{8} \frac{(5/8)^{\sqrt{2(8+1)}-1}}{1-5/8} = 3.125 \frac{0.635^{3.24}}{0.375} \\ &= 1.91 \text{ minutes of waiting for an available escort.} \end{aligned}$$

Thus, 8 escorts suffice.

[4*pts*] e) If UTD operates with 6 escorts, write an expression for the probability that a request will find all escorts busy. Do not attempt to compute the probability.
ANSWER: $r = p/a = 5$. That probability with $m = 6$ is

$$\frac{5^6/6!}{\sum_{i=1}^{6} 5^i/i!}.$$

[Spring 10] PlaPharma (PP) is a pharmaceutical company headquartered in Plano. It has 3 new products to be produced at 2 plants. Plants are in Frisco (F) and Grapevine (G). For product i, let x_{iF} and x_{iG} be the production amount at plant F and G. We are presented with a linear programming formulation for production planning:
Maximize $5(x_{1F}+x_{1G})+7(x_{2F}+x_{2G})+3(x_{3F}+x_{3G})$
ST: $3x_{1F}+4x_{2F}+2x_{3F}\leq 30$; $4x_{1G}+6x_{2G}+2x_{3G}\leq 40$ Work hours capacity in F and G, respectively.
$x_{1F}+x_{1G}\leq 7$; $x_{2F}+x_{2G}\leq 5$; $x_{3F}+x_{3G}\leq 9$ Sales potential of product 1, 2 and 3, respectively.
$x_{1F},x_{1G},x_{2F},x_{2G},x_{3F},x_{3G}\geq 0$.
All parts below are independent.

[2*pts*] a) (   ) This formulation assumes that same amount of profit is made from a product no matter where it is produced. Either write True (T) or False (X) into the parentheses. No explanation is necessary. ANSWER: T

[3*pts*] b) What is profit made from the sale of 1 unit of product 1? How many hours does it take to produce 1 unit of product 2 in Grapevine plant? What is the maximum number of units of product 3 that can be sold in the market? Write 3 numbers in the order of questions, no explanation is necessary: ..; ..; ... ANSWER: 5; 6; 9.

[5*pts*] c) The new vice president of operations wants to reduce the product offerings to avoid undue diversification of the product line. Modify the LP formulation to ensure that **at most two products** are produced.

ANSWER: Let $y_i=1$ if product i is produced; it is 0 otherwise. Modify the sales potential constraints:
$x_{1F}+x_{1G}\leq 7y_1$ Sales potential of product 1 if it is produced.
$x_{2F}+x_{2G}\leq 5y_2$ Sales potential of product 2 if it is produced.
$x_{3F}+x_{3G}\leq 9y_3$ Sales potential of product 3 if it is produced.
Add the new constraints: $y_1+y_2+y_3\leq 2$ and $y_1,y_2,y_3\in\{0,1\}$. Students need to write only the modifications above. For clarity, we present the complete formulation:

Maximize $5(x_{1F}+x_{1G})+7(x_{2F}+x_{2G})+3(x_{3F}+x_{3G})$
ST: $3x_{1F}+4x_{2F}+2x_{3F}\leq 30$; $4x_{1G}+6x_{2G}+2x_{3G}\leq 40$
$x_{1F}+x_{1G}\leq 7y_1$; $x_{2F}+x_{2G}\leq 5y_2$; $x_{3F}+x_{3G}\leq 9y_3$
$y_1+y_2+y_3-2\leq 0$
$x_{1F},x_{1G},x_{2F},x_{2G},x_{3F},x_{3G}\geq 0$, $y_1,y_2,y_3\in\{0,1\}$.

[5*pts*] d) There is a cost of undue diversification stemming from loss of efficiencies in the production. This cost is estimated to be 10 (in the units that the profit is measured in LP formulation). It is incurred only when all three new products are produced. Modify the LP formulation to **appropriately account for the cost**.

ANSWER: Let $y_i=1$ if product i is produced; it is 0 otherwise. Let $\bar{y}=1$ when all three products are produced. That is $\bar{y}=1$ if and only if $y_1+y_2+y_3=3$.
Insert new constraints: $y_1+y_2+y_3-2\leq\bar{y}$ and $\bar{y}\in\{0,1\}$. Deduct the cost of 10 from the objective: Maximize $5(x_{1F}+x_{1G})+7(x_{2F}+x_{2G})+3(x_{3F}+x_{3G})-10\bar{y}$. Modify the sales potential constraints as in c).
Since we are maximizing, we want $\bar{y}=0$. However this is not possible when $y_1+y_2+y_3-2=1$, which forces $\bar{y}=1$ by the constraint $y_1+y_2+y_3-2\leq\bar{y}$. Only when $y_1+y_2+y_3=3$, we have $\bar{y}=1$ and we pay the cost. Students need to write only the modifications above. For clarity, we present the complete formulation:

