# Basestock Model

## Chapter 13

# Learning Goals

- Basestock policy:Inventory management when the leftover inventory is not salvaged but kept for the next season/period
- Demand during lead time
- Inventory position vs. inventory level

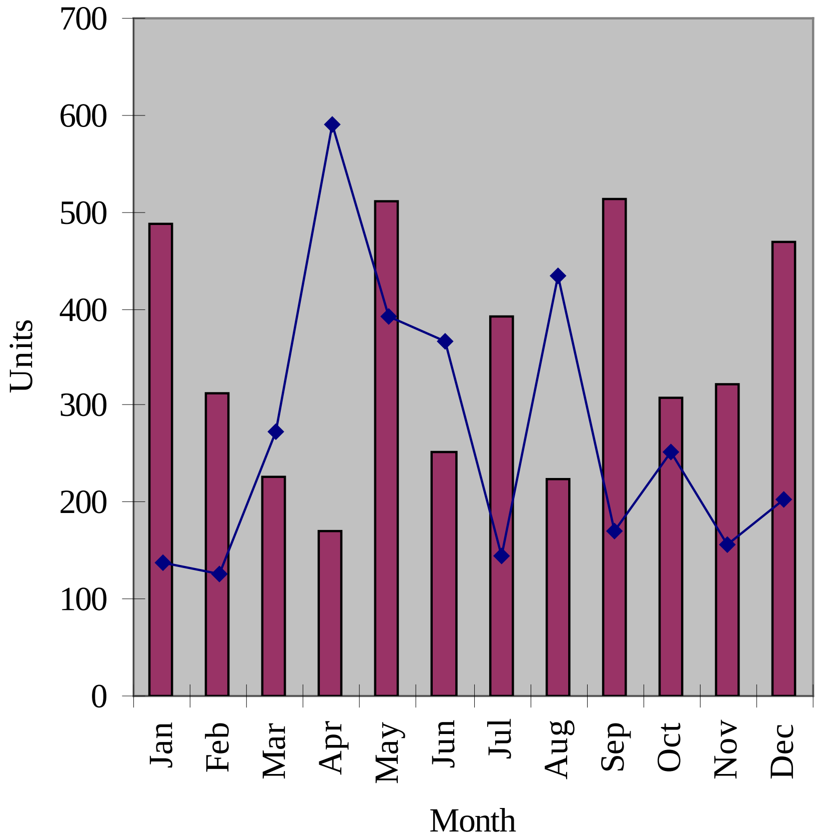
# Medtronic's InSync pacemaker supply chain


- Supply chain:
  - One distribution center (DC) in Mounds View, MN.
  - About 500 sales territories throughout the country.
    - Consider Susan Magnotto's territory in Madison, Wisconsin.

- Objective:
  - Because the gross margins are high, develop a system to minimize inventory investment while maintaining a very high service target, e.g., a 99.9% in-stock probability or a 99.9% fill rate.

# InSync demand and inventory, the DC Normal distribution

**DC receives pacemakers with a delivery lead time of 3 weeks.**




DC shipments (columns) and end of month inventory (line)

Average monthly demand = 349 units

Standard deviation of demand = 122.28

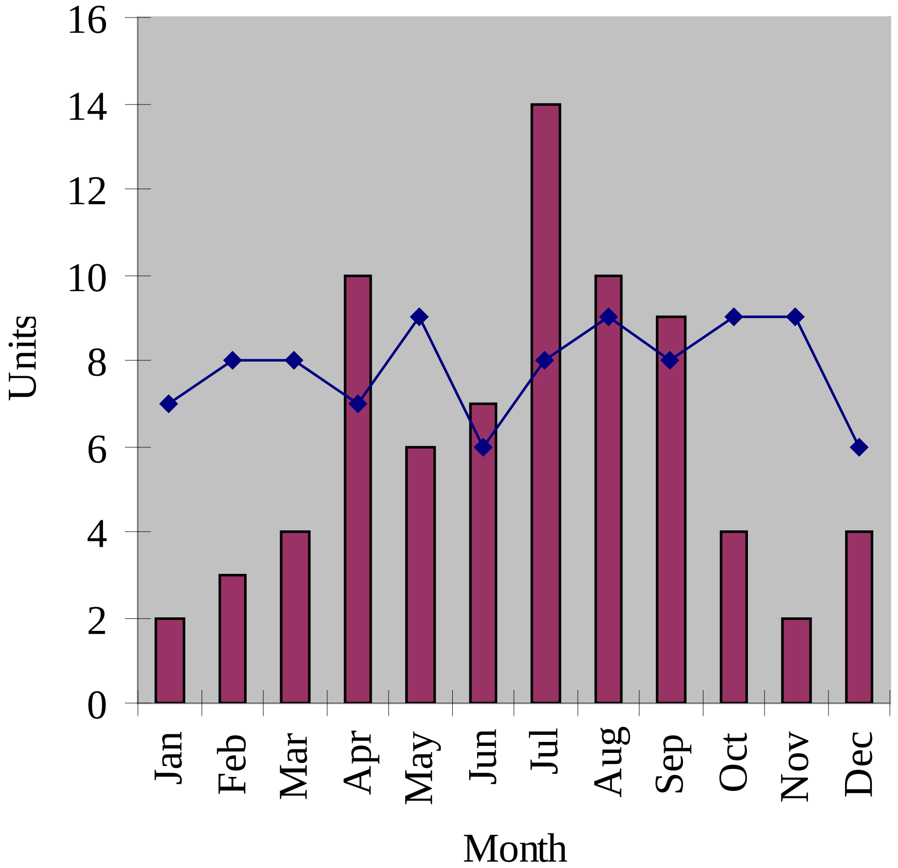
Average weekly demand = 349/4.33 = 80.6

Standard deviation of weekly demand =

$$122.38 / \sqrt{4.33} = 58.81$$

(The evaluations for weekly demand assume 4.33 weeks per month and demand is independent across weeks.)

# InSync demand and inventory, Susan's territory




Susan's shipments (columns) and end of month inventory (line)

Total annual demand = 75 units

Average daily demand = 0.29 units (75/260), assuming 5 days per week.

