Contracts and Derivatives

Outline

- **Forwards, Futures and Swaps**
- **Options: Call, Put, Collar, Covered Call, Covered Put, Swaption**
- **Hedging**
- **Weather Derivatives**

Based on

Energy Derivatives. Chapter 2 of **Managing Energy Risk**. M. Burger, B, Graeber, G, Schindlmayr, Wiley, 2008.

Price and Derivative Modelling. Chapter 9 of **Electricity Markets**. C. Harris. 2006.

Forwards

Why a forward or another derivative?

- To rearrange the price risk between buyer (Cirro electricity retailer) and seller (Luminant electricity generator) in advance
  - Cirro needs 93,000 MWh during peak periods of August for Ercot retail market
  - August wholesale price can range over \$80/MWh – \$4,500/MWh, despite current price of \$40/MWh
  - Cirro buys a forward from Luminant, terms can be of the following kind
    - Delivery of 500 MWh per hour during 6 peak hours on each day of August, in total this yields 500*6*31=93,000 MWh.
    - Price is fixed at \$100/MWh for each MWh regardless of the delivery time.
    - For this sort of certainty supply/price Cirro pays (side payment) \$1,000,000 to Luminant
  - Cirro replaces the price uncertainty of \$80-4500/MWh with price certainty of \$100 by paying \$1M extra to Luminant.

A bit on terminology:

- Forwards involve only a seller-buyer pair.
  - Luminant sells the forward → is short in the forward contract
  - Cirro buys the forward → is long in the forward contract
  - Contract price is \$1 million, or \$10.75=1,000,000/93,000 per MWh
  - (Total) delivery price is \$9.3 million=93,000*100, or \$100 per MWh.
  - Term of the contract is from now until (the last) delivery.

Forward Example

Conceptual - Who profits?

Contract parameters known now

- K: Contract price per unit
- f: Forward (delivery) price per unit
- T: Term of the contract

Parameter that is safe to assume to be known

- r: Interest rate, continuous compounding

Random Variable, price in the future

- $p(T,T)$

Buyer pays: K now and f in the future to obtain the commodity whose value is $p(T,T)$ in the future.

- Buyer's cost now K
- Buyer's benefit in the future $E(p(T,T)-f)$
- Continuous discounting $\exp(-rT)$

Buyer profits if $K < \exp(-rT)\, E\,(p(T,T)-f)$, or

- $K + \exp(-rT) f < \exp(-rT)\, E\,(p(T,T))$.

Seller profits if $K > \exp(-rT)\, E\,(p(T,T)-f)$

Fair price: $K = \exp(-rT)\, E\,(p(T,T)-f)$

Numerical - Who profits?

Contract parameters known now

- K: 1,000,000/93,000=\$10.75/MWh
- f: \$100/MWh, see swaps for multiple Aug prices
- T: 120 days =1/3 years

Parameter that is safe to assume to be known

- 10% annual interest = discount by 1/1.1
- $1/1.1 = 0.9 = \exp(-r1)$ or $r = -\ln(0.9) = 0.1$
- r: 0.1 per year so that $1/1.1 = \exp(-0.1)$

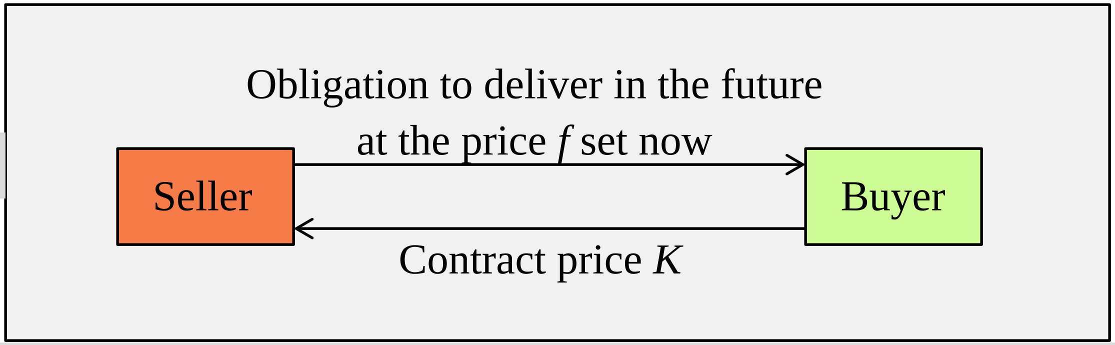
Variable

- $p(T,T)$ has "a distribution" with mean \$120/MWh and standard deviation \$10/MWh.
- More on distributions when necessary.

