OPRE 6302. OM : Queues

1 Solved Exercises

1.1 Statement of Exercises

1. Textbook 7.5.

1.2 Solutions

ANSWER for Exercise 1:

Let us summarize the problem parameters: Inter-arrival time: 30 customers per hour = 1 customer / 2 min. Then $a = 2$ min, $CV_a = 1$. Process time: $p = 1.7$ min, $CV_p = \sigma_p/p = 3$ min / 1.7 min = 1.765.
a) Waiting time = 1.7 min * [(1.7/2)/(1-1.7/2)] * [(12 + 1.7652)/2] = 19.82 min.
b) Utilization = 1.7 min / 2 min = 0.85. Idle time = 0.15 * 8 hrs = 72 min. Since it takes 1.5 min to sort, 48=72/1.5 videos can be sorted per day.
c) The thruput rate is $R := 1/a$=0.5 customers per min. Thus, the average number of customers in line waiting $I = RT_q = 0.5$ (19.8) = 9.9 customers. In addition, an average of 0.85 customers (equal to the utilization u) are being served at any given time. So the average number of customers at the check-out desk = 9.9 + .85 = 10.75.
d) Because only 90% of customers go through checkout, the inter-arrival time of paying customers changes: a=27 customers per hour = 1 customer / 2.22 min.
Waiting time = 1.7 min * [(1.7/2.22)/(1-1.7/2.22)] * [(12 + 1.7652)/2] = 11.38 min.
e) The average person waits 19.81 minutes, and 30 customers arrive in one hour, so there are approximately 19.82(30)=594.6 mins of wait time per hour. This costs the store 0.75(594.6) = $445.95 for wait time per hour. If 2 servers are used, we can apply the waiting time formula, and using the same methodology, calculate a cost of 0.75(0.88)(30) = $19.79 for wait time per hour. Finally, if 3 servers are used, the cost is 0.75(0.162)(30) = $3.65 for wait time per hour. Adding any more employees would not be cost effective. The most cost effective number of employees is 3. □

2 Exercises

1. **[2 barbers at a barber shop]** Speedy and Picky are two barbers at a barber shop. They have one chair for each of the at most two customers having a hair cut. They also have two more chairs for waiting customers. If there are 4 customers in the shop, any arriving new customer chooses to go to another barber shop rather than waiting for a long time. Thus, the number of customers in the shop can be at most 4. Ted Handsome gets frequent hair cuts at this barbershop and he has driven by the shop 16 times in the last 6 months. Each time he has recorded the number $\tilde{I}$ of customers in the shop to obtain:

$\tilde{I}$	0	1	2	3	4
Frequency	once	four times	six times	four times	once

Naturally, the number $\tilde{I}$ of customers differ on each occasion so it is random. Ted is planning to have a hair cut today as well.

a) How many people should Ted expect to find in the barber shop today if we assume that the past observations are good indicators of the future? To answer this, treat $\tilde{I}$ as random variable and then compute the expected value I of $\tilde{I}$, i.e., $I = \mathrm{E}(\tilde{I})$.
b) Let $\tilde{I}_q$ be the number of customers waiting (in the queue). Clearly, $\tilde{I}_q$ is at most 2. Using the frequencies for $\tilde{I}$ provided above, compute the frequencies for $\tilde{I}_q$, i.e., fill in for the question marks below. Also compute the expected value I_q of $\tilde{I}_q$, i.e., $I_q = \mathrm{E}(\tilde{I}_q)$.

$\tilde{I}_q$	0	1	2
Frequency	?	?	?

c) Let $\tilde{I}_p$ be the number of customers served (having hair cut). Clearly, $\tilde{I}_p$ is at most 2. Using the frequencies for $\tilde{I}$ provided above, compute the frequencies for $\tilde{I}_p$, i.e., fill in for the question marks below.

$\tilde{I}_q$	0	1	2
Frequency	?	?	?

Also compute the expected value I_p of $\tilde{I}_p$, i.e., $I_p = \mathrm{E}(\tilde{I}_p)$. Do you expect to get $I = I_p + I_q$, explain.

2. [**Exponential interarrival and activity times**] Consider the waiting time formula in a queue:

$$\text{Time in queue} = \text{Activity time}\ \frac{\text{Utilization}}{1 - \text{Utilization}}\ \frac{CV_a^2 + CV_p^2}{2}.$$

In this exercise, we shall simplify this formula and write it in a different form when the interarrival and activity times are exponentially distributed.
a) Use the coefficient of variation (CV) of exponential distribution to simplify the formula above.
b) By expressing the utilization in terms of "expected activity time p" and "expected interarrival time a", put the simplified formula in part a) into the following form:

$$\text{Time in queue} = \frac{1}{a-p}[?].$$

Find the term in square brackets above and express it in English.

3. [**Exponential interarrival times**] Consider the waiting time formula in a queue:

$$\text{Time in queue} = \text{Activity time}\ \frac{\text{Utilization}}{1 - \text{Utilization}}\ \frac{CV_a^2 + CV_p^2}{2}.$$

In this exercise, we shall simplify this formula and write it in a different form when the interarrival times are exponentially distributed.
a) Use the coefficient of variation (CV) of exponential distribution to simplify the formula above.
b) By expressing the utilization in terms of "expected activity time p" and "expected interarrival time a", put the simplified formula in part a) into the following form:

$$\text{Time in queue} = \frac{1}{a-p}[?].$$

Find the term in square brackets above and express it in English.
c) What does the term in square brackets reduce to if the activity time is constant and so it is equal to its expected value p.

4. **[Top 9 reasons for queuing for the presidential elections]** According to the article titled "To Queue or not to Queue" in the 2008 Spring issue of Analytics journal (http://analytics.informs.org), there are 9 reasons for unexpected queues in the 2008 presidential election:

   (a) Underestimated voter turnout

   (b) The limited election budget

   (c) High-level of activities aimed at mobilizing voters

   (d) The demographical (age, education level, gender) changes in the electorate

   (e) Untested early voting options

   (f) Lack of voter training to operate voting machines

   (g) Weather

   (h) Deployment of voting machines and workers WITHOUT QUEUING ANALYSIS

   In a simple queue, we have the number of servers m, the activity time p and its coefficient of variation CV_p, the interarrival time a and its coefficient of variation CV_a.
   a) Explain if and how the top 9 reasons above effect m, p, CV_p, a and CV_a. (Do not let my question discourage you from voting in the election. On the contrary, please vote not only to fulfill your civic duty but also to have a say in important policy decisions. While waiting in the voting queue, remember my question $\ddot{\smile}$.)
   b) Add a 10th reason to the list above. (Send your 10 reasons to David Letterman show on CBS.)