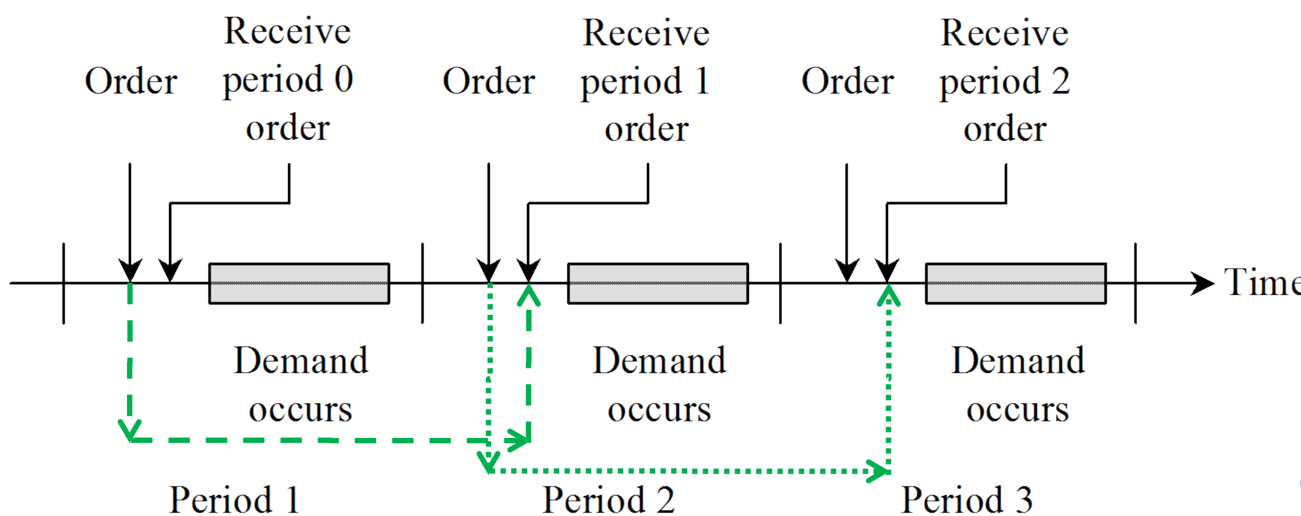
Poisson demand distribution works better for slow moving items

# Order Up-To (=Basestock) Model

# Sequence of events: Timing in the basestock (=order up-to) model

- Time is divided into periods of equal length, e.g., one hour, one month.
- During a period the following sequence of events occurs:
  - A replenishment order can be submitted.
  - Inventory is received.
  - Random demand occurs.
- Lead time $l$: a fixed number of periods after which an order is received. Recall the production planning example of LP notes.

An example with $l = 1$



# Newsvendor model vs. Order up-to model

- Both models have uncertain future demand, but there are differences…

| | Newsvendor | Order up-to |
|---|---|---|
| Inventory obsolescence | After one period | Never |
| Number of replenishments | One | Unlimited |
| Demand occurs during replenishment | No | Yes |

- Newsvendor applies to short life cycle products with uncertain demand and the order up-to applies to long life cycle products with uncertain demand.

# The Order Up-To Model:

# Model design and implementation

# Order up-to model definitions

- *On-hand inventory* = the number of units physically in inventory ready to serve demand.
- *Backorder* = the total amount of demand that has not been satisfied:
  - All backordered demand is eventually filled, i.e., there are no lost sales.
- *Inventory level* = *On-hand inventory* - *Backorder*.

- *On-order inventory / pipeline inventory* = the number of units that have been ordered but have not been received.
- *Inventory position* = *On-order inventory* + *Inventory level*.
- *Order up-to level S*
  - the maximum inventory position we allow.
  - sometimes called the *base stock level*.
  - this is the target inventory level we want to have in each period before starting to deal with that period's demand.

# Why inventory position but not inventory level?

- Suppose orders are daily (=period), like cleaning aisle at a Wal Mart
- On Monday morning at 8 am, the inventory level is 2 plastic 1 gallon containers for Clorox regular bleach.
- The ideal number of containers to start the day with is 10 and daily demand is 1.
- Lead time is slightly over 1 day. Monday's order is received on Tuesday at 9 am.
- Should we order 10-2=8 containers on Monday?
  - Yes, if the pipeline inventory is 0.
  - No, if the pipeline inventory is 1. This pipeline inventory arrives at 9 am on Monday. Effectively this makes the inventory available to us 3 rather 2 containers for most of Monday.
  - No, if the pipeline is 2, 3, or, 10 or more containers.
- We have already ordered the pipeline inventory which must be taken into account when issuing new orders:
      » inventory position = inventory level + pipeline inventory

