Models

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Model Airplanes

When I was in grade school we used to build model airplanes out of kits. The frame was made out precut pieces of balsa wood, each having been carefully pinned according to the plans along a preprinted arc to obtain the appropriate curvature, and then cemented, piece to piece, with airplane glue. The fuselage was made out of tissue paper, first glued to the balsa frame, trimmed, then dampened with water to shrink it taut, and finally, when dry, lacquered and painted to make it stiff and realistic. The engine was merely a long rubber band that ran the internal length of the fuselage, from propeller block at the nose to tail, wound up by rotating the propeller many times and then let loose to unwind for a flight of perhaps ten seconds at best. If you were a really ambitious model builder you followed the instructions very carefully: you were supposed to sand off any excess glue on the frame so as leave no imperfections at all.

What was "model" about the model airplane? The Zippy in the kit I remember building was smaller than a real Zippy (I presume there was somewhere in the world of real airplanes an actual Zippy); it was lighter; it was made out of totally different materials. But it did capture two essential features of the putatively real Zippy: appearance and flight. The model looked a lot like a 'plane, and it could (briefly) fly.

Nevertheless, the model wasn't the Zippy. It wasn't the thing in itself. It was a model Zippy. It lacked seats, ailerons, proper windows, and doors among many other real-life details, and focused on only a few important features. What features are important depends on the model user. In my case, aged about ten, appearance and flight sufficed. At age three or four, crudely shaped wings, a body and a throaty airplane engine noise might have been enough. A few years older and I would have wanted a combustion engine and radio control. But, however complex, none of these model Zippys would have been a Zippy.

What constrains the construction of a model Zippy?

First, the user and his needs. What aspects of the actual complex airplane and its surroundings is the user most interested in emulating, testing, playing or tinkering with? An engineer needs a different model than a child does. Second, engineering and construction: how to put together the model reliably and effectively, with as much accuracy in the key features as possible. Third, science: deep down below, even though the Wright Brothers probably didn't know the partial differential equations of fluid flow, the possibility of heavier-than-air flight is built upon the science of mechanics and aerodynamics, on Newton's laws and the Navier-Stokes equations.

Models in Physics

Scientific models are different. Resemblance isn't enough. Scientific models aim at *divination* – foretelling the future, and controlling it, and there are two different approaches physicists use in creating such models.

Fundamental Models

The first approach is to aim at what physicists call a fundamental model, which by nature describes the *dynamics* behind events in the world. A fundamental model consists of a system of principles, usually formulated mathematically, that are then used to draw *causal* inferences about future behavior. *Dynamics* and *causality* are the essential characteristics. A fundamental model, particularly a successful one, is more of a theory than a model. To be a little pedantic, fundmantal models proclaim that "These are the laws of the uni verse." They describe the dynamics in God's terms; they seek to state eternal truths, like Moses descending from the mountain.

An example all physicists are familiar with is Newton's laws, which state that

$$F = ma$$

$$F = \frac{GMm}{r^2}$$

These are laws of cause and effect. Mass causes gravitational force. Force produces acceleration. Newton's theory isolates the appropriate variables to use, and specifies a causal relationship between them.

The gap between a successful theory and the part of the universe it describes is virtually non-existent: the theory is the universe, not a model of the universe; the universe is the theory.

Phenomenological Models

The second type of model is what physicists call a phenomenological model. Phenomenological models are also used to make predictions, but they don't state absolute principles; instead they make pragmatic analogies between things you'd like to understand and things you already do understand from more fundamental models. The analogies can be descriptive and useful, but analogies are self-limiting and there is often a toylike quality to them. In physics, one doesn't delude oneself into thinking of analogies as truth.

Phenomenological models don't say "This is a law." Instead, they say "Approximately, you can think of this part of the world as being a lot like this other kind of thing you already understand more deeply." Phenomenological models describe the world in man's language rather than God's.

A good example is the liquid drop model of the nucleus, where you think of an atomic nucleus as behaving much like an oscillating drop of fluid, even though you know that at a more reductionist level it's composed of protons and neutrons, themselves particles. Calibrating the liquid drop's parameters to match the known properties of the nucleus, you can then use the model to compute and predict values of other yet unmeasured properties.