- $10.75 < \underbrace{19.34 = \exp(-0.1/3)\,(120-100)}_{\text{Buyer profits}}$

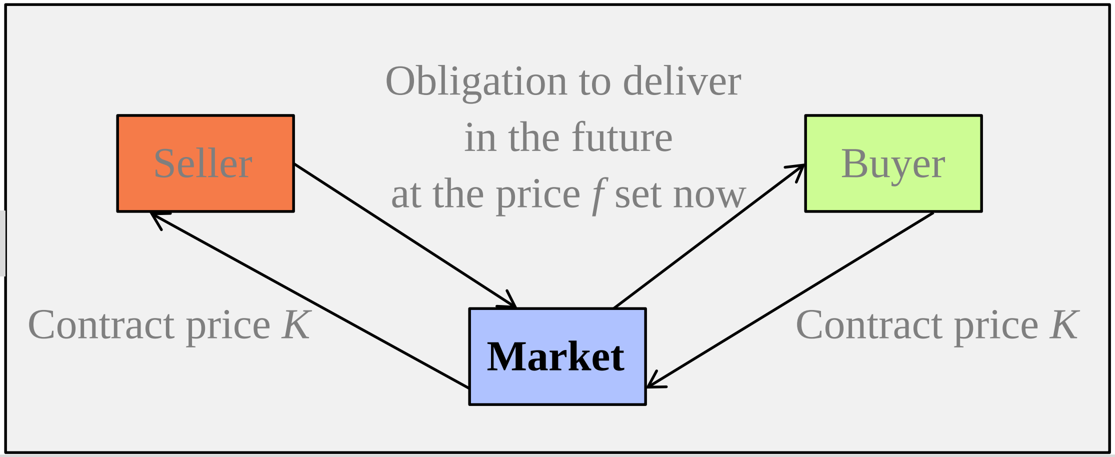
Forward vs. Future

Forward Contract:



Sellers are producers (physical owners) that deliver the commodity (oil, electricity), or financial institutions or hedging/trading companies.

Futures Market:



Sellers are often financial entities that speculate on the price.

- Forward contracts are typically between private counterparties and cover longer term.
- Market (CME, NYMEX) provides liquidity (frequent transactions).

Profiting from Oil Price Drop: Case of PointState Capital

Background: Jun’14 Oil price \$107/bbl and Dec’14 oil price \$50/bbl.

- PointState Capital, a hedge fund, starts 2014 with \$5.8 B in assets and profits \$1 B from the price drop.

  Long: own forward contracts to buy oil in the future at the current high price

  - by taking a contrarian’s perspective then as the market was expecting price increases due to Russia-Ukraine-Crimea issue and the associated shipment curtailments.

  Source: K. Burton, K. Bit, S. Foxman. 2015. Druckenmiller Alums at PointState Make \$1 Billion on Oil. Bloomberg, Jan 21 issue.

PointState CEO Zach Schreiber
Sohn Investment Conf., NYC, May 5, 2014

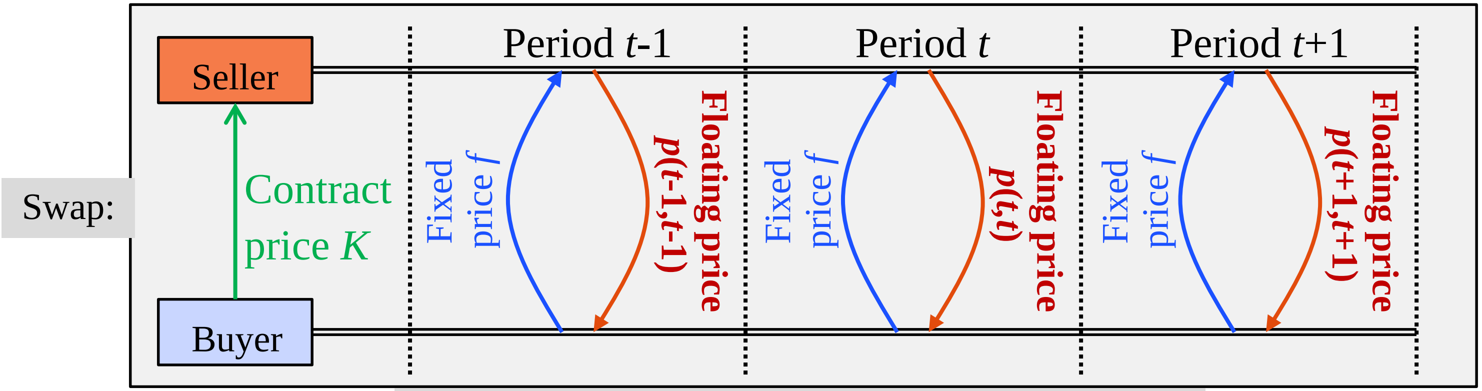
Question: How can you use futures market to short oil to make a \$1 billion profit?