# Order up-to model implementation

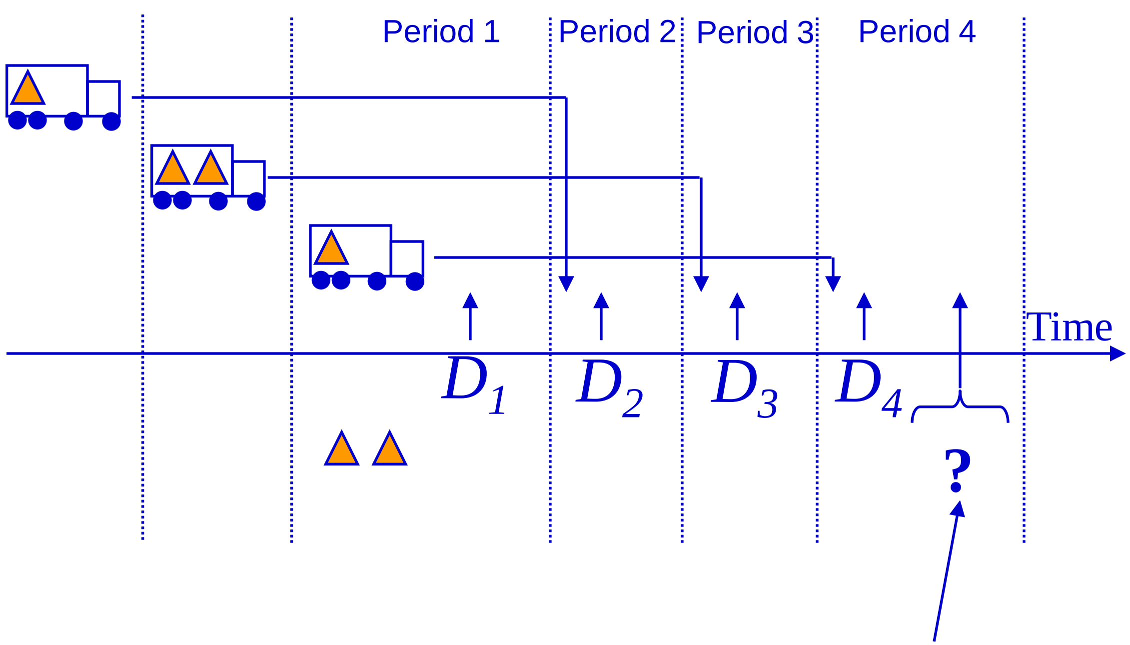
- *Each period's order quantity = S – Inventory position*
  - Suppose *S* = 4.
    - » If a period begins with an inventory position = 1, then three units are ordered.
      - (4 – 1 = 3 )
    - » If a period begins with an inventory position = -3, then seven units are ordered
      - (4 – (-3) = 7)
    - » If a period begins with an inventory position = 5, then no units are ordered
      - (4 – 5 = -1), how can the inventory position be 5 when S=4?
- A period's order quantity = the previous period's demand:
  - Suppose *S* = 4.
    - » If demand were 10 in period 1, then the inventory position at the start of period 2 is 4 – 10 = -6, which means 10 units are to be ordered in period 2.
  - The order up-to model is a *pull system* because inventory is ordered in response to demand.
  - But S is determined by the forecasted demand.

# The Basestock Model:

## Performance measures

# What determines the inventory level?

- Short answer:
  - *Inventory level* at the end of a period = *S* minus demand over *l* +1 periods.
- Example with *S* = 6, *l* = 3, and 2 units on-hand at the start of period 1
  - Pipeline: 1 to arrive in Period 2; 2 to arrive in Period 3; 1 to arrive in Period 4
  - Inventory level + Pipeline=6= *S*




Keep in mind:

**Before meeting demand in a period, Inventory level + Pipeline = S.**

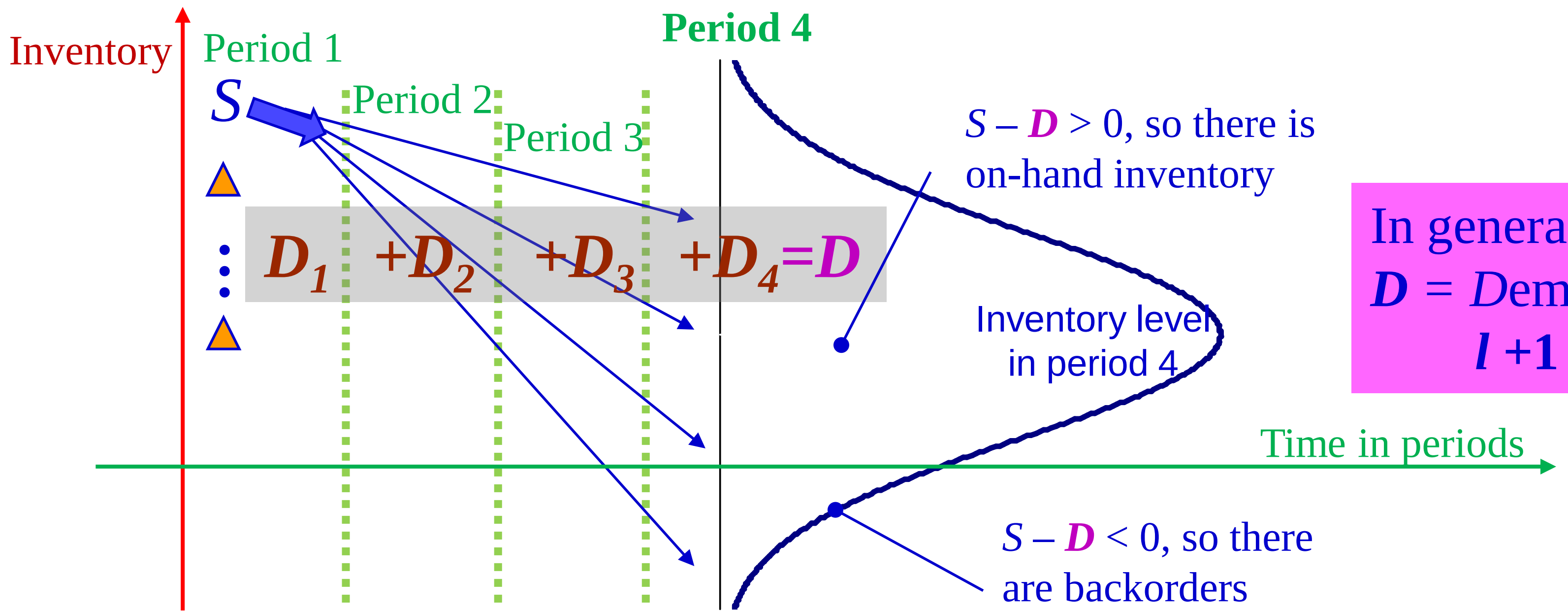
**All inventory on-order at the start of period 1 arrives before meeting the demand of period 4**

**Orders in periods 2-4 arrive after period 4; They are irrelevant.**

***All demand is satisfied so there are no lost sales.***

***Inventory level at the end of period 4*** $= 6 - D_1 - D_2 - D_3 - D_4 = S - D_1 - D_2 - D_3 - D_4$

# Expected on-hand inventory and backorder




In general, $\boldsymbol{D}$ = $D$emand over $\boldsymbol{l}$ **+1** periods

- This is like a Newsvendor model in which the order quantity is *S* and the demand distribution is demand over *l* +1 periods.
- Bingo,
  - *Expected on-hand inventory* at the end of a period can be evaluated like *Expected left over inventory* in the Newsvendor model with *Q* = *S*.
  - *Expected backorder* at the end of a period can be evaluated like *Expected lost sales* in the Newsvendor model with *Q* = *S*.

