OPRE 6201 Quiz Fall 2003

This is an open textbook and open lecture notes exam. You may use a calculator although leaving quantities in fractional forms is perfectly acceptable and preferable. Cellular communication devices (phones, laptops, palm pilots, etc.) **cannot** be used during this exam. **Do not forget** to define any variables you introduce. Unless otherwise stated, x_i always denotes the ith decision variable. Good luck!

NAME (please print): _____

Question	Out of	Points
1	20	
2	20	
3	20	
4	20	
Total	80	

1. Put **T** before a statement if you think that statement is true. Otherwise put **X**.

   1. ( ) Forecasting models can be classified as optimization models. X
   2. ( ) A bounded LP must have a bounded feasible region. X
   3. ( ) Basic solutions are sometimes feasible. T
   4. ( ) An LP can be unbounded and infeasible. X
   5. ( ) An LP with multiple optimal solutions has multiple objective functions. X
   6. ( ) $2x_1 - 7x_2 = 5$ is or can be written as linear inequalities. T

      A super True / False question:

   7. ( ) To solve the cheese problem of HW1, we can add the constraint $y_j \leq x_j$ instead of $y_j \leq d_{j+1}$. T

      If true, justify. If false, find a numerical example that satisfies the constraints but holds the cheese for more than 1 month.

      Constraints $y_j - x_j \leq 0$ and $y_j - d_{j+1} \leq 0$ are equivalent because $y_j - x_j = y_{j-1} - d_j$.

Each T/X is 2 points, the last is 8 points.

2. Consider the feasible region $2x_1 + x_2 \geq 20$, $3x_1 + 4x_2 \geq 60$, $x_1 + x_2 \geq 10$, $x_1 \leq 15$, $x_1 - x_2 \leq 10$ and $x_1, x_2 \geq 0$

a) Draw the feasible region. Identify all the Basic Feasible Solutions on your drawing and write their coordinates.

The feasible region has 4 corner points at (0,20), (4,12), (100/7,30/7) and (15,5). It is unbounded in the x_2 direction.

b) Draw the objective function "Minimize $Z = 3x_1 + 5x_2$" at the optimal point in your graph of a).

c) Provide the values for the decision variables at the optimal point. Provide the objective function value at the optimal point.

(100/7,30/7). 450/7

a) is 10, b) is 6 and c) is 4 points.

3. The Apex Television company has to decide on the number of 27 and 20 inch sets to be produced at one of its factories. Market research indicates that at most 40 of the 27 inch sets and 10 of the 20 inch sets can be sold per month. The maximum number of work hours available is 800 hours per month. A 27 inch set requires 15 work hours and a 20 inch set requires 10 work hours . Each 27 inch set produces a profit of $120 and the same number is $80 for 20 inch sets.

a) Formulate an LP to maximize the profit:

B: Number of 27 inch sets produced per month. S: Number of 20 inch sets produced per month.

Max $120B + 80T$
Subject to:
$B \leq 40$
$S \leq 10$
$15B + 10S \leq 800$
$B, S \geq 0$

b) Through commercials, TV set demand can be increased. For every $20 spent for commercials, 1 more 27 inch TV **and** 2 more 20 inch TV can be sold. Formulate an LP to maximize the profit with a budget of $400 for commercials.

C: Commercial budget spent for TVs.

Max $120B + 80T - C$
Subject to:
$B \leq 40 + C/20$
$S \leq 10 + C/10$
$C \leq 400$
$15B + 10S \leq 800$
$B, S \geq 0$

a) is 12 points, b) is 8 points.

4. Two cities generate waste and their wastes are sent to incinerators (=furnaces) for burning. Daily waste production and distances among cities and incinerators are below:

	Waste produced tons/day	Distance to incinerator A in miles	B in miles
City 1	500	30	20
City 2	400	36	42

Incineration reduces each ton of waste to 0.2 tons of debris, which must be dumped at one of the two landfills. It costs \$3 per mile to transport a ton of material (either debris or waste). Distances (in miles) among incinerators and landfills are shown below:

	Capacity tons/day	Incineration cost dollars/ton	Distance to landfills Northern in miles	Southern in miles
Incinerator A	500	40	5	8
Incinerator B	600	30	9	6

Incineration capacity and cost is based on the amount of waste input.

a) Formulate an LP that can be used to minimize the total cost of disposing waste of both cities.

Decision variables: Let $x_{i,j}$ be the waste sent from city i to incinerator j, $i \in \{1,2\}$ and $j \in \{A,B\}$. Let $y_{j,k}$ be the waste sent from incinerator i to ladfill j, $j \in \{A,B\}$ and $k \in \{S,N\}$. Some of these are consequential

Objective function: Minimize $3(30x_{1A}+20x_{1B}+36x_{2A}+42x_{2B}+5y_{AN}+8y_{AS}+9y_{BN}+6y_{BS})+40(x_{1A}+x_{2A})+30(x_{1B}+x_{2B})$

Constraints:

Waste produced per day: $500 = x_{1A} + x_{1B}$, $400 = x_{2A} + x_{2B}$.

Incinerator capacities: $x_{1A} + x_{2A} \leq 500$, $x_{1B} + x_{2B} \leq 600$.

Waste reduction: $0.2(x_{1A} + x_{2A}) = y_{AN} + y_{AS}$, $0.2(x_{1B} + x_{2B}) = y_{BN} + y_{BS}$.

Nonnegativity constraints.

b) After a quick look at the distance between cities and incinerators and also between incinerators and landfills, we conclude if incinerators are moved into the cities the problem data will change as:

	Waste produced tons/day	Distance to incinerator A	B
City 1	500	0	30
City 2	400	30	0

	Capacity tons/day	Incineration cost dollars/ton	Distance to landfills Northern	Southern
Incinerator A	500	40	35	38
Incinerator B	600	30	51	48

Explain why such moves may reduce transportaion costs. Now modify your LP to minimize total costs.

These moves save transportation costs because after burning only a small portion of waste remains for transportation.

Only the objective function needs modification.

Objective function: Minimize $3(0x_{1A}+30x_{1B}+30x_{2A}+0x_{2B}+35y_{AN}+38y_{AS}+51y_{BN}+48y_{BS})+40(x_{1A}+x_{2A})+30(x_{1B}+x_{2B})$

c) By moving the incinerators into cities, the solution in b) will improve the optimal objective value of the formulation in a). Would you approve the move or is there anything the analysis is missing, why? Answer in at most 4 sentences.

The move cannot be approved. It can cause tremendous health problems for city population if the incinerators release the air without filtering. If additional filters are to be built, they cost extra. Neither health nor extra filter costs is captured in our formulations.

The answers to this part can be very creative. What is to remember is that the models capture only some aspects of the real life so before implementing their solution, we must take a look at the "bigger picture".

a) is 12, b) is 6 and c) is 2 points.