The gap between a successful phenomenological model and the part of the universe it describes is finite, actually quite large. The model is an approximation, an effigy, a realistic-looking wax apple, Parrhasius's painted curtain that fooled his fellow artist, a wonderful resemblance but not the thing itself.

Models in Finance

What's the point of a model in finance?

It takes only a little experience to see that it's not the same as the point of a model in physics or applied mathematics. Here's a simple but prototypical financial model. How do you estimate the price of a seven-room apartment on Park Ave if someone tells you the market price of a typical two-room apartment in Battery Park City? Most likely, you figure out the price per square foot of the two-room apartment. Then you multiply by the square footage of the Park Ave apartment. Finally you make some rule-of-thumb corrections for location, park views, light, facilities and so on.

The model's critical parameter is the implied price per square foot. You calibrate the model to Battery Park City. Then you use it to interpolate or extrapolate to Park Ave. The price per square foot is *implied* from the market price of the Battery Park City apartments; it's not the construction price per square foot because there are other variables – exposure, quality of construction, neighborhood – that are subsumed into that one number.

The Aim of Financial Models

The way property markets use implied price per square foot illustrates the functions of financial models more generally.

Models are used to rank securities by value.

Implied price per square foot can be used to rank and compare many similar but not identical apartments. Apartments have manifold properties, as outlined above. Implied price per square foot provides a simple one-dimensional scale on which to *begin* ranking apartments by value. The single number given by implied price per square foot doesn't truly reflect the value of the apartment; it provides a starting point after which more qualitative factors must be taken into account.

Similarly, yield to maturity for bonds allows you to compare the values of many similar but not identical bonds, each with a different coupon and/or maturity, by mapping their yields onto a linear scale. The same applies to Price/Earnings (P/E) for stocks and option-adjusted spread (OAS) for mortgages or callable bonds. All of these metrics reduce a many-dimensioned problem to a single one. The Black-Scholes implied volatility of options provides a similar way of collapsing multiple-qualitied instruments (strike, expiration, underlyer …) onto a single value scale and then making pragmatic modifications to it.

Models are used to interpolate from liquid prices to illiquid ones

In finance, models are used less for divination than to interpolate or extrapolate from the known dollar prices of liquid securities to unknown dollar values of illiquid securities – in this case from the Battery Park City price to the Park Ave value. Models in finance don't predict the future; instead they allow you to compare different prices in the present. Similarly, OAS is used to interpolate from relatively liquid bonds to less liquid ones. Correspondingly, the Black-Scholes model proceeds from a known stock price and a riskless bond price to the unknown price of a hybrid security, an option, similar to the way one estimates the value of fruit salad from its constituent fruits, or, inversely, the way one estimates the price of one fruit from the price of the other fruits and the salad.

None of these metrics are strictly accurate or correct, but they all provide immensely helpful ways of beginning to estimate value.

Models transform intuitive linear quantities into nonlinear dollar values

In physics a theory predicts the future. In finance a model is also a means of translating intuition into dollar values. The apartment-value model transforms price per square foot into the dollar value of the apartment. It's intuitively easier to start from price per square foot (or per room) because it captures much of the variability of apartment prices. Similarly, P/E describes much of the variability of share prices. It's less easy to develop intuition about yield to maturity, option-adjusted spread, default probability or return volatility than it is to think about price per square foot. Nevertheless, all of these parameters are clearly related to value and easier to think about than dollar value itself. They are intuitively graspable, and the more sophisticated you get, the richer your intuition. Models develop by leapfrogging from simple intuitive mental concepts (volatility, for example) to the mathematics that describes it (geometric Brownian motion and the Black-Scholes model) to richer mental concepts (the volatility smile) to experience-based intuition about them to newer models (stochastic volatility models, for example) that incorporate the newer concept.

In contrast with fundamental or phenomenological model, the gap between a successful financial model and the correct value is nearly indefinable, because the fair value is finance's *fata morgana*, undefined by prices, which themselves are not stationary, and so model success is temporary at best. If fair value were strictly calculable, there would be no markets.