Maximize $5(x_{1F}+x_{1G})+7(x_{2F}+x_{2G})+3(x_{3F}+x_{3G})-10\bar{y}$
ST: $3x_{1F}+4x_{2F}+2x_{3F}\leq 30$; $4x_{1G}+6x_{2G}+2x_{3G}\leq 40$
$x_{1F}+x_{1G}\leq 7y_1$; $x_{2F}+x_{2G}\leq 5y_2$; $x_{3F}+x_{3G}\leq 9y_3$
$y_1+y_2+y_3-2\leq\bar{y}$
$x_{1F},x_{1G},x_{2F},x_{2G},x_{3F},x_{3G}\geq 0$, $y_1,y_2,y_3,\bar{y}\in\{0,1\}$.

Note that by setting $\bar{y}=0$, we obtain the formulation in c).

[5*pts*] e) The new VP wants use only one of the plants to produce 3 products. Apparently producing new products at these plants require some investment and PlaPharma is willing to make that investment only at one of the plants. Modify the LP formulation to ensure that **exactly one plant** is utilized to produce new products.

ANSWER: Let $z_F=1$ when Frisco plant is used. If $Z_F=0$, Grapevine plant is used. We add constraint $z_F\in\{0,1\}$ to ensure that exactly one of the plants is used. We also need to modify the plant capacity constraints:
$3x_{1F}+4x_{2F}+2x_{3F}\leq 30z_F$ Work hours capacity constraint in Frisco if this plant is used.
$4x_{1G}+6x_{2G}+2x_{3G}\leq 40(1-z_F)$ Work hours capacity constraint in Grapevine if this plant is used. Students need to write only the modifications above. For clarity, we present the complete formulation:

Maximize $5(x_{1F}+x_{1G})+7(x_{2F}+x_{2G})+3(x_{3F}+x_{3G})$
ST: $3x_{1F}+4x_{2F}+2x_{3F}\leq 30z_F$; $4x_{1G}+6x_{2G}+2x_{3G}\leq 40(1-z_F)$; $z_F\in\{0,1\}$
$x_{1F}+x_{1G}\leq 7$; $x_{2F}+x_{2G}\leq 5$; $x_{3F}+x_{3G}\leq 9$; $x_{1F},x_{1G},x_{2F},x_{2G},x_{3F},x_{3G}\geq 0$.

[Spring 08] RichChick is an east-Richardson company that produces two types of Chicken cutlets for sale to supermarkets. Cutlets are called **S**ilverlet and **B**rownlet. Each cutlet consists of white meat and dark meat. Silverlet sells for \$12/kg and must consist of at least 80% white meat. Brownlet sells for \$10/kg and must consist of at least 40% white meat. At most 300 kg of Silverlet and 500 kg of Brownlet can be sold in the next month. Three types of chicken are used to manufacture the cutlets: **F**arm-grown chicken, Free **R**ange chicken and **O**rganic-fed chicken. Each chicken's cost, and its dark and white meat yield are below:

Chicken type	Farm-grown	Free-Range	Organic-fed
Cost in \$	6	7	8
White meat yield in kg	1	2	2
Dark meat yield in kg	2	1	1

[4*pts*] a) To formulate a LP to maximize RichChick's profit in the next month, define decision variables.

ANSWER: F, R, O: Number of Farm-grown, free-Range, Organic-fed chicken purchased. D_S, D_B: Kilograms of Dark meat used in Silverlet, Brownlet. W_S, W_B: Kilograms of White meat used in Silverlet, Brownlet.

[4*pts*] b) Profit is revenue minus costs, express RichChick's profit in terms of decision variables.

ANSWER: Max $12(W_S + D_S) + 10(W_B + D_B) - 6F - 7R - 8O$

[4*pts*] c) Write constraints so that RichChick does not attempt to sell more cutlets than next month's demand.

ANSWER: $W_S + D_S \le 300$ and $W_B + D_B \le 500$

[4*pts*] d) Write constraints so that RichChick does not attempt to use more white or dark meat than it buys in the form of Farm-grown chicken, Free Range chicken and Organic-fed chicken.

ANSWER: $W_S + W_B \le F + 2R + 2O$ and $D_S + D_B \le 2F + R + O$

[4*pts*] e) Finish your formulation by adding any constraints you find necessary.