1. In Jun 2014, the forward price for Dec delivery is \$100/bbl. You can sell 20 million units of a futures contract in Jun 2014 to a buyer with
   a. your obligation to deliver oil in Dec at the price of f=\$100/bbl
   b. the buyer’s obligation to buy at the price f.
   c. no contract price $K = 0$.
2. In Dec 2014, the price is \$50/bbl so you can buy 20 million barrels from the spot market by paying \$1 billion. When you sell these barrels of oil to the buyer of the future contract, you are entitled to receive \$2 billion. The difference of \$1 billion is your profit.

Trading is a child play
after assuming price uncertainty away

Swap: Repeated Forwards

- A swap is between a buyer (payer) and a seller (receiver). A buyer pays contract price K to the seller. Buyer can buy the commodity multiple times at fixed price f from the seller while the market price is floating at … $p(t-1,t-1), p(t,t), p(t+1,t+1)$ …
- Amount exchanged in each period as well as the price f can vary.



Energy (coal, oil, gas, electricity) is sold over time.

- In the energy domain, power generators need continuous supply of fuel (e.g., coal and gas) and can engage in swap contracts with fuel suppliers to fix the price of the fuel and hence to eliminate their risk exposure to fuel price fluctuations.
  - Outside energy domain, interest rate swaps are common. In an interest rate swap, a company promises to periodically pay Libor-based interest to another while it receives a fixed rate interest from the other.
- Fair price of a 3-period swap: $K = \exp(-r)\mathrm{E}\{p(t-1,t-1) - f\} + \exp(-2r)\mathrm{E}\{p(t,t) - f\} + \exp(-3r)\mathrm{E}\{p(t+1,t+1) - f\}$.
  - If $K <$ right-hand side, buyer profits. If $K >$ right-hand side, seller profits.

Until now, all evaluations have required only the expected value of the price rather than its distribution. So the price distribution has been unnecessary.

- Price Uncertainty (Distribution)
- Options
- Combinations: Derivates of derivatives of derivates of

Forward Price Evolution
Exponential Brownian Motion

Forward price $p(t\text{-}1,T)$ — t-1

Forward price $p(t,T)$ — t

T — Spot price $p(T,T)$

Multiplicative Evolution

$$\frac{p(t,T)}{p(t-1,T)} = \varepsilon_{t-1,T} = \varepsilon_{T-(t-1)}$$

$$\frac{p(t+1,T)}{p(t,T)} = \varepsilon_{T-t}$$

.....

$$\frac{p(T-1,T)}{p(T-2,T)} = \varepsilon_{2}$$

$$\frac{p(T,T)}{p(T-1,T)} = \varepsilon_{1}$$

$$\begin{aligned} p(T,T) &= \varepsilon_1 p(T-1,T) \\ &= \varepsilon_1\varepsilon_2 p(T-2,T) \\ &= .... \\ &= \varepsilon_1\varepsilon_2....\varepsilon_{T-t} p(t,T) \end{aligned}$$