Fata Morgana is the Italian name for Morgan le Fay, the fairy half-sister of King Arthur (both "fata" and "fay" mean fairy.) The other uses of the phrase derive from this use, all meaning an illusion, or illusory prospect:

The qualities of models in different fields is summarized in the table below.

Field	Model Aims
Physics	Reproduction, Divination
Hobbyists	Resemblance
Finance	Ranking, Interpolation, Intuition

The Foundations of Financial Engineering

Science – mechanics, electrodynamics or molecular biology, for example – seeks to discover the fundamental principles that describe the world, and is usually reductive. Engineering is about using those principles, constructively, for a purpose.

Mechanical engineering is concerned with building devices based on the principles of mechanics (Newton's laws) suitably combined with empirical rules about more complex forces (friction, for example) that are too difficult to derive from first principles. Electrical engineering is the study of how to create useful electrical devices based on Maxwell's equations and solid state physics. Bio-engineering is the art of building prosthetics and other biologically active devices based on the principles of biochemistry, physiology and molecular biology.

What about financial modeling, financial engineering or quantitative finance? In a logically consistent world, financial engineering, layered above a base of solid financial science, would be the study of how to create functional financial devices – convertible bonds, warrants, credit default swaptions, etc. – that perform in desired ways, not just at expiration, but throughout their lifetime. Which brings us to financial science, the study of the fundamental laws of financial objects, be they stocks, interest rates, or whatever else a theory uses as atomic constituents. Here, unfortunately, lie dragons.

Brownian motion, the underpinning of much of quantitative finance, is indeed science, but it's accurate science only for small particles bumped around by invisible atoms. For stocks, the standard theory of geometric Brownian motion is an idealization that, while it captures some of the essential features of price uncertainty, is not a very good description of the detailed characteristics of their price distributions. Markets are plagued and blessed with anomalies that disagree with standard and non-standard theories. So, while we in financial engineering are rich in techniques (stochastic calculus, optimization, the Hamilton-Jacobi-Bellman equation, and so on), we don't (yet? ever?) have the right laws of science to exploit.

What solid laws and concepts do we have for building our ranking and translation models? In truth, only one.

The One Law of Financial Modeling

According to a legend, Hillel, a famous Jewish sage, was asked to recite the essence of God's laws while standing on one leg. "Do not do unto others as you would not have them do unto you," he is supposed to have said. "All the rest is commentary. Go and learn."

You can summarize the essence of quantitative finance on one leg too: "If you want to know the value of a security, use the known price of another security that's as similar to it as possible. All the rest is modeling. Go and build."

"Security" here refers not only to a single security but also a portfolio of securities. The wonderful thing about this law — it's valuation by analogy – when compared with almost everything else in economics, is that it dispenses with utility functions, the unobservable hidden variables whose ghostly presence permeates most of faux-quantitative economic theory. Financial economists refer to their essential principle as the law of one price, or the principle of no riskless arbitrage, which states that:

Any two securities with identical future payoffs, no matter how the future turns out, should have identical current prices.

The law of one price, this valuation by analogy, is the only genuine law we have in quantitative finance, and it is not a law of nature[1]. It's a general reflection on the practices of human beings, who, when they have enough time and enough information, will grab a bargain when they see one. The law usually holds in the long run, in well-oiled markets with enough savvy participants, but there are always short- or even longer-lived and persistent exceptions.

How do you use the law of one price to determine value? If you want to estimate the unknown value of a target security, you must find some other replicating portfolio, a collection of more liquid securities that, collectively, has the same future payoffs as the target, no matter how the future turns out. The target's value is then simply the value of the replicating portfolio.

Where do models enter? It takes a model to show that the target and the replicating portfolio have identical future payoffs under all circumstances. To demonstrate payoff identity, (1) you must specify what you mean by "all circumstances," for each security, and (2) you must find a strategy for creating a replicating portfolio that, in each future scenario or circumstance, will have payoffs identical to those of the target. That's what the Black-Scholes options model does; it tells you exactly how to replicate or manufacture fruit salad – an option – out of fruit – stock and bonds. The appropriate price should be the cost manufacture.

Valuation by analogy can suffer from "lack of an analog". After all we are not talking about valuing apartments on the West Side, of which thousands are being sold annually. That is not true for many financial products that are illiquid.