# Stockout and in-stock probabilities, on-order inventory and fill rate

- The stockout probability is the probability that at least one unit is backordered in a period:

$$\begin{aligned} Stockout\ probability &= Prob\{Demand\ over\ (l+1)\ periods > S\} \\ &= 1 - Prob\{Demand\ over\ (l+1)\ periods \le S\} \end{aligned}$$

- The in-stock probability is the probability all demand is filled in a period:

$$\begin{aligned} In\text{-}stock\ probability &= 1 - Stockout\ probability \\ &= Prob\{Demand\ over\ (l+1)\ periods \le S\} \end{aligned}$$

- *Expected on-order inventory = Expected demand over one period* x *lead time*
  - This comes from Little's Law. Note that it equals the expected demand over *l* periods, not *l* +1 periods.

- The fill rate is the fraction of demand within a period that is NOT backordered:

$$Fill\ rate = 1 - \frac{Expected\ backorder}{Expected\ demand\ in\ one\ period}$$

# Demand over *l*+1 periods

- DC:
  - The period length is one week, the replenishment lead time is three weeks, $l = 3$
  - Assume demand is normally distributed:
    - Mean weekly demand is 80.6 (from demand data)
    - Standard deviation of weekly demand is 58.81 (from demand data)
    - Expected demand over $l$ +1 weeks is (3 + 1) x 80.6 = 322.4
    - Standard deviation of demand over $l$ +1 weeks is $\sqrt{3+1} \times 58.81 = 117.6$

- Susan's territory:
  - The period length is one day, the replenishment lead time is one day, $l$ =1
  - Assume demand is Poisson distributed:
    - Mean daily demand is 0.29 (from demand data)
    - Expected demand over $l$+1 days is 2 x 0.29 = 0.58
    - Recall, the Poisson is completely defined by its mean (and the standard deviation is always the square root of the mean)

# DC's *Expected backorder* with $S = 625$

- *Expected backorder* is analogous to the *Expected lost sales* in the Newsvendor model:
  - Suppose order up-to level $S = 625$ at the DC
  - Normalize the order up-to level: $z = \dfrac{S-\mu}{\sigma} = \dfrac{625-322.4}{117.6} = 2.57$
  - Lookup $L(z)$ in the Standard Normal Loss Function Table: $L(2.57)=0.0016$
  - Convert expected lost sales, $L(z)$, for the standard normal into the expected backorder with the actual normal distribution that represents demand over $l$+1 periods:

$$Expected\ backorder = \sigma \times L(z) = 117.6 \times 0.0016 = 0.19$$

  - Therefore, if $S = 625$, then on average there are 0.19 backorders at the end of any period at the DC.

# Other DC performance measures with $S$ = 625

$$\textit{Fill rate} = 1 - \frac{\textit{Expected backorder}}{\textit{Expected demand in one period}} = 1 - \frac{0.19}{80.6} = 99.76\%.$$

So 99.76% of demand is filled immediately (i.e., without being backordered).

$$\text{Expected on-hand inventory} = S - \left(D - \max\{D - S, 0\}\right)$$

Demand that can be serviced

$$\textit{Expected on-hand inventory} = S\textit{-Expected demand over } (l+1) \textit{ periods}$$
$$+ \textit{Expected backorder}$$
$$= 625 - 322.4 + 0.19 = 302.8.$$

So on average there are 302.8 units on-hand at the end of a period.

$$\textit{Expected on-order inventory} = \textit{Expected demand in one period} \times \textit{Lead time}$$
$$= 80.6 \times 3 = 241.8.$$

So there are 241.8 units on-order at any given time.

# The Order Up-To Model:

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## Choosing an order up-to level *S* to meet a service target

# Choose $S$ to hit a target in-stock with normally distributed demand

- Suppose the target in-stock probability at the DC is 99.9%:
  - From the Standard Normal Distribution Function Table, $\Phi(3.08)=0.9990$; $\Phi$ is the cumulative density for Standard Normal
  - So we choose $z = 3.08$
  - To convert $z$ into an order up-to level:

$$S = \mu + z \times \sigma = 322.4 + 3.08 \times 117.6$$
$$= 685$$

  - Note that $\mu$ and $\sigma$ are the parameters of the normal distribution that describes demand over $l + 1$ periods.
  - Or, use S=norminv(0.999,322.4,117.6)

# Choose *S* to hit a target fill rate with normally distributed demand

- Find the *S* that yields a 99.9% fill rate for the DC.
- Step 1: Evaluate the target lost sales

$$1\text{-Fill rate} = \frac{\text{Standard deviation of demand over } (l+1) \text{ periods} * L(z)}{\text{Expected demand in one period}}$$

$$L(z) = \left(\frac{\text{Expected demand in one period}}{\text{Standard deviation of demand over } (l+1) \text{ periods}}\right)(1 - \text{Fill rate})$$

$$= \left(\frac{80.6}{117.6}\right)(1 - 0.999) = 0.0007$$

- Step 2: Find the *z* that generates that target lost sales in the Standard Normal Loss Function Table:
  - $L(2.81) = L(2.82) = L(2.83) = L(2.84) = 0.0007$
  - Choose $z = 2.84$ to be conservative (higher *z* means higher fill rate)
- Step 3: Convert z into the order up-to level: S=322.4+2.84*117.62=656

# Summary

- Basestock policy: Inventory management when the leftover inventory is not salvaged but kept for the next season/period
- Expected inventory and service are controlled via the order up-to (basestock) level:
  - The higher the order up-to level the greater the expected inventory and the better the service (either in-stock probability or fill rate).
- Demand during lead time
- Inventory position vs. inventory level