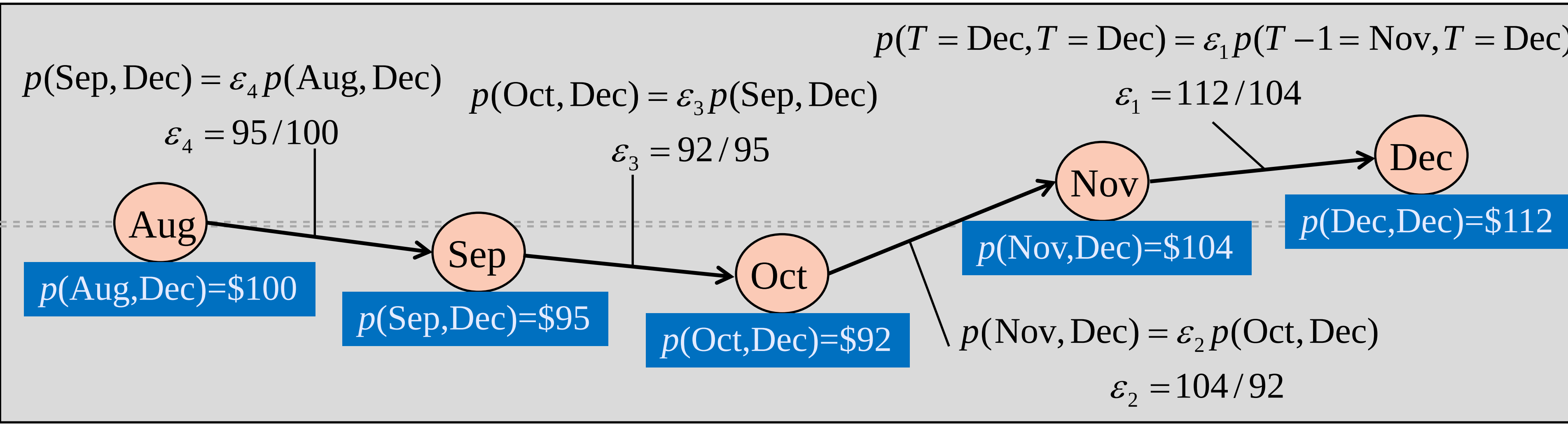
$$\ln p(T,T) - \ln p(t,T) = \sum_{i=1}^{T-t} \ln \varepsilon_i$$

Each $\ln \varepsilon_i$ Normal.

Price is Exponential Brownian Motion

Forward Price Evolution Example

- Forward prices for December from months Aug-Dec:



- Forward prices for December from months Aug-Dec:

$$\ln\varepsilon_1=\ln(112/104);\ \ln\varepsilon_2=\ln(104/92);\ \ln\varepsilon_3=\ln(92/95);\ \ln\varepsilon_4=\ln(95/100).$$

- Forward prices for November from months Jul-Nov, using similar process data not shown explicitly:

$$\ln\varepsilon_1=\ln(105/101);\ \ln\varepsilon_2=\ln(101/90);\ \ln\varepsilon_3=\ln(90/93);\ \ln\varepsilon_4=\ln(93/98).$$

- 4 normal distributions fit to 4 logarithms of epsilons. Good fit ⇒ exponential Brownian motion.

N. Meade. 2010. Oil prices — Brownian motion or mean reversion? *Energy Economics*, Vol. 32, Iss. 6: 1485-1498:

Our evaluation of the two modelling approaches … geometric Brownian motion and mean reversion showed that both models ceased to be plausible for horizons of three months or less. Geometric Brownian motion is not supportable as a long term model because of time varying volatility and the returns exhibit many jumps of a magnitude completely inconsistent with a Gaussian [Normal] density function. … investigat[ing] several alternative non-Gaussian densities a mixture of two Gaussians was most successful at capturing the jump-diffusion process, providing plausible density forecasts up to a year ...

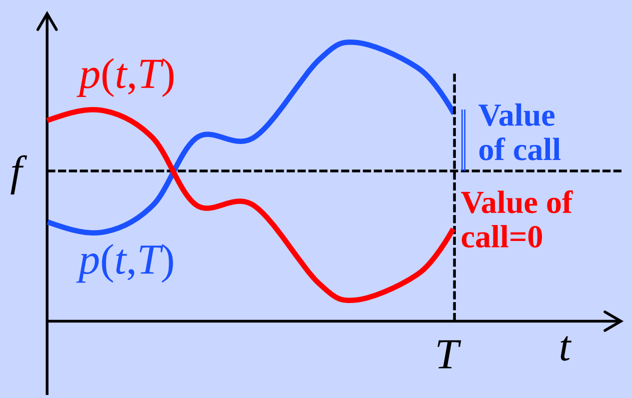
Options: Call and Put

Random Variable, price in the future $p(T,T)$
- f: Strike price (to buy or sell, see below)
- T: Term of the option
- K: Price of option charged by the seller

Parameter that is safe to assume to be known
- r: Interest rate

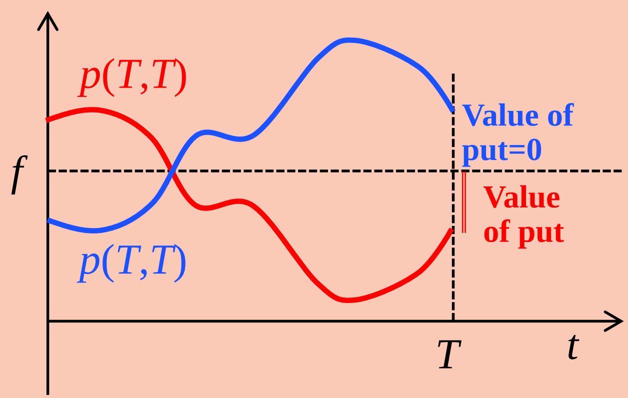
Call Option

Ability (option without obligation) to purchase (call) commodity at fixed price f at time T.