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[1] The time value of money, the benefits of diversification and the value of the right to choose are other useful principles. I thank Marcos Carreira for these comments.

The tricky part of building these models is specifying what you mean by "under all circumstances." In Black-Scholes all circumstances means a future in which stock returns are normally distributed and stock prices move continuously. Of course, that's not exactly true. Trying to specify "all circumstances" reminds me of the 1960s movie *Bedazzled*, starring Peter Cook and Dudley Moore. It retells the story of Faust, in which Dudley Moore plays a short-order cook in a Wimpy's in London who sells his soul to the devil in exchange for seven chances to specify the circumstances under which he can achieve his romantic aims with the Wimpy's waitress he covets. Each time the devil asks him to specify the romantic scenarios under which he believes he will succeed, he can't get them quite specific enough. He says he wants to be alone with the waitress in a beautiful place where they're both in love with each other. He gets what he wants – a snap of the devil's fingers and he and his mutually beloved are instantaneously transported a country estate, where he's a guest of the owner, her husband, whom her principles will not allow her to betray. In the final episode he wishes for them to be alone together and totally in love in a quiet place where no one will bother them. He gets his wish: the devil makes them both nuns in nunnery where everyone has taken a vow of silence. This is the same difficulty you have specifying future scenarios in financial models – like the devil, markets always outwit you. Even if they are not strictly random, their vagaries are too rich to capture in a few sentences or equations.

Model Risk

Risk is future uncertainty. A coin flip is risky. You know the current state of the coin, but not the future one. You can however, perform an infinite number of mental flips and reliably calculate the probability distribution of heads and tails, which will match a physical coin's probability distribution to the extent that the coin is separable from its surroundings and uninfluenced by them.

In this sense, a liquid stock price is risky. You (more or less) know the current price and have no idea about the direction of its future change. But you cannot perform an infinite number of mental stock price moves with any reliability; the stock, the market and the world are not clearly separable and influence each other, so that the probability distribution of stock prices can not be accurately known (and may not be time-invariant). The history of the world doesn't affect a coin flip. The history of the world does have a bearing on the next change in a stock's price. The risk of a stock price is qualitatively different from the risk of a coin flip.

Financial models interpolate from liquid to illiquid prices by analogy, and must necessarily change over time as the economic environment changes or as market participants become more sophisticated. The Black-Scholes model, for example, used to be regarded as adequate for valuing exotic options prior to the market crash of 1987, but is now often replaced by a range of extended models that incorporate local volatility, stochastic volatility or jumps. One doesn't know the truly correct current model, let alone the future one, and so the correct model is uncertain not only in the future but in the present too[2]. The word "risk" therefore inaccurately describes the indeterminate nature of

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[2] Emanuel Derman, *Markets and Models*, RISK, 14(7), 48-50, 2001.

financial models. If you want to call describe this state of ignorance as risk, then don't forget that it's a shorthand term for uncertainty, for something much vaguer than probabilistic risk. There is no ensemble of models each with an associated known probability of being right[3].

Idolatry

The greatest danger in financial modeling is therefore the age-old sin of idolatry. Financial markets are alive but a model is a limited human work of art. A model may be entrancing but no matter how hard you try, you will not be able to breath true life into it. To confuse the model with the world is to embrace a future disaster driven by the belief that humans obey mathematical rules.

Financial modelers must therefore compromise, must firmly decide what small part of the financial world is of greatest current interest, decide on its key features, and make a mock-up of only those. A model cannot include everything. If you are interested in everything you are interested in too much. A successful financial model must have limited scope; you must work with simple analogies; in the end, you are trying to rank complex objects on a low-dimensional scale. In physics there may one day be a Theory of Everything; in finance and the social sciences, you're lucky if there is a useable theory of anything.

Models are best regarded as a collection of parallel inanimate thought universes you can explore. Each universe should be consistent, but the actual financial and human world, unlike the world of matter, is going to be vastly more complex and vivacious than any model you make of it. You are always trying to shoe-horn the real world into one of the models you have to see how usefully it approximates the key features you're concerned with .