ANSWER: Silverlet must have at leat 80% white meat:

$$\frac{W_S}{W_S + D_S} \ge 0.8.$$

Brownlet must have at least 40% white meat:

$$\frac{W_B}{W_B + D_B} \ge 0.4.$$

Nonnegativity constraints: $F, R, O, W_S, D_S, W_B, D_B \ge 0$

Remark: If you define S and B as the amount of Silverlet and Brownlet produced, then you must also add the constraints

$$S = W_S + D_S \qquad B = W_B + D_B.$$

Only after these adding these constraints can you replace $W_S + D_S$ by S and $W_B + D_B$ by B in the formulation above. If you forget $S = W_S + D_S$ or $B = W_B + D_B$, your output S and B becomes independent of the input D_S, D_B, W_S, W_B.

[Spring 08] UTDDrill is a drill bit producer established by some UTD MBA students. UTDDrill buys 5 cm long blank drill bits and shapes them with drill patterns (like twist, masonry, lip, spur) by using a lathe. Then UTD Drill packages 12 different drills in to a single box and sells them to the Lowe's Home Improvement stores in Texas. UTDDrill requires 3000 blank drill bits every month and buys each bit at \$0.05 and each box at \$0.60. It costs \$12 to UTDDrill to initiate an order from any of its suppliers. The holding costs are based on an annual interest rate of 12%.

[8*pts*] a) Determine the optimal number of drill bits that UTDDrill should purchase and the time between these purchases.

ANSWER: R= 3000/month. $P=\infty$. K=1200 cents. $h=5(0.01)=0.05$ cents. Then

$$\textbf{EOQ} = \sqrt{\frac{2KR}{h}} = \sqrt{\frac{2\cdot 1200\cdot 3000}{0.05}} = \sqrt{2\cdot 1200\cdot 3000\cdot 20} = \sqrt{2\cdot 12\cdot 3\cdot 2}*1000 = 12,000.$$

Time between orders is

$$\textbf{Length of an Inventory Cycle} = Q/R = 12000/3000 = 4 \text{ months}.$$

[4*pts*] b) What is the annual holding and ordering cost for blank drill bits?

ANSWER:

$$\textbf{Total monthly cost} = C(Q=EOQ;P=\infty) = \underbrace{\frac{1}{2}EOQ\ h}_{Inventory\ holding\ cost\ per\ month} + \underbrace{\frac{KR}{EOQ}}_{Set\ up\ cost\ per\ month}$$
$$= \frac{1}{2}12000\cdot 0.05 + \frac{1200\cdot 3000}{12000} = 300+300 = 600 \text{ cents},$$

or since optimal order is used, Total monthly cost=$\sqrt{2KRh}=\sqrt{2\cdot 1200\cdot 3000\cdot 0.05}=\sqrt{1200\cdot 300}=\sqrt{36}\cdot 100=600$. Then the annual cost is \$72= $6\cdot 12$.

[4*pts*] c) Determine the optimal number of package boxes that UTDDrill should purchase.

ANSWER: R= 3000/12 per month. $P=\infty$. K=1200 cents. $h=60(0.01)=12\cdot 0.05$ cents. Then

$$\textbf{EOQ} = \sqrt{\frac{2KR}{h}} = \sqrt{\frac{2\cdot 1200\cdot 3000/12}{12\cdot 0.05}} = \frac{1}{12}\sqrt{2\cdot 1200\cdot 3000\cdot 20} = \frac{1}{12}\sqrt{2\cdot 12\cdot 3\cdot 2}*1000 = 1,000.$$

[4*pts*] d) The forecaster at UTDDrill has made a mistake in computing the monthly blank drill bit demand which is actually 5000 per month. What is the annual holding and ordering cost for blank drill bits when the purchase quantity in a) is used with the correct demand of 5000 per month? Basically, update your computations in b).

ANSWER:

$$\textbf{Total monthly cost} = C(Q=EOQ;P=\infty) = \underbrace{\frac{1}{2}EOQ\ h}_{Inventory\ holding\ cost\ per\ month} + \underbrace{\frac{KR}{EOQ}}_{Set\ up\ cost\ per\ month}$$
$$= \frac{1}{2}12000\cdot 0.05 + \frac{1200\cdot 5000}{12000} = 300+500 = 800 \text{ cents}.$$

Then the annual cost is \$96= $8\cdot 12$. Note that here we are computing the cost of ordering suboptimally in 12000 units. The formula $\sqrt{2KRh}$ cannot be used when the order quantity is suboptimal.

[Spring 07] Assigning tasks to workers. Consider the following 8 tasks:

Task	1	2	3	4	5	6	7	8
Time	15	25	15	20	15	20	50	15

They are to be assigned to 3 workers (**A**braham, **B**en and **C**andy) on a conveyor-paced assembly line.

[10*pts*] a) Provide a formulation to maximize the capacity of the line. Hint: Consider minimizing the cycle time denoted by C. Please define all the variables that appear in your formulation. Do not attempt to solve the formulation.