Put Option

Ability to sell (put) commodity at fixed price f at time T.



Non-discounted Value of Call = E max$\{p(T,T)-f,0\}$

Non-discounted Value of Put = E max$\{f-p(T,T),0\}$

These evaluations have E (xpected value) of a nonlinear term (max) and require the price distribution.

Option Profits

Call Option

Buyer profits:

- $K <$ exp($-rT$) E (max{$p(T,T)$-f,0})

Seller profits:

- $K >$ exp($-rT$) E (max{$p(T,T)$-f,0})

Fair price:

- $K =$ exp($-rT$) **E (max{$p(T,T)$-f,0})**

- Call option is an insurance for the consumer (of oil or electricity) to provide a ceiling on the market price. No matter how high the market price is, the consumer owning call options can still buy the commodity at the ceiling price f.

Put Option

Buyer profits:

- $K <$ exp($-rT$) E (max{f-$p(T,T)$,0})

Seller profits:

- $K >$ exp($-rT$) E (max{f-$p(T,T)$,0})

Fair price:

- $K =$ exp($-rT$) **E (max{f-$p(T,T)$,0})**

- Put option is an insurance for the producer (of oil or electricity) to provide a floor on the market price. No matter how low the market price is, the producer owning put options can still sell the commodity at the floor price f.

If $p(T,T)$ is exponential Brownian motion, non-discounted option values

E(max{$p(T,T)$-f,0}) and **E(max{f-$p(T,T)$,0})**

can be evaluated by using the celebrated Black-Scholes formula. This is the building block of financial engineering. Black, Scholes and Merton got Nobel Prize in Economics in 1997.

Call Option Numerical Example

In the comparisons of

K and $\exp(-rT)\, E(\max\{p(T,T)-f,0\})$

The non-discounted option value

$E(\max\{p(T,T)-f,0\})$

is important. Finding out this value is called evaluation of an option

Southwest Airlines uses jet fuel whose prices increase with the crude oil prices. To hedge against the increase in jet fuel prices, Southwest Airlines buys a **call** option on the price of crude oil. The strike price is set at \$100 per barrel while Southwest assess the crude oil price distribution as

\$75 wp 5%, \$80 wp 10%, \$85 wp 15%, 90 wp 20%
\$95 wp 20%, \$100 wp 15%, \$105 wp 10%, 110 wp 5%,

where wp stands for with probability. What is the per barrel non-discounted value of this option?

The non-discounted value of the **call** option

wp 5%, $\max\{p(T,T)-f,0\}= \max\{110-100,0\}=10$
wp 10%, $\max\{p(T,T)-f,0\}= \max\{105-100,0\}= 5$
wp 15%, $\max\{p(T,T)-f,0\}= \max\{100-100,0\}= 0$
wp 20%, $\max\{p(T,T)-f,0\}= \max\{95-100,0\}= 0$
wp 50%, $\max\{p(T,T)-f,0\}= 0$

$E(\max\{p(T,T)-f,0\})$ is a weighted average=
$=(0.05)10+(0.10)5+(0.85)0=1.$

The non-discounted value of this option on crude oil is \$1 per barrel.

If Southwest Airlines wants the **call** option above for 30,000 barrels of crude oil, what is the fair contract price with a maturity of 1 year and continuous annual interest rate of r=0.1?

Total value a year from now is \$30 thousands. We need the present value of this amount a year in advance. The present value of this amount is exp(-0.1) (30) = (0.905)(30) = 27.145 thousands. The fair contract price is \$27,145.

Put Option Numerical Example

In the comparisons of

K and $\exp(-rT)$ E $(\max\{f-p(T,T),0\})$

The non-discounted option value

E $(\max\{f-p(T,T),0\})$

is important. Finding out this value is called evaluation of an option

An oil exploration company borrows $40 million from Bank of Texas to fund its drilling program. The value of the oil obtained from the drilling program can be thought as a collateral for the loan. Thinking in this line, Bank of Texas *suggests/forces* the exploration company to buy a **put** option for crude oil at the strike price of $90 per barrel on Nov 1. The exploration company assess the crude oil price distribution as

$75 wp 5%, $80 wp 10%, $85 wp 15%, 90 wp 20%
$95 wp 20%, $100 wp 15%, $105 wp 10%, 110 wp 5%,

where wp stands for with probability. What is the per barrel non-discounted value of this option?