The right way to engage with a model is, like a fiction reader, to temporarily suspend disbelief, and then push it as far as you can. The success of the theory of options valuation, the best model economics can offer, is the story of a Platonically simple theory, taken more seriously than it deserves and then used extravagantly, with hubris, as a crutch to human thinking. "If a fool would persist in his folly, he would become wise," wrote Blake in *The Marriage of Heaven and Hell*. That's what options markets have done with options theory.

A little hubris is good. But catastrophes strike when hubris evolves into idolatry. Somewhere between these two extremes, a little north of common sense but still south of idolatry, lies the wise use of conceptual models.

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[3] This point has also been made by R. Cont, *Model Uncertainty and its Impact on the Pricing of Derivative Instruments*, Mathematical Finance, 16(3), 519–547, 2006.

Appendix: Avoiding Modeling Errors[4]

- Expect neither miracles nor formulas to be used mechanically or electronically; expect only aids to rational thinking.

- Many financial phenomena are not mathematically modelable in a useful way. You're worse off thinking you have a model and relying on it than simply realizing you don't, and then allowing for a margin of error.

- Since financial models translate intuitive qualities into dollar values, and since financial data are often sparse, unreliable and unstationary, models should be relatively simple. Complexity in the face of lack of highly detailed knowledge is pointless.

- Concepts first, then mathematical formulation, then calibration.

- Markets are by definition vulgar, and correspondingly the most useful models are wisely vulgar too, using variables that the crowd uses to describe the phenomena they observe. In physics it pays to drop down deep, several levels below what you can observe (think of Kepler, Newton, Einstein, Schrodinger), formulate an elegant principle, and then rise back to the surface again to work out the observable consequences. In finance, which lacks deep scientific principles, it's better to use models that have as direct as possible a path between observation and consequences. (Of course, over time crowds and markets get smarter and the definition of vulgarity changes to encompass increasingly sophisticated concepts.)

- Markets learns from experience and proceed to adopt new paradigms and make new mistakes. What's a good model today may be inappropriate tomorrow. Be sure to understand what your model is assuming and ignoring.

- Regard models as interdisciplinary endeavors. Financial models are generally not back-of-the-envelope formulas handed over to "coders" to turn into executable instructions. Modeling is multi-disciplinary: it touches on the practicality of doing business, on financial theory, on mathematical modeling and computer science, on computer implementation and on the construction of user interfaces. Models end up as computational computer programs embedded in human and machine interfaces that are themselves computer programs. The risks lie in the knowledge of the business, the applicability of the financial model, the mathematics and numerical analysis used to solve it, the computer science used to implement and present it, and in the transmission of information and knowledge accurately from one part of the model, in the larger sense of the word, to the next. It helps to be knowledgeable in all of these

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[4] For a more detailed discussion of modeling errors and how to avoid them, see Emanuel Derman, *Model Risk*, RISK 9(5) 139-145, 1996. See also Chapter 15 of *Risk Management* Michel Crouhy, Robert Mark, and Dan Galai, McGraw Hill, 2000.

areas in order to notice an error and then diagnose it.

To avoid errors, it's important to have modelers, programmers and users who all work closely together, understand each other's domains well enough to know what constitutes a warning symptom, and have a good strategy for testing a model and its limits. Too much specialization is harmful.

- Test complex models in simple cases first. Often there are simple special-case known solution; a deep-in-the-money call option is a forward, for example. Test the model at these boundaries first.

  Don't ignore small numerical discrepancies. Track down their origin. Small disagreements often serve as warnings of potentially large disagreements and disastrous errors under other scenarios.

- Usable financial models are software, and are subject to all the problems of writing, debugging and maintaining large scale software systems.

- Be reluctant to give up content for the sake of analytical elegance.

- Work closely with end users (traders, salespeople …) to embed the software in their environment in the most convenient way.

- Informed and patient users who clearly comprehend both the model and the method of solution are invaluable in discovering model errors.

- Efficient testing requires a good user interface.

- Diffuse a new model slowly outwards through the organization, from its developers to developers of other models to the end users.

- Pride of ownership: One of the best defences against modeling error is to ensure that both models and systems are built by people who love what they do, take pride in their work and are rewarded for doing it well.