# The Order Up-To Model:

## Computations with Poisson Demand

## The rest is not included in OPRE 6302 exams

# Performance measures in Susan's territory

- Look up in the Poisson Loss Function Table expected backorders for a Poisson distribution with a mean equal to expected demand over $l$+1 periods:

| Mean demand = 0.29 | | | Mean demand = 0.58 | | |
|---|---|---|---|---|---|
| $S$ | $F(S)$ | $L(S)$ | $S$ | $F(S)$ | $L(S)$ |
| 0 | 0.74826 | 0.29000 | 0 | 0.55990 | 0.58000 |
| 1 | 0.96526 | 0.03826 | 1 | 0.88464 | 0.13990 |
| 2 | 0.99672 | 0.00352 | 2 | 0.97881 | 0.02454 |
| 3 | 0.99977 | 0.00025 | 3 | 0.99702 | 0.00335 |
| 4 | 0.99999 | 0.00001 | 4 | 0.99966 | 0.00037 |
| 5 | 1.00000 | 0.00000 | 5 | 0.99997 | 0.00004 |

$F(S)$ = *Prob* {*Demand is less than or equal to S*}

$L(S)$ = loss function = *expected backorder* = expected amount demand exceeds $S$

- Suppose $S$ = 3:
  - Expected backorder = 0.00335
  - In-stock = 99.702%
  - Fill rate = 1 – 0.00335 / 0.29 = 98.84%
  - Expected on-hand = $S$–demand over $l$+1 periods+backorder = 3–0.58+0.00335 = 2.42
  - Expected on-order inventory = Demand over the lead time = 0.29

# What is the Poisson Loss Function

- As before we want to compute the lost sales=E(max{D-Q,0}), but when D has a Poisson distribution with mean μ
- The probability for Poisson demand is given as

$$P(D=d)=\frac{\mu^d e^{-\mu}}{d!} \quad \text{for } d \in \{0,1,2,3,.....\}$$

- Or, use Excel function Poisson(d,μ,0)
- Then the lost sales is

$$E(\max\{D-Q,0\})=\sum_{d=0}^{\infty}\max\{d-Q,0\}\frac{\mu^d e^{-\mu}}{d!}=\sum_{d=Q}^{\infty}(d-Q)\frac{\mu^d e^{-\mu}}{d!}$$

- You can use Excel to approximate this sum for large Q and small μ.
- Or, just look up the Table on p. 383 of the textbook.

# Choose S to hit a target in-stock with Poisson demand

- Recall:
  - Period length is one day, the replenishment lead time is one day, $l = 1$
  - Demand over $l + 1$ days is Poisson with mean 2 x 0.29 = 0.58
- Target in-stock is 99.9%

| S | *Probability* { *Demand over l+1 periods* <= *S* } |
|---|---|
| 0 | 0.5599 |
| 1 | 0.8846 |
| 2 | 0.9788 |
| 3 | 0.9970 |
| 4 | 0.9997 |
| 5 | 1.0000 |

These probabilities can be found in the Poisson distribution function table or evaluated in Excel with the function Poisson(*S*, *0.58*, 1)

- In Susan's territory, $S = 4$ minimizes inventory while still generating a 99.9% in-stock:

# Choose *S* to hit a target fill rate with Poisson demand

- Suppose the target fill rate is 99.9%
- Recall,

$$\text{Fill rate} = 1 - \frac{\text{Expected backorder}}{\text{Expected demand in one period}}$$

- So rearrange terms in the above equation to obtain the target expected backorder:

$$\text{Target expected backorder} = \text{Expected demand in one period} \times (1 - \text{Fill rate})$$

- In Susan's territory:

$$\text{Target expected backorder} = 0.29 \times (1 - 0.999) = 0.00029$$

- From the Poisson Distribution Loss Function Table with a mean of 0.58 we see that $L(4) = 0.00037$ and $L(5) = 0.00004$,
- So choose $S = 5$

# The Order Up-To Model:

## Appropriate service levels

# Justifying a service level via cost minimization

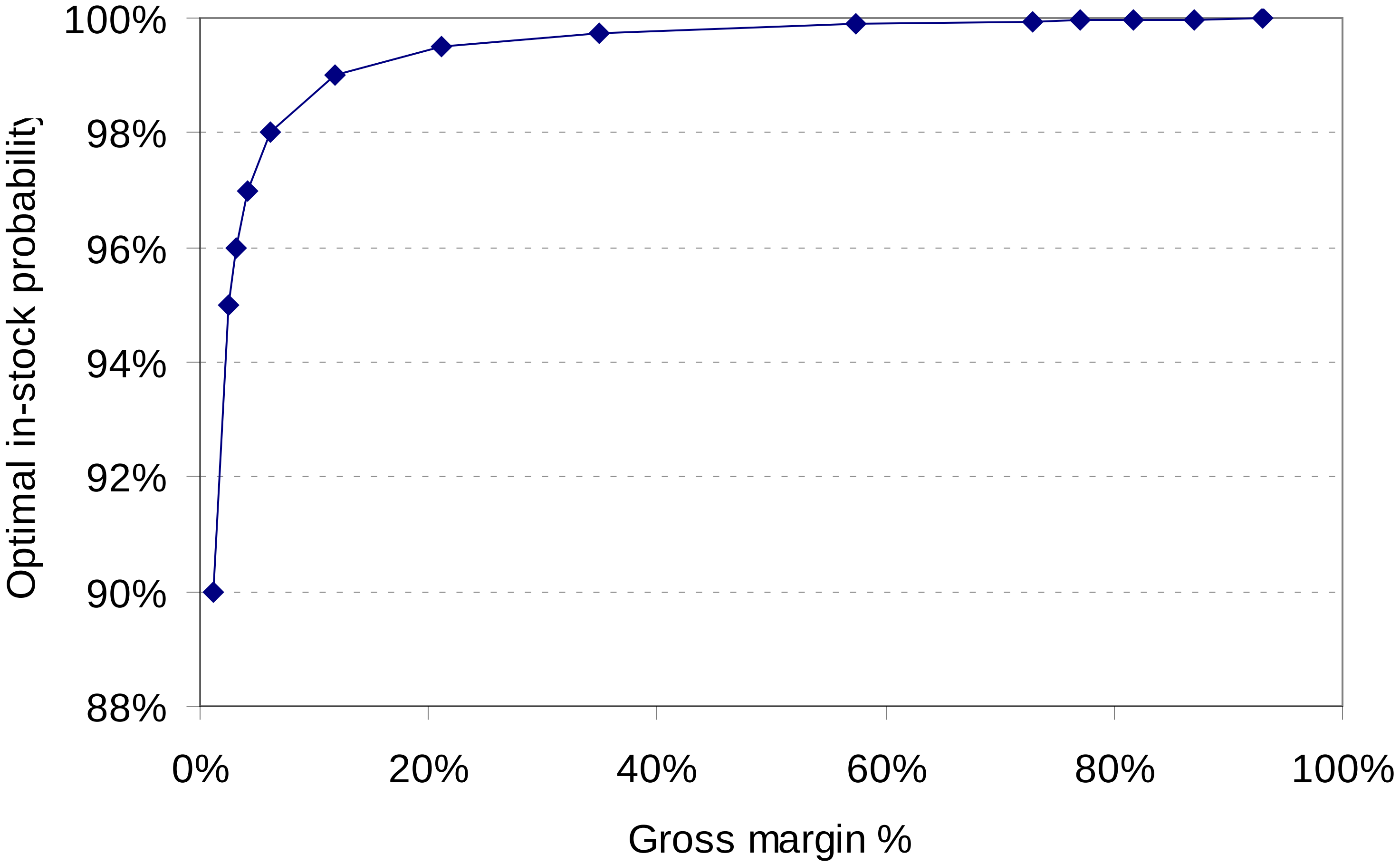
- Let $h$ equal the holding cost per unit per period
  - e.g. if $p$ is the retail price, the gross margin is 75%, the annual holding cost is 35% and there are 260 days per year, then $h = p \times (1 - 0.75) \times 0.35 / 260 = 0.000337 \times p$
- Let $b$ equal the penalty per unit backordered
  - e.g., let the penalty equal the 75% gross margin, then $b = 0.75 \times p$
- "Too much-too little" challenge:
  - If $S$ is too high, then there are holding costs, $C_o = h$
  - If $S$ is too low, then there are backorders, $C_u = b$
- Cost minimizing order up-to level satisfies

