OPRE 6302. OM : Basestock

1 Solved Exercises

1.1 Statement of Exercises

1. [Put Option for 2 Periods] A finance manager at Opt-Out-Options (o^3) is valuing a put option for aluminum in April. A put option allows the option buyer the right – but not the obligation – to sell a stock to the seller of the option at a certain time for a certain price (the strike price). The option seller commits to buy the stock if the option buyer wants to sell. Consider a put option that will be exercised by o^3 in June, two months from now. The option is for an aluminum stock whose current value is $\mu = 40$, but the stock value changes every month. The amount of change in each month is normally distributed with mean zero and standard deviation 2.
   a) How much should o^3 pay for this option if its strike price is $S = 50$?
   b) The finance manager at o^3 finds the put option value to be smaller than expected. While rechecking her assumptions, she finds out that aluminum stocks are more likely to lose their value in the next to months than gaining more value because of an economic recession. Thus, the amount of stock price change in each month is normally distributed with mean -3 and standard deviation 2. With this correct representation of negative drift in the stock prices, how much should o^3 pay for this option if its strike price is $S = 50$?
   c) Another finance manager bets that Chinese aluminum consumption will drive the prices up. Thus, the amount of stock price change in each month will be normally distributed with mean 4 and standard deviation 2. With this positive drift in the stock prices, how much should o^3 pay for this option if its strike price is $S = 50$?
   d) Comment if a put option is more valuable when an economic recession or an expansion is likely.

2. Spice problem 11.5 on page 278 of the textbook.

1.2 Solutions

ANSWER for Exercise 1:

a) The option says that the buyer can sell the stock at the strike price of 50 to the seller. The buyer will do so if the stock price two months later is less than 50. Otherwise, the buyer will sell the stock in the market without exercising the put option. We need a handle on the price of the stocks two months from now. Since the stock value changes by $N(0, 2^2)$ every month of the two months, we can call these changes as D_1 and D_2 for month 1 and 2, respectively. Equipped with this notation, the price two month later is $\mu + D_1 + D_2$.

The value of the option is $S - \mu - D_1 - D_2$ if it is exercised, i.e., $S > \mu - D_1 - D_2$. Otherwise, it is zero. Then the value of option is

$$\max\{S - \mu - D_1 - D_2, 0\},$$

which is a random variable. Now, in advance of observing D_1, D_2, we can only talk about the expected value of the option:

$$\mathrm{E}\max\{S - \mu - D_1 - D_2, 0\}.$$

Note that the expected value of the put option has the same expression as the left over inventory at the end of a period. Namely, think of $S - \mu$ as the basestock value and think of D_1, D_2 as periodic demands so that $\mathrm{E}\max\{S - \mu - D_1 - D_2, 0\}$ can be interpreted as the left over inventory. Simplifying the leftover inventory expression, we obtain $\mathrm{E}\max\{10-D,0\}$ where $D=D_1+D_2$ so it has a Normal distribution with mean 0 and standard deviation $\sqrt{2^2+2^2}$.

To compute the leftover inventory, we can consider stockouts. For any number a, we have $a=\max\{a,o\}-\max\{-a,0\}$, or equivalently $\max\{-a,0\}=\max\{a,o\}-a$. Taking $-a=10-D$, we obtain $\mathrm{E}\max\{10-D,0\}=\mathrm{E}\max\{D-10,0\}-\mathrm{E}(D-10)$, or

$$\begin{aligned}\underbrace{\mathrm{E}\max\{10-D,0\}}_{\textit{Expected leftover inventory}} &= \underbrace{\mathrm{E}\max\{D-10,0\}}_{\textit{Expected stockout}}+\underbrace{10-\mathrm{E}(D)}_{\textit{Safety stock}}\\ &= \mathrm{E}\max\{D-10,0\}+10.\end{aligned}$$

What remains is to compute $\mathrm{E}\max\{D-10,0\}$, which has a an associated z value of $10-0/\sqrt{8}\approx 3.54$. Then

$$\begin{aligned}\mathrm{E}\max\{D-10,0\} &= \sigma[normdist(z,0,1,0)-z(1-normdist(z,0,1,1))]\\ &= \sqrt{8}[normdist(3.54,0,1,0)-3.54(1-normdist(3.54,0,1,1))]\\ &= \sqrt{8}*4.98*10^{-5}\\ &\approx 0.\end{aligned}$$

Then the value of the put option is $\mathrm{E}\max\{10-D,0\}=10$.

b) Similar to part a), we want to compute $\mathrm{E}\max\{10-D,0\}$ where $D=D_1+D_2$ is Normally distributed with mean (-3)+(-3) and standard deviation $\sqrt{2^2+2^2}$. Then the associated z value becomes $(10-(-6))/\sqrt{8}\approx 5.66$.

