

## OPRE 6201 : Quiz 2 on 12 July 2004

This is an open textbook and open lecture notes exam. You may use a calculator although leaving quantities in fractional forms is perfectly acceptable and preferable. **Do not forget** to define any variables you introduce. Unless otherwise stated,  $x_i$  always denotes the  $i$ th decision variable. Good luck ...

NAME (please print): \_\_\_\_\_

[10pts] 1. Consider the nonnegative variables  $x_1, x_2, x_3$  subject to

$$x_1 + x_2 - x_3 \leq 2$$

$$x_1 - x_2 - x_3 \leq 3$$

a) How many basic solutions have  $x_3 = 0$ , that is how many basic solutions are in the form  $(x_1, x_2, x_3 = 0, \dots)$ ? Make a table of all those basic solutions with  $x_3 = 0$ , indicate if they are feasible.

We add slacks  $s_1$  and  $s_2$  to inequalities to make them equality. Then we have  $n = 5$  variables,  $m = 2$  equations so we set  $n - m = 3$  variables to zero to find a basic solution. But the question asks us to fix  $x_3 = 0$ . Thus, we choose 2 variables among the remaining to set equal to zero. This can be made in six ways as enumerated in the table below:

$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	Feasible ?
0	0	0	2	3	Yes
0	2	0	0	5	Yes
0	-3	0	5	0	No
2	0	0	0	1	Yes
3	0	0	-1	0	No
2.5	-0.5	0	0	0	No

b) Consider the objective function “Maximize  $2x_1 + x_2$ ”. A co-worker of yours claims:

- Since the objective function does not include  $x_3$ , in the optimal solution  $x_3$  can be taken as zero. Thus, we can restrict our search for the optimal solution to the basic feasible solutions listed in part a).

If these statements are true, what will be the optimal solution? If you think the statements are true, write “True” on your paper. If you think the statements are false, find a feasible solution with a better objective value than those basic solutions listed in part a).

If you think the statements are true, you would find the optimal solution at  $x_1 = 2, x_2 = x_3 = 0$  with the objective value of  $2 \times 2 + 1 \times 0 = 4$ . However, the statements are false because  $x_1 = 3, x_2 = 0, x_3 = 1$  is a feasible solution with objective value 6.

[10pts] 2. Consider

$$\text{Max } 5x_1 + x_2 + 3x_3 + 4x_4$$

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$$x_1 - 2x_2 + 4x_3 + 3x_4 \leq 20$$

$$-4x_1 + 6x_2 + 5x_3 - 4x_4 \leq 40$$

$$2x_1 - 3x_2 + 3x_3 + 8x_4 \leq 50$$

$$x_1, x_2, x_3, x_4 \geq 0$$

a) Construct the initial Simplex tableau for this problem. What are your basic variables?

$z$	$x_1$	$x_2$	$x_3$	$x_4$	$s_1$	$s_2$	$s_3$	rhs
1	-5	-1	-3	-4	0	0	0	0
0	1	-2	4	3	1	0	0	20
0	-4	6	5	-4	0	1	0	40
0	2	-3	3	8	0	0	1	50

$s_1, s_2, s_3$  are basic variables.

b) Enter the nonbasic variable that would give you the best improvement in the objective and do a **single** Simplex iteration. Provide the next tableau below.

$z$	$x_1$	$x_2$	$x_3$	$x_4$	$s_1$	$s_2$	$s_3$	rhs
1	0	-11	17	11	5	0	0	100
0	1	-2	4	3	1	0	0	20
0	0	-2	21	8	4	1	0	120
0	0	-1	-5	2	-2	0	1	10

c) Consider the initial Simplex tableau in a). Starting with that initial tableau, an iteration is made to obtain the tableau below:

$z$	$x_1$	$x_2$	$x_3$	$x_4$	$s_1$	$s_2$	$s_3$	rhs
			0		0.75	0	0	
			1		0.25	0	0	
			0		-1.25	1	0	
			0		-0.75	0	1	

Not all entries are filled in this tableau. What are the nonbasic variables in the tableau? What are the values of objective function and basic variables?

In the current tableau, basic variables are  $x_3, s_2, s_3$ . The remaining variables  $x_1, x_2, x_4, s_1$  are nonbasic. In the initial tableau  $s_1, s_2, s_3$  are basic variables. Thus, the first iteration pushed  $x_3$  into the basis while  $s_1$  left. Such an iteration yields the rhs values (top to bottom) of 15, 5, 15, 35. Then  $z = 15$ ,  $x_3 = 5$ ,  $s_2 = 15$  and  $s_3 = 35$ .

[10pts] 3. A feasible region has the following basic feasible solutions:  $(x_1, x_2) = (0, 1)$ ,  $(x_1, x_2) = (1, 0)$ ,  $(x_1, x_2) = (-1, -1)$  and  $(x_1, x_2) = (0, -1)$ .

a) Draw the feasible region.

Feasible region has the following four corners:  $(x_1, x_2) = (0, 1)$ ,  $(x_1, x_2) = (1, 0)$ ,  $(x_1, x_2) = (-1, -1)$  and  $(x_1, x_2) = (0, -1)$ . All you need to do is connect the points.

b) Maximize  $-x_1 + 2x_2$  graphically over the feasible region. Find the optimal objective value and the optimal solution.

Bring the objective function from top, it touches the feasible region at  $(0,1)$  first. Max is at  $(0,1)$ , objective value is 2.

c) Minimize  $-2x_1 + x_2$  graphically over the feasible region. Find the optimal objective value and the optimal solution.

Bring the objective function from right, it touches the feasible region at  $(1,0)$  first. Min is at  $(1,0)$ , objective value is -2.

d) Find a single linear objective function such that both  $(1,0)$  and  $(0,1)$  simultaneously maximize it.

Max  $x_1 + x_2$  is maximized at  $(1,0)$  and  $(0,1)$ .