$$Prob\left\{Demand\ over\ \left(l+1\right)\ periods \le S\right\} = \frac{C_u}{C_o + C_u} = \frac{b}{h+b} = \frac{(0.75 \times p)}{(0.000337 \times p) + (0.75 \times p)}$$
$$= 0.9996$$

- Optimal in-stock probability is 99.96% because

# The optimal in-stock probability is usually quite high

- Suppose the annual holding cost is 35%, the backorder penalty cost equals the gross margin and inventory is reviewed daily.



# The Order Up-To Model:

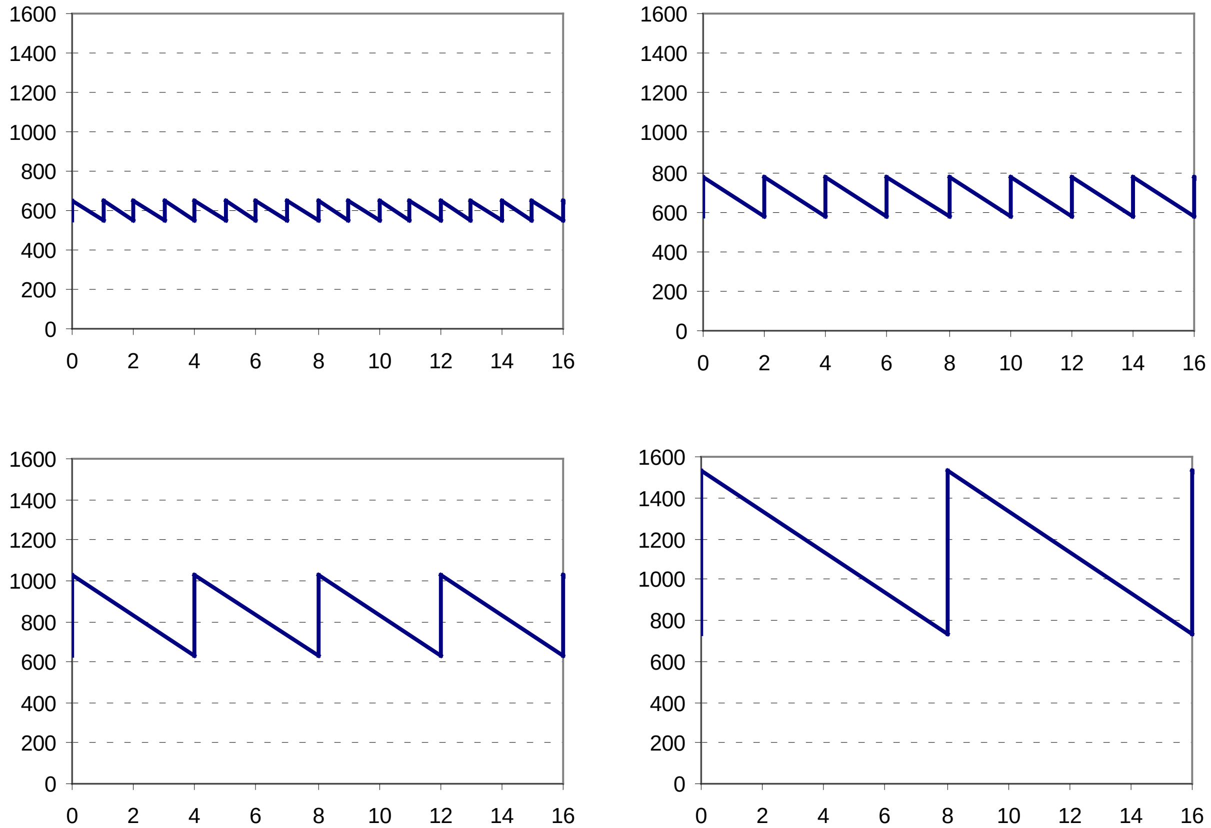
## Controlling ordering costs

# Impact of the period length

- Increasing the period length leads to larger and less frequent orders:
  - The average order quantity = expected demand in a single period.
  - The frequency of orders approximately equals 1/length of period.
- Suppose there is a cost to hold inventory and a cost to submit each order (independent of the quantity ordered)…
- … then there is a tradeoff between carrying little inventory (short period lengths) and reducing ordering costs (long period lengths)