ANSWER: [2*pts*] Let $x_{iA} = 1$ if the ith task is assigned to Abraham; Otherwise $x_{iA} = 0$. Similarly define x_{iB} and x_{iC}. Let the total time of the tasks assigned to Abraham, Ben and Candy be t_A, t_B and t_C. Note that $C = \max\{t_A, t_B, t_C\}$, so $C \geq t_A$, $C \geq t_B$ and $C \geq t_C$. We use these inequalities in our formulation:

[2*pts*] Minimize C
Subject to
[2*pts*] $C - t_A \geq 0$, $C - t_B \geq 0$, $C - t_C \geq 0$: Definition of cycle time.
[2/3*pts*] $t_A - 15x_{1A} - 25x_{2A} - 15x_{3A} - 20x_{4A} - 15x_{5A} - 20x_{6A} - 50x_{7A} - 15x_{8A} = 0$: Definition of the total
[2/3*pts*] $t_B - 15x_{1B} - 25x_{2B} - 15x_{3B} - 20x_{4B} - 15x_{5B} - 20x_{6B} - 50x_{7B} - 15x_{8B} = 0$: time of activities
[2/3*pts*] $t_C - 15x_{1C} - 25x_{2C} - 15x_{3C} - 20x_{4C} - 15x_{5C} - 20x_{6C} - 50x_{7C} - 15x_{8C} = 0$: assigned to A, B, C
[2*pts*] $x_{iA} + x_{iB} + x_{iC} = 1$ for $i = 1 \ldots 8$: Every item must be assigned to exactly one of the workers.
[0*pts*] $x_{iA}, x_{iB}, x_{iC} \in \{0, 1\}$
[0*pts*] $C, t_A, t_B, t_C \geq 0$.

Definition of variables, the objective and each type of the constraint (there are 3 important ones) worth 2 points.

[3*pts*] b) In part a), there are no precedence relations. If there had been precedence relations among tasks, would the capacity have been more or less. Please provide a brief argument.

ANSWER: Adding constraints to the formulation above to represent the precedence relations can only remove the existing feasible solutions. If some solutions are eliminated, we may not be able to decrease C as much as we can without the precedence constraints. Hence, the objective value, the optimal C, can increase with teh constraints. In other words, the capacity of the system may decrease or remain constant. However, it cannot increase with the precedence relations.

[Spring 07] CzePhone is a new service company that provides European mobile phones to American visitors to Czech Republic. The company currently has 80 phones available at the Prague airport. There are - on average - 25 customers per day requesting a phone. These requests arrive throughout the 24 hours the store is open. The corresponding coefficient of variation is 1. Customers keep their phones on average 72 hours. The standard deviation of this time is 100 hours.

Since CzePhone currently has no competitor in Prague airport providing equally good service, customers are willing to wait for the telephones. Yet, during the waiting period, customers are provided a free calling card. Based on prior experience, CzePhone found that the company incurred a cost of $1 per hour per waiting customer, independent of day or night.

[4*pts*] a. What is the average number of telephones CzePhone has in its store?

ANSWER: Consider the phones as servers. Thus $m = 80$, $a = 24$ hours/25 customers $= 0.96$ hours and $p = 72$ hours. So, $u = p/(m * a) = 72/(80 * 0.96) = 94\%$. This means that 75=0.94*80 phones are in use, and 5 phones are available in th store on average.

[4*pts*] b. How long does a customer, on average, have to wait for the phone?

ANSWER: $CV_a = 1$ and $CV_p = 100/72 = 1.39$. Evaluating

$$T_q = \left(\frac{p}{m}\right)\left(\frac{u^{\sqrt{2(m+1)}-1}}{1-u}\right)\left(\frac{CV_a^2 + CV_p^2}{2}\right),$$

at $p = 72$, $a = 0.96$, $m = 80$, $u = 0.94$, $CV_a = 1$ and $CV_p = 1.39$,

$$\begin{aligned} T_q &= \frac{72}{80}\frac{0.94^{\sqrt{2(80+1)}-1}}{1-0.94}\frac{1^2+1.39^2}{2} \\ &= 0.9\frac{0.94^{11.73}}{0.06}1.47 \\ &= 0.9 \cdot 8 \cdot 1.47 = 10.58 \text{ hours} \end{aligned}$$

[4*pts*] c. What are the total monthly (30 days) expenses for telephone cards?

ANSWER: First calculate I_q, the average number of people in the queue. From the Little's formula: $I_q = (1/a)T_q = 10.58/0.96 = 11.02$. So we can multiply 1 * 24 hours/day * 30 days * 11.02 people in the queue = $7934.4.

[5*pts*] d. Assume CzePhone could buy additional phones at $1000 per unit. What is the payback period (the time the phone pays for $1000 investment) for one additional phone?

