OPRE 6302. OM : Capacity

1 Solved Exercises

1.1 Statement of Exercises

1. Textbook 3.4.

2. UTD Cafetria has 4 cashiers. The first cashier can process cash, credit card and comet transactions. The second can process cash and comet card transactions. The third and fourth can process only cash transactions. Each of a cash transaction and a credit card transaction takes 30 seconds while a comet card transaction takes only 15 seconds.
   a) What is the capacity of UTD Cafeteria cashiers?
   b) Can these cashiers process 4 cash, 2 credit card and 4 comet card transactions every minute?
   c) If there are on average 4 cash, 2 credit card and 2 comet card transactions per minute during the lunch time of 12:00-13:00, what is the utilization of the cashiers?
   d) If the customers can be convinced to use their comet cards, there would be on average 10 comet card transactions per minute during the lunch time of 12:00-13:00. If all the cashiers are equipped to handle these comet card transactions, what is the utilization?
   e) With the efficiency gains obtained by comet card use in d), the cafeteria plans to drop the number of cashiers from 4 to 3. The cafeteria offers 5% discount for comet card purchases, what would the implied percentage of cashier costs in the costs of goods sold by the cafeteria?

1.2 Solutions

ANSWER for Exercise 1:

The day starts with 25,000 gallons of milk in inventory. From 8 a.m. onwards, 5,000 gallons are produced per hour while 10,000 gallons are to be shipped. Thus, the inventory is depleted at a rate of 5000 gallons per hour, which leaves no milk after 5 hours, or at 1 p.m. From then on, clients will have to wait. This situation gets worse and worse until 6pm when the last client arrives. Then the inventory is short of 25,000 gallons or the inventory level is -25,000 gallons.

Let us draw the inventory level I_t in time over $[8, 23]$, where 23 is 11 p.m. $I_8 = 25,000$. I_t decreases over $8, 18$ at the rate of 5000 gallons/hour. $I_{18} = -25,000$. It increases over $[18, 23]$ at the rate of 10,000 gallons/hour. $I_{20.5} = 0$, that is the backlog is cleared at 8:30 p.m. Continuing to produce until 11 p.m. to start the next day with 25,000 gallons, we have $I_{23} = 25,000$.

a) 1 p.m.

b) Clients will stop waiting when the backlog of 25,000 accumulated by 6 p.m. clears. Since we are doing this at a rate of 5,000 gallon per hour, clients will stop waiting at 11 p.m., i.e., after 5 more hours.

c) At 6 p.m., we have a backlog of 25,000 gallons, which is equivalent to 20 trucks.

d) The trucks of the clients wait for milk when the inventory is short, that is when $I_t < 0$. Shortage lasts from 1 p.m. until 8:30 p.m., over $[13, 20.5]$. The negative inventory level constitutes a triangle whose base is $[13, 20.5]$, or 7.5 hours. The height of the triangle is -25,000 gallons=20 trucks. Out of 20 trucks some wait 7.5 hours (exactly those which arrived at 1:01 p.m.), some wait 1 minute (exactly those which arrived 8:29 p.m. An average truck waits 7.5/2 hours. There are 20 trucks waiting at 6 p.m. So the total waiting time is 20(7.5/2)=75 trucks $\times$ hour. Note that this result is exactly the area of the triangle defined by $I_t < 0$. The total waiting cost then is 75(50)=3750 dollars per day. □

ANSWER for Exercise 2:
a) Since the cash and credit card transactions take same amount of time, we can group them under cash-credit category. A cash-credit transaction takes 30 seconds. The first cashier can process cash-credit and comet transactions. The second can process cash-credit and comet card transactions. The third and fourth can process only cash-credit transactions. The capacity of the third and the fourth cashiers together is 4 cash-credit transactions per minute. The capacity of the first and the second cashiers are 4 cash-credit or 8 comet transactions. Summing up all these, the total capacity is either [8 cash-credit and 0 comet] transactions, or [4 cash-credit and 8 comet] transactions every minute, or any combinations (weighted averages) of these two.
b) 4 cash and 2 credit transactions are equivalent to 6 cash-credit transactions. The question is asking if the cashiers can process 6 cash-credit and 4 comet card transactions every minute. If the first cashier processes 4 comet card transactions and the remaining cashiers can process 6 cash-credit transactions. Then the cashiers can process 6 cash-credit and 4 comet card transactions every minute. Another way to see this is to write

$$[6 \text{ cash-credit and } 4 \text{ comet}] = 0.5[8 \text{ cash-credit and } 0 \text{ comet}] + 0.5[4 \text{ cash-credit and } 8 \text{ comet}],$$

so [6 cash-credit and 4 comet] is a combination of [8 cash-credit and 0 comet] and [4 cash-credit and 8 comet]. Another combination is

$$[7 \text{ cash-credit and } 2 \text{ comet}] = 0.75[8 \text{ cash-credit and } 0 \text{ comet}] + 0.25[4 \text{ cash-credit and } 8 \text{ comet}].$$

This combination can be achieved by assigning 2 comet card and 1 cash-credit transactions to the first cashier while the other cashiers process 2 cash-credit transactions.
c) 4 cashiers have a total capacity of 4 minutes in every minute. 4 cash, 2 credit card and 2 comet card transactions require 4(0.5)+2(0.5)+2(0.25)=3.5 minutes in every minute. The total capacity is 4 minutes, the work requires 3.5 minutes. Then the utilization is 3.5/4=7/8=87.5%.
d) 4 cashiers can process 16 comet card transactions so the utilization drops to 10/16=62.5%.
e) If the cafeteria passes all the savings due to 1 fewer cashier to the patrons, we infer that the cost of 1 cashier is 5% of the cost of goods sold. □