

1 Solved Exercises

1.1 Statement of Exercises

1. [Hair Cut] Ted Handsome takes a good care of his hair. Every two weeks, he has his hair cut. He prefers a hair salon in McKinney. This salon has 4 barbers. Recently, Ted found out that he usually needs to wait before his hair cut. While waiting, he timed barbers to find out that it takes about 5 minutes for a customer to be finished. Moreover, every 10 minutes another customer arrives. Find the probability that Ted has to wait for his haircut.
2. Textbook 7.4.

1.2 Solutions

ANSWER for Exercise 1:

In this case, we have $m = 4$ servers. The interarrival time is $a = 10$ minutes. Since 4 barbers finish one hair cut in 5 minutes, one barber will finish a hair cut in 20 minutes. Then the processing time is $p = 20$ minutes. Finally, note that $r = p/a = 20/10 = 2$. We can use

$$P_m(r) = \frac{r^m/m!}{\sum_{i=0}^m r^i/i!},$$

to evaluate

$$\begin{aligned} P_4(2) &= \frac{(2)^4/4!}{(2)^0/0! + (2)^1/1! + (2)^2/2! + (2)^3/3! + (2)^4/4!} = \frac{16}{24 + 12 + 48 + 32 + 16} \\ &= 0.121 \end{aligned}$$

Ted Handsome finds all 4 barbers busy with probability 0.121. □

ANSWER for Exercise 2:

20 cars want to fill in per hour. This leaves us with an interarrival time of $a=60/20$ minutes=3 minutes. The service time is $p=5$ minutes ($r = p/a=5/3=1.667$) and we have $m=6$ servers.

a) All servers will be busy with a probability of 0.0056, this number comes from the Erlang loss table on p.374 of the textbook. Alternatively, you can use

$$P_m(r) = \frac{r^m/m!}{\sum_{i=0}^m r^i/i!},$$

to evaluate

$$P_6(5/3) = \frac{(1.67)^6/6!}{(1.67)^0/0! + (1.67)^1/1! + (1.67)^2/2! + (1.67)^3/3! + (1.67)^4/4! + (1.67)^5/5! + (1.67)^6/6!}.$$

b) The flow rate thru the gas station is $19.89=20(1-0.0056)$ cars per hour. □