ANSWER: Now $m = 81$ so $u = 72/(81 * 0.96) = 92.6\%$.

$$\begin{aligned} T_q &= \frac{72}{81}\frac{0.926^{\sqrt{2(81+1)}-1}}{1-0.926}\frac{1^2+1.39^2}{2} \\ &= 0.889\frac{0.926^{11.8}}{0.074}1.47 \\ &= 0.889 \cdot 5.455 \cdot 1.47 = 7.13 \text{ hours} \end{aligned}$$

Then $I_q = (1/a)T_q = 7.13/0.96 = 7.43$. So we can multiply 1 * 24 hours/day * 30 days * 7.43 people in the queue = $5349.6. With an extra phone, we save 7934.4-5349.6=2584.8 in a month. It takes about 1000/2584.8 months = 11.6 days to recover $1000 investment.

This question unfortunately involves some algebra. There will be partial credit if you have the correct steps but incorrect numerical answers.

[Spring 06] The airport branch of a car rental company operates 24 hours per day and maintains a fleet of 75 SUVs. The interarrival time between requests for an SUV is 2.4 hours, on average, with a standard deviation of 2.4 hours. There is no indication of a systematic arrival pattern over the course of the day. Assume that, if all SUVs are rented, customers are willing to wait until there is an SUV available. An SUV is rented, on average, for 4 days, with a standard deviation of 1 day.
[3*pts*] a. Let us treat this case as a queue problem. Let the SUVs be the servers. What are m, a, p, CV_a, CV_p?

ANSWER: We know that $a = 2.4$ hours, $p = 96$ hours, $CV_a = (2.4/2.4) = 1$, $CV_p = (24/96) = 0.25$, and $m = 75$ cars.

[5*pts*] b. What is the average number of SUVs parked in the company's lot? Hint: What is the number of SUVs utilized?

ANSWER: To determine the number of cars on the lot, we can look at the utilization rate of our "servers" $= (1/a)/(m/p) = 53.3\%$. Therefore, on average 53.3% of the cars are in use or 40 cars, so on average 35 cars are in the lot.

[7*pts*] c. Through a marketing survey, the company has discovered that if it reduces its daily rental price of $80 by $25, the average demand would increase to 12 rental requests per day and the average rental duration becomes 3 days. Is this price decrease warranted? Provide an analysis by comparing the daily revenue under both pricing schemes. Assume that the standard deviations of the interarrival and activity times do not change.

ANSWER: If the average demand is increased to 12 rentals per day, then $a = 2$ hours. If the average rental duration is to 4 days, then $p = 72$ hours. So utilization rate becomes 48%. This means that 36 cars are rented on average.

With the initial rate average revenue per day =80*40= $3,200. With the proposed rate average revenue per day 55*36=$1,980. Therefore, the company should not make the proposed changes.

[Spring 06] A company produces two products (1,2) using two machines (M,N). Product 1 requires processes on **both** machines M and N. On the contrary product 2 can be produced on **either** machine M or N. Processing times (in minutes) on each machine are

Product	Machine N	Machine M
1	15	18
2	20	25

Each machine works for 8 hours every day. Due to marketing limitations the number of Product 1 sold must be at least the number of Product 2 sold. When sold, each unit of Product 1 and 2 contributes to profit \$16 and \$20.
[10*pts*] a) Provide an LP to maximize daily contribution to profit.

ANSWER: Let x_1 be the number of Product 1 produced, and x_{2M} and x_{2N} be the number of Product 2 produced on machines M and N.
Max $16x_1 + 20(x_{2M} + x_{2N})$
ST
$15x_1 + 20x_{2N} \leq 8(60)$ Capacity constraint for machine N
$18x_1 + 25x_{2M} \leq 8(60)$ Capacity constraint for machine M
$x_1 \geq x_{2M} + x_{2N}$
$x_1, x_{2M}, x_{2N} \geq 0$

[5*pts*] b) Suppose that marketing limitation is lifted. Then compute how many more Product 2 can be produced by producing one fewer Product 1. Basically think of using capacity to produce Product 2 as opposed to Product 1. In this case, compare the reduction in profit due to Product 1 against the increase in profit due to Product 2. Finally argue that Product 1 will not be produced in the optimal solution without the marketing limitations.

ANSWER: With one less Product 1, 15 mins and 18 mins capacity are released on Machines N and M. This capacity can be used to produce 15/20 and 18/25 Product 2 on machines N and M. The net effect to profit is -16+(15/20)20+(18/25)20 and is positive. Reducing Product 1 production increases profit so no Product 1 is produced in the optimal solution.