The non-discounted value of the **put** option

wp 5%, $\max\{f-p(T,T),0\}= \max\{90-75,0\}=15$
wp 10%, $\max\{f-p(T,T),0\}= \max\{90-80,0\}=10$
wp 15%, $\max\{f-p(T,T),0\}= \max\{90-85,0\}= 5$
wp 20%, $\max\{f-p(T,T),0\}= \max\{90-90,0\}= 0$
wp 50%, $\max\{f-p(T,T),0\}= 0$

E $(\max\{f-p(T,T),0\})$ is a weighted average=

$=(0.05)15+(0.10)10+(0.15)5+(0.70)0=2.5.$

The non-discounted value of this option on crude oil is $2.5 per barrel.

If the oil exploration company want to buy the **put** option above on May 1 for 100,000 barrels of its production, what is the fair contract price with continuous annual interest rate of r=0.1?

Total value on Nov 1 is $0.25 million. We need the present value of this amount six month (1/2 years) in advance. The present value of this amount is exp(-0.1/2) (0.25) = (0.951)(0.25) = 0.2378 millions. The fair contract price is $237,800.

The exploration company budgeted only $100,000 for the **put** option. Would the strike price be higher or lower for a put option whose fair price is $100,000?

Strike price will be lower.

Call – Put = Forward Option Combinations:

Value of Call Option
exp($-rT$) E (max{$p(T,T)-f$,0})

Value of Put Option
exp($-rT$) E (max{$f-p(T,T)$,0})

[Value of Call Option] – [Value of Put Option]

= exp($-rT$) E [max{$p(T,T)-f$,0} - max{$f-p(T,T)$,0}]

= exp($-rT$) E [max{$p(T,T)-f$,0} + min{$-f+p(T,T)$,0}]

= [Value of Forward Contract] = exp($-rT$) E [$p(T,T)-f$]

- Buying a call option and selling a put option at the same strike price f is equivalent to buying a forward contract.
- If you have a scheme to find the value of a put option, you can use that scheme to get the [Value of Call Option] = [Value of Forward Contract]+[Value of Put Option].

- [Value of Put Option] – [Value of Call Option] = – [Value of Forward Contract]

  = exp($-rT$) E [max{$f-p(T,T)$,0}-max{$p(T,T)-f$,0}] = exp($-rT$) E [max{$f-p(T,T)$,0} + min{$-p(T,T)+f$,-0}]

  = exp($-rT$) E [$f-p(T,T)$]
- Buying a put option and selling a call option at the same strike price f is equivalent to selling a forward contract.
- What if strike prices differ?

Collar = Put – Call
Combinations: Collar to Bound Market Prices

Collar is a combination of buying a put option and selling a call option

- Put option: We can sell at least f_P.
- Call option: Another party can buy at f_C from us where $f_C > f_P$.

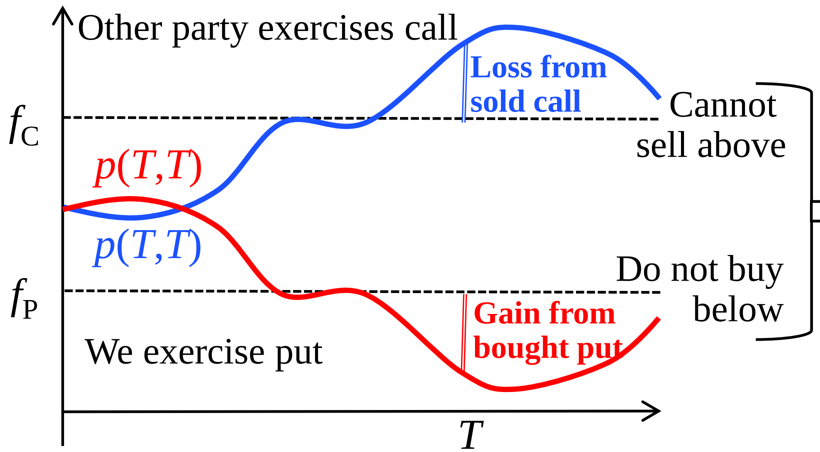
Profits depending on future price $p(T,T)$