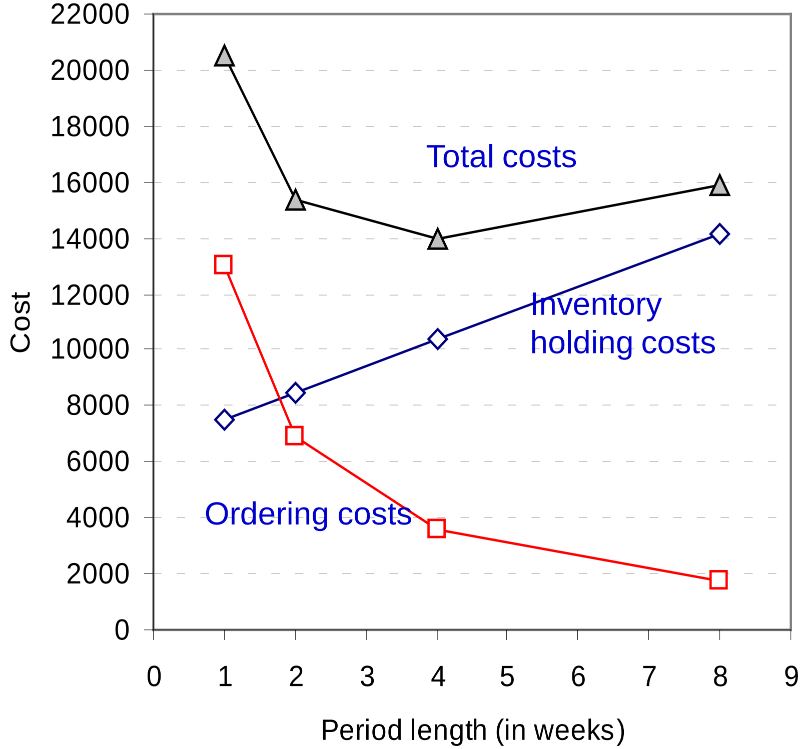
# Example with mean demand per week = 100 and standard deviation of weekly demand = 75.

- Inventory over time follows a “saw-tooth” pattern.
- Period lengths of 1, 2, 4 and 8 weeks result in average inventory of 597, 677, 832 and 1130 respectively:



# Tradeoff between inventory holding costs and ordering costs

- Costs:
  - Ordering costs = $275 per order
  - Holding costs = 25% per year
  - Unit cost = $50
  - Holding cost per unit per year =
    25% x $50 = 12.5



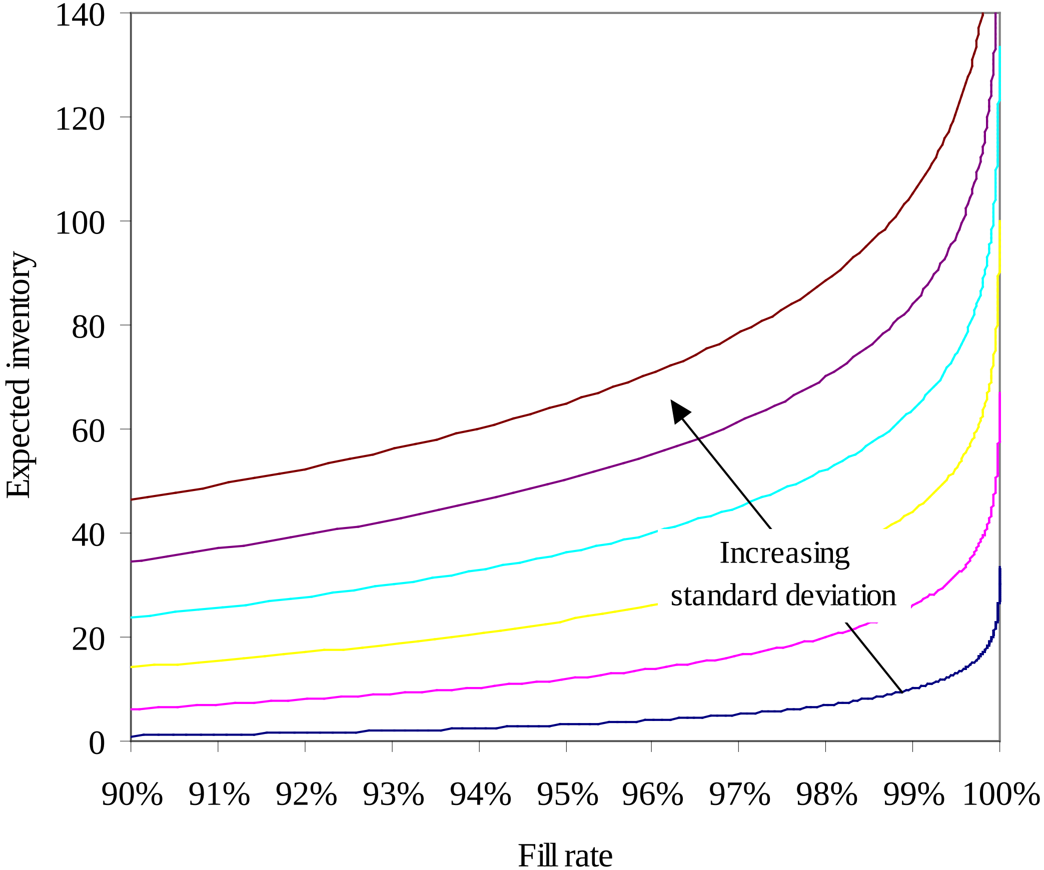

- Period length of 4 weeks minimize costs:
  - This implies the average order quantity is 4 x 100 = 400 units

- EOQ model: $Q = \sqrt{\frac{2 \times K \times R}{h}} = \sqrt{\frac{2 \times 275 \times 5200}{12.5}} = 478$