Additional Questions

1. Three US Olympic teams and their trainers will fly back from Sydney to San Fransisco with a plane that can carry 100 people. This will be a nonstop flight lasting 20 hours. Three teams are Swimming, Gymnastics and Cycling. These teams have the following number of members and trainers: Swimming 42 and 12; Gymnastics 22 and 14; Cycling 34 and 16. There must be at least one swimming trainer accompanying every three swimmers on the plane. Similarly, there must be at least one gymnastics trainer for every two gymnasts on the plane. Cyclists tend to be older and can travel by themselves without trainers. Swimming and cycling associations are equally paying for the trip and they first require that at least the 70% of the seats are allocated to swimmers, cyclists and their trainers. Second, the total number of swimmers and their trainers must equal to the total number of cyclist and their trainers.

a) Provide an LP formulation to minimize the number of people that cannot be put on this flight.

ANSWER:

Let x_s, x_g, x_c, t_s, t_g, t_c be the number of swimmers, gymnasts, cycles, and their trainers put on the plane.

Min $140 - (x_s + x_g + x_c + t_s + t_g + t_c)$

ST :

$$x_s - 3t_s \leq 0$$
$$x_g - 2t_g \leq 0$$
$$x_s + t_s + x_c + t_c \geq 70$$
$$x_s + t_s - x_c - t_c = 0$$
$$0 \leq x_s \leq 42 \ , \ 0 \leq x_g \leq 22 \ , \ 0 \leq x_c \leq 34$$
$$0 \leq t_s \leq 12 \ , \ 0 \leq t_g \leq 14 \ , \ 0 \leq t_c \leq 16$$

b) What is the optimal value of the objective in a)? Justify your answer. You can answer this without solving the formulation.

ANSWER:

Consider $x_s = 23$, $t_s = 12$, $x_g = 20$, $t_g = 10$, $x_c = 34$ and $t_c = 1$, this solution is feasible and yields an objective value of 40. You can pick another solution and discover that it also gives an objective value of 40 (consider $x_s = 24$, $t_s = 12$, $x_g = 18$, $t_g = 10$, $x_c = 30$ and $t_c = 6$). Indeed any feasible solution has an objective value of 40. Moreover, we can not reduce the objective value below 100, because the plain takes 100 people and we have 140 athletes.

c) Suppose that leaving out a gymnast costs three times as much as leaving out a swimmer or a cyclist. And also suppose that the cost of leaving out trainers is negligible. Modify your answer to a) to minimize the cost of people left behind (not put on the plane).

ANSWER:

Modify the objective function as Min $(42 - x_s) + 3(22 - x_g) + (34 - x_c)$.

2. Farmer Billy Bauer has two farms in Dallas to grow wheat and barley. There are differences in the yields and costs of growing crops due to soil conditions at two farms:

	McKinney Farm	Addison Farm
Barley yield/acre	400 bushels	700 bushels
Cost/acre of barley	\$90	\$80
Wheat yield/acre	350 bushels	300 bushels
Cost/acre of wheat	\$110	\$100

McKinney and Addison farms have 70 and 120 acres for cultivation. At least 20000 bushels of barley and 30000 bushels of wheat must be grown. Provide an LP to minimize the cost of meeting wheat and barley demand.

ANSWER:

Let BM: Area in acres dedicated for Barley production at McKinney. BA: Area in acres dedicated for Barley production at Addison. Similarly define WM and WA.

Min $90BM + 80BA + 110WM + 100WA$
ST:
$BM + WM \leq 70$
$BA + WA \leq 110$
$400BM + 700BA \geq 20000$
$350WM + 300WA \geq 30000$
$BM, BA, WM, WA \geq 0$

3. PlanoTurkey produces two types of turkey cutlets for sale to fast food restaurants. Each type of cutlet consists of white meat and dark meat. Cutlet 1 sells for \$8/kg and must consist of at least 70% white meat. Cutlet 2 sells for \$6/kg and must consist of at least 60% white meat. At most 50 kg of cutlet 1 and 30 kg of cutlet 2 can be sold for Thanksgiving. Two types of turkey used to manufacture the cutlets are purchased from an Addison farm. Each type 1 turkey costs \$10 and yields 5 kg of white meat and 2 kg of dark meat. Each type 2 turkey costs \$8 and yields 3 kg of white meat and 3 kg of dark meat. Formulate a LP to maximize PlanoTurkey's profit.

a) Define decision variables.

ANSWER:

T_1: Number of type 1 turkey purchased. D_1: Kilograms of dark meat used in cutlet 1. W_1: Kilograms of white meat used in cutlet 1. Define T_2, D_2, W_2 similarly.

b) Profit is revenue minus costs, express the profit in terms of decision variables.

ANSWER:

Max $8(W_1 + D_1) + 6(W_2 + D_2) - 10T_1 - 10T_2$

c) Write constraints so that no more cutlets than demand is sold.

ANSWER:

$W_1 + D_1 \leq 50$ and $W_2 + D_2 \leq 30$

d) Write constraints so that PlanoTurkey does not use more white or dark meat than it buys from the Addison farm.