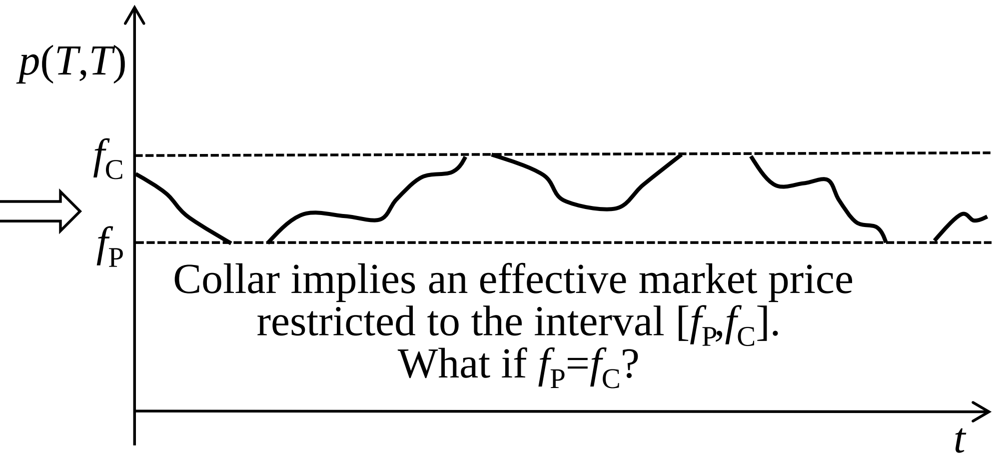
- If $p(T,T) \le f_P < f_C$, we exercise put to earn $f_P - p(T,T)$, the other party does not exercise call.
- If $f_P < p(T,T) \le f_C$, neither we nor the other party exercises put or call.
- If $f_P < f_C \le p(T,T)$, we do not exercise put but the other party exercises call and our incurred loss is $p(T,T) - f_C$.

- Value of Collar = [Value of Put] – [Value of Call] = E max$\{f_P - p(T,T), 0\}$ - E max$\{p(T,T) - f_C, 0\}$.
- Typically collars are costless, i.e., are set such that the fair contract price is zero, i.e., K=0. Then

$$\mathrm{E}\max\{p(T,T) - f_C, 0\} = \mathrm{E}\max\{f_P - p(T,T), 0\}.$$

For a price process that is symmetric around its mean (e.g., Brownian), K=0 implies $f_C = f_P$.





Protective Put = Forward + Put
Combinations: Protective Put Contracts

Protective Put Contract is a combination of buying a forward and buying a put

- Forward contract: We will buy at f_F.
- Put option: We buy at at f_P where $f_P > f_F$.

Profits depending on future price $p(T,T)$

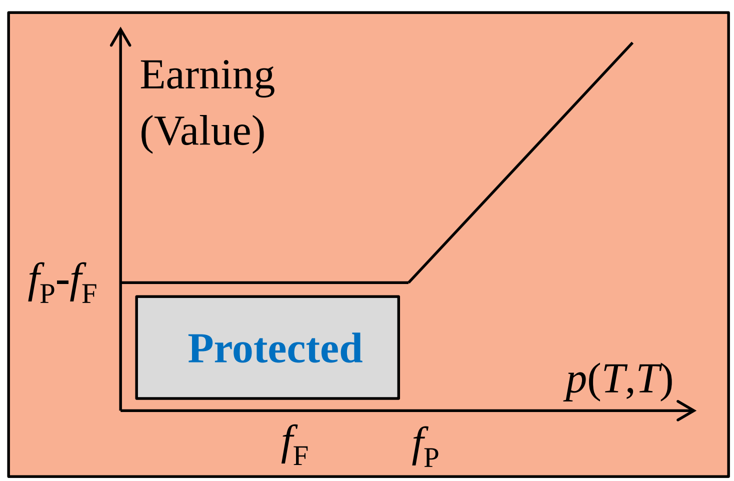
- If $p(T,T) \le f_F < f_P$, then we exercise the put option and earn $f_P - f_F$.
- If $f_F < p(T,T) \le f_P$, then we exercise the put option and earn $f_P - f_F$.
- If $f_F < f_P \le p(T,T)$, then we do not exercise the put option and earn $p(T,T) - f_F$.

- Non-discounted Value of Covered Call

  $= \mathrm{E}\, p(T,T) - f_F + \mathrm{E} \max\{f_P - p(T,T), 0\}$

  $= \mathrm{E} \max\{f_P, p(T,T)\} - f_F$

  $\ge f_P - f_F$.