# The Order Up-To Model:

## Managerial insights

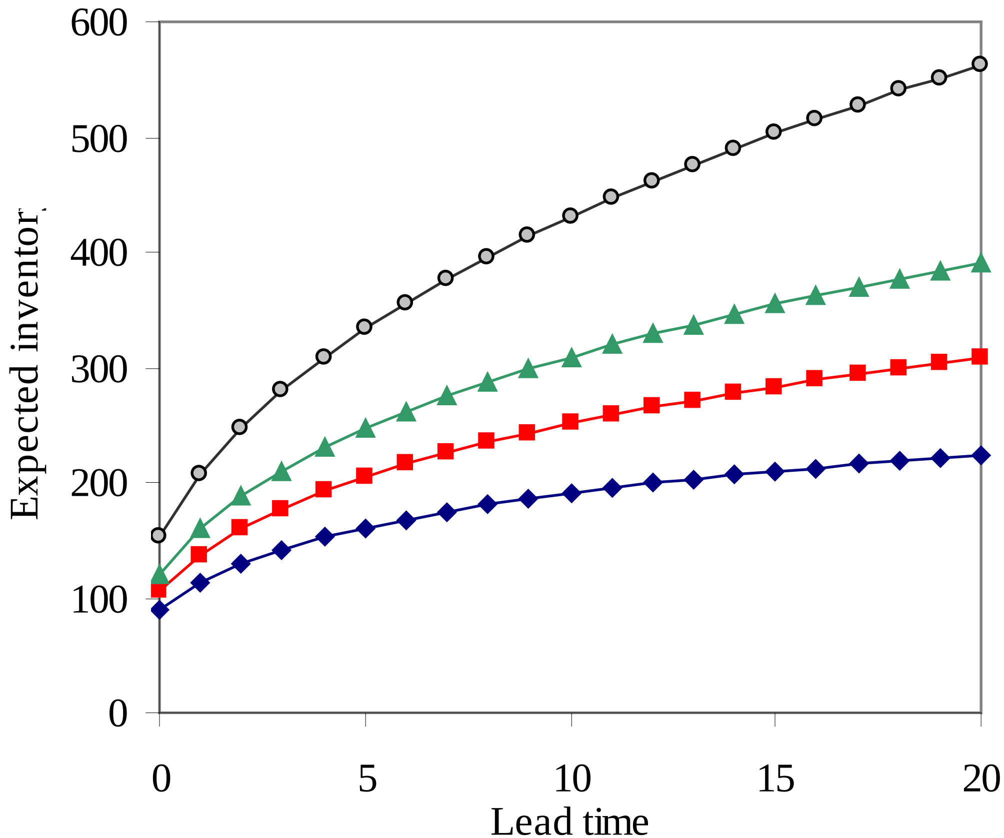
# Better service requires more inventory at an increasing rate




- More inventory is needed as demand uncertainty increases for any fixed fill rate.
- The required inventory is more sensitive to the fil rate level as demand uncertainty increases

The tradeoff between inventory and fill rate with Normally distributed demand and a mean of 100 over (l+1) periods. The curves differ in the standard deviation of demand over (l+1) periods: 60,50,40,30,20,10 from top to bottom.

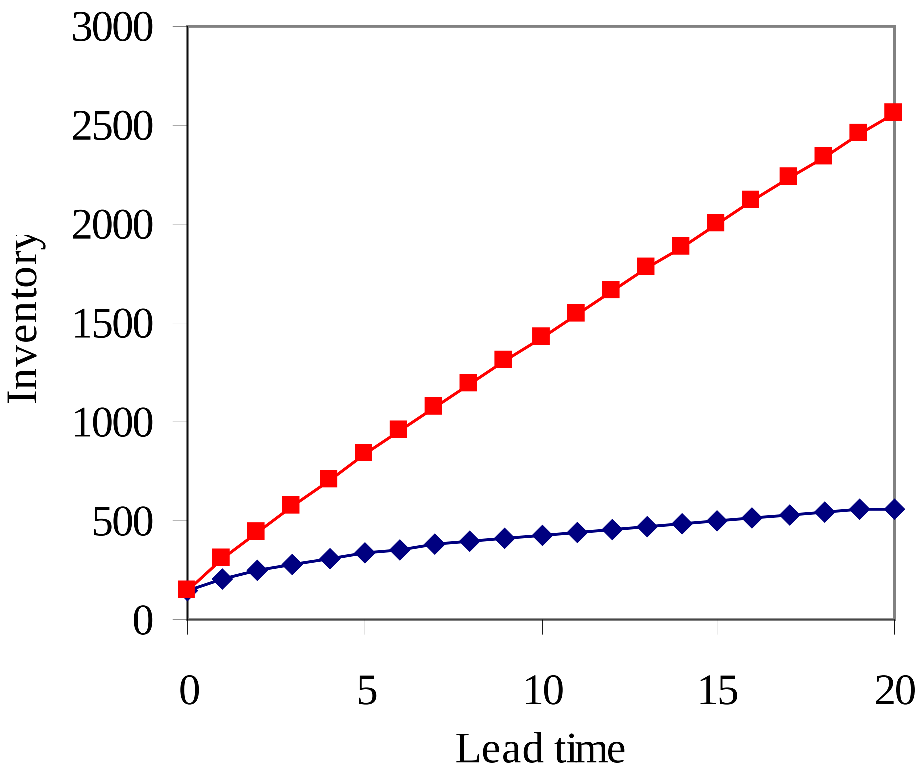
# Shorten lead times and to reduce inventory




The impact of lead time on expected inventory for four fill rate targets, 99.9%, 99.5%, 99.0% and 98%, top curve to bottom curve respectively. Demand in one period is Normally distributed with mean 100 and standard deviation 60.

- Reducing the lead time reduces expected inventory, especially as the target fill rate increases

# Do not forget about pipeline inventory




Expected inventory (diamonds) and total inventory (squares), which is expected inventory plus pipeline inventory, with a 99.9% fill rate requirement and demand in one period is Normally distributed with mean 100 and standard deviation 60

- Reducing the lead time reduces expected inventory and pipeline inventory
- The impact on pipeline inventory can be even more dramatic that the impact on expected inventory