ANSWER:

$W_1 + W_2 \leq 5T_1 + 3T_2$ and $D_1 + D_2 \leq 2T_1 + 3T_2$

e) Finish your formulation by adding any contraints you find necessary.

ANSWER:

Cutlet 1 must have at leat 70% white meat:

$$\frac{W_1}{W_1 + D_1} \geq 0.7$$

.

Cutlet 2 must have at leat 60% white meat:

$$\frac{W_1}{W_1 + D_1} \geq 0.6$$

.

Nonnegativity constraints: $T_1, D_1, W_1, T_2, D_2, W_2 \geq 0$

4. The Apex Television company has to decide on the number of 27 and 20 inch sets to be produced at one of its factories. Market research indicates that at most 40 of the 27 inch sets and 10 of the 20 inch sets can be sold per month. The maximum number of work hours available is 800 hours per month. A 27 inch set requires 15 work hours and a 20 inch set requires 10 work hours . Each 27 inch set produces a profit of $120 and the same number is $80 for 20 inch sets.

a) Formulate an LP to maximize the profit:

ANSWER:

B: Number of 27 inch sets produced per month. S: Number of 20 inch sets produced per month.

Max $120B + 80T$
Subject to:
$B \leq 40$
$S \leq 10$
$15B + 10S \leq 800$
$B, S \geq 0$

b) Through commercials, TV set demand can be increased. For every $20 spent for commercials, 1 more 27 inch TV **and** 2 more 20 inch TV can be sold. Formulate an LP to maximize the profit with a budget of $400 for commercials.

ANSWER:

C: Commercial budget spent for TVs.

Max $120B + 80T - C$
Subject to:
$B \leq 40 + C/20$
$S \leq 10 + C/10$
$C \leq 400$
$15B + 10S \leq 800$
$B, S \geq 0$

5. To celebrate the ending of the term, suppose that you go to a restaurant with the following menu:

	Salad	Soup	Steak	Chicken	Rice	Pasta	Fish	Fries	Cheesecake	Pie	Coffee
Calories	400	300	1200	1100	800	900	1000	400	500	400	100
Price	5	4	14	12	7	8	13	3	6	6	2

Suppose that you have $25 and do not want to consume more than 5000 calories and less than 2500 calories.

(8 points) a) Your objective is to look rich, i.e. to order as many items as possible. Provide a formulation to achieve this objective.

ANSWER:

Let $x_i = 1$ if ith item is ordered, 0 otherwise.

Maximize $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$

St

$400x_1 + 300x_2 + 1200x_3 + 1100x_4 + 800x_5 + 900x_6 + 1000x_7 + 400x_8 + 500x_9 + 400x_{10} + 100x_{11} \leq 5000$

$400x_1 + 300x_2 + 1200x_3 + 1100x_4 + 800x_5 + 900x_6 + 1000x_7 + 400x_8 + 500x_9 + 400x_{10} + 100x_{11} \geq 2500$

$5x_1 + 4x_2 + 14x_3 + 12x_4 + 7x_5 + 8x_6 + 13x_7 + 3x_8 + 6x_9 + 6x_{10} + 2x_{11} \leq 25$

$x_i \in \{0, 1\}$

(6 points) b) Write a constraint that does not allow ordering cheesecake and pie together.

ANSWER:

$x_9 + x_{10} \leq 1$

(6 points) c) Write a constraint so that coffee is ordered when cheesecake **or** pie is ordered.

ANSWER:

$2x_{11} \geq x_9 + x_{10}$

6. To celebrate the ending of the term, suppose that you go to a restaurant with the following menu:

	Salad	Soup	Steak	Chicken	Rice	Pasta	Fish	Fries	Cheesecake	Pie	Coffee
Calories	400	300	1200	1100	800	900	1000	400	500	400	100
Price	5	4	14	12	7	8	13	3	6	6	2

Suppose that you do not want to consume more than 4000 calories and less than 2000 calories but at the same time you will order at least 5 items.

(8 points) a) Provide a formulation that minimizes your expense.

ANSWER:
Let $x_i = 1$ if ith item is ordered, 0 otherwise.

Minimize $5x_1 + 4x_2 + 14x_3 + 12x_4 + 7x_5 + 8x_6 + 13x_7 + 3x_8 + 6x_9 + 6x_{10} + 2x_{11}$ St
$400x_1 + 300x_2 + 1200x_3 + 1100x_4 + 800x_5 + 900x_6 + 1000x_7 + 400x_8 + 500x_9 + 400x_{10} + 100x_{11} \leq 4000$
$400x_1 + 300x_2 + 1200x_3 + 1100x_4 + 800x_5 + 900x_6 + 1000x_7 + 400x_8 + 500x_9 + 400x_{10} + 100x_{11} \geq 2000$
$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} \geq 5$
$x_i \in \{0, 1\}$

(6 points) b) Write a constraint that forces you to order the steak if you are ordering the soup and allow to order the steak without the soup.