$$\begin{aligned}\mathrm{E}\max\{D-10,0\} &= \sigma[normdist(z,0,1,0)-z(1-normdist(z,0,1,1))]\\ &= \sqrt{8}[normdist(5.66,0,1,0)-5.66(1-normdist(5.66,0,1,1))]\\ &= \sqrt{8}*1.26*10^{-9}\\ &\approx 0.\end{aligned}$$

Then the value of the put option is $\mathrm{E}\max\{10-D,0\}=\mathrm{E}\max\{D-10,0\}+10-\mathrm{E}(D)=0+10-(-6)=16$.

c) Similar to part b), we want to compute $\mathrm{E}\max\{10-D,0\}$ where $D=D_1+D_2$ is Normally distributed with mean (4)+(4) and standard deviation $\sqrt{2^2+2^2}$. Then the associated z value becomes $(10-8)/\sqrt{8}\approx 0.71$.

$$\begin{aligned}\mathrm{E}\max\{D-10,0\} &= \sigma[normdist(z,0,1,0)-z(1-normdist(z,0,1,1))]\\ &= \sqrt{8}[normdist(0.71,0,1,0)-0.71(1-normdist(0.71,0,1,1))]\\ &= \sqrt{8}*0.14\\ &\approx 0.4\end{aligned}$$

Then the value of the put option is $\mathrm{E}\max\{10-D,0\}=\mathrm{E}\max\{D-10,0\}+10-\mathrm{E}(D)=0.4+10-(8)=2.4$.

d) By comparing parts b) and c), we see that the value of the put option drops when the stock prices are likely to increase. A put option is more valuable when a recession is expected.

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ANSWER for Exercise 2:

a) Leadtime is four weeks, we ned demand during $l+1$=5 weeks. Quarterly demand is given in the question and there are 4*4.33=13 weeks per quarter. Mean demand during 5 weeks is $\mu=(5/13)*415=159.62$ and the standard deviation of the demand during 5 weeks is $\sigma=\sqrt{5/13}*154=95.51$.

When the fill rate is 0.9925, 0.75% of the demand is backordered, that is 31.92*0.0075=0.2394 units of backorder per week. Then we must have $0.2394=95.51*L(z)$ or $L(z)=0.00241$. We have $L(2.43)=0.0025$

and $L(2.44) = 0.0024$, so let us pick $z = 2.44$. Then the basestock (order-up-to) level is $S = 159.62 + (2.44)95.51 = 392.66$.

b) We have Norminv(0.9925, 159.62, 95.51)=391.94 from Excel, so $S = 391.94$. We also have Norminv(0.9925, 0, 1)=2.43, so $S = 159.62 + (2.43)95.51 = 391.71$, practically the same as 391.94. We find the same z-value of $z = 2.43$ from the normal distribution tables. This value also gives $S = 391.71$. As expected, different methods can yield (due to computational accuracy) slightly different but virtually the same answer.

c) The holding cost is 0.75 and the backorder penalty cost is 50 per ounce. The critical ratio is $50/(0.75+50) = 0.9852$. The rest is repeating the steps in b). We have Norminv(0.9852, 159.62, 95.51)=367.39 from Excel, so $S = 367.39$. We also have Norminv(0.9925, 0, 1)=$2.175 \approx 2.18$, so $S = 159.62 + (2.18)95.51 = 367.83$, practically the same as 367.39. We find the same z-value of $z = 2.18$ from the normal distribution tables. This value also gives $S = 367.83$.

□

2 Exercises

1. **[Call Option for 2 Periods]** A finance manager at Opt-Out-Options (o^3) is valuing a call option for aluminum in April. The buyer of the call option has the right – but not the obligation – to buy a stock from the seller of the option at a certain time (the expiration date) for a certain price (the strike price). The option seller commits to sell the stock if the option buyer wants to buy. Consider a call option that will be exercised by o^3 in June, two months from now. The option is for an aluminum stock whose current value is $\mu = 40$, but the stock value changes every month. The amount of change in each month is normally distributed with mean 4 and standard deviation 2. How much should o^3 pay for this option if its strike price is $S = 50$?

2. **[Blood Inventory]** The Plano Presbyterian Hospital keeps an inventory of A Rh positive blood bags of 1 liter each. The hospital targets to have 10 bags every morning and estimates its daily demand to be normally distributed with mean of 8 liters and a standard deviation of 1 liter. The hospital places orders to the regional Red Cross DC every morning to replenish its blood inventory but receives these orders with a lead time of 1 day.
   a) Suppose we are on Wed morning and experienced demands of 10 and 6 bags of blood on Mon and Tue, what should the order size be on Wed morning?
   b) If we have pipeline inventory of 4 bags and an inventory position of 2 bags on a day, what is the inventory level on that day?
   c) What is the in-stock probability with the parameters given in the question statement above?
   d) What is the expected backorder with the parameters given in the question statement above?