ANSWER:
$x_3 \geq x_2$

(6 points) c) Write a constraint so that chicken is not ordered when steak **and** fish are ordered.

ANSWER:
Correct answer: $x_4 \leq 2 - x_3 - x_7$
A wrong answer: $x_3 + x_7 \geq x_4 - 1$. This allows for $x_3 = 1, x_7 = 1, x_4 = 1$ so it is wrong.

7. Choco is a small chocolate manufacturer producing only Black (B), Milk (M) and Strawberry (S) chocolate. Manufacturing costs per ton for B, M and S are 800, 700 and 900 dollars. Marketing needs at least 8, 14 and 10 tons of B, M and S chocolate for the next month and guarantees a revenue of 3300, 3000 and 2800 dollars for each ton of B, M and S. Total labor capacity for the next month is 2400 hours but each ton of B, M and S consume 50, 60, 80 hours of capacity. Finally finance department enforces a minimum profit limit of 60,000 dollars per month.

(10 points) a) Let C be the total production costs, and B, M, S be the total tons of Black, Milk and Strawberry chocolate produced in the next month. Write an LP that minimizes C subject to limitations given above.

ANSWER:

Minimize C
St
$C = 800B + 700M + 900S$
$B \geq 8$, $M \geq 14$, $S \geq 10$
$3300B + 3000M + 2800S - C \geq 60,000$
$50B + 60M + 80S \leq 2400$
$B, M, S \geq 0$

(5 points) b) Compute the contribution to margin for each ton of B, M, S chocolates and compare with labor hours. Is there a chocolate type which is dominated both in unit contribution to margin and labor hours, explain? Would your LP indicate a positive production quantity for that type, why?

ANSWER:

Contribution to margins are 2500, 2300 and 1900, and labor hours are 50, 60 and 80 per ton of B, M and S. Clearly M and S are dominated in profitability. LP has $M \geq 14$, $S \geq 10$, so it will set $M = 14$, $S = 10$.

(5 points) c) Use the labor hours constraint to find the smallest number U tons such that $B \leq U$, $M \leq U$ and $S \leq U$ in any feasible solution to your LP. I am just asking for the smallest number U such that if B, M and S are feasible then $B \leq U$, $M \leq U$ and $S \leq U$.

ANSWER:

If no M or S produced, we have $B \leq 2400/50 = 48$. Similarly, $M \leq 40$ and $S \leq 30$ so $U = \max\{48, 40, 30\} = 48$.

Another but more complicated way is to involve demand constraints:
$50B \leq 2400 - 60 \cdot 14 - 80 \cdot 10$
$60M \leq 2400 - 50 \cdot 8 - 80 \cdot 10$
$80S \leq 2400 - 60 \cdot 14 - 50 \cdot 8$
Let $U = \max\{B, M, S\}$. Both ways get the full mark.

8. Refer to the statement of the previous problem. The marketing group at Choco is convinced that forcing manufacturing to produce above some predetermined quantities is not a good idea. They have stopped requiring at least 8, 14, 10 tons of B, M and S, and have gave freedom to manufacturing to set these numbers to zero if it is profitable to do so. On the other hand manufacturing group has just realized that production costs are not entirely proportional to production quantities. Actually there are fixed costs that are paid to start up the production for each chocolate, these costs are independent of production quantities. Fixed cost for a chocolate is incurred only if that chocolate is produced (in positive quantities). Fixed costs are 10,000, 12,000 and 8000 dollars for chocolates B, M and S.

(12 points) a) Define appropriate variables and provide a formulation to minimize the total cost in light of new marketing policy and new fixed costs.

ANSWER:

Let y_B, y_M and y_S be binary variables becoming 1 if B, M and S are produced, respectively.

Minimize C
St
$C = 800B + 700M + 900S + 10,000y_B + 12,000y_M + 8,000y_S$
$B \leq 48y_B$, $M \leq 48y_M$, $S \leq 48y_S$
$3300B + 3000M + 2800S - C \geq 60,000$
$50B + 60M + 80S \leq 2400$
$B, M, S \geq 0$, $y_B, y_M, y_S \in \{0, 1\}$

Note we computed 48 tons as the smallest uniform upper bound on B, M and S from the 3.c. But you do not have to use this bound. Any number larger than 48 will work as well.

(8 points) b) Write a constraint to make sure that: at least 16 tons of Milk chocolate must be produced only when no Black chocolate is produced.

ANSWER:

$M \geq 16(1 - y_B)$