Covered Call = Forward – Call Combinations: Covered Call Contracts

Covered Call Contract is a combination of buying a forward and selling a call

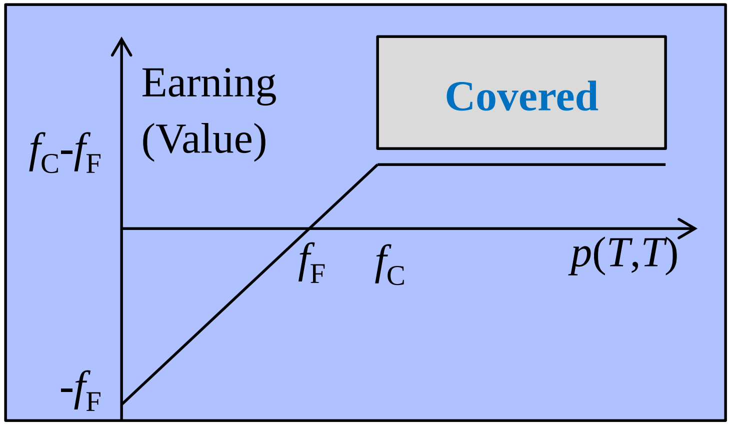
- Forward contract: We will buy at f_F.
- Call option: Another party can buy at f_C from us where $f_C > f_F$.

Profits depending on future price $p(T,T)$

- If $p(T,T) \leq f_F < f_C$, then the other party does not exercise the call option and we earn $p(T,T) - f_F$.
- If $f_F < p(T,T) \leq f_C$, then the other party does not exercise the call option and we earn $p(T,T) - f_F$.
- If $f_F < f_C \leq p(T,T)$, then the other party exercises the call option and we earn $f_C - f_F$.

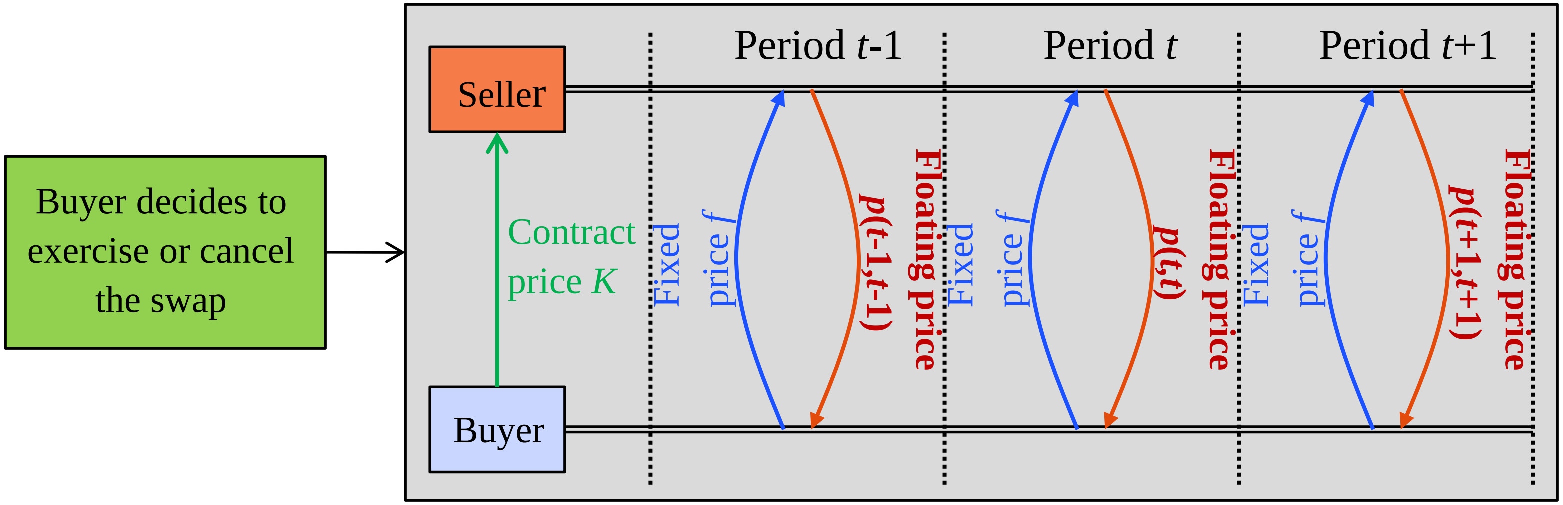
- Non-discounted Value of Covered Call

$$= \mathrm{E}\, p(T,T) - f_F - \mathrm{E} \max\{p(T,T) - f_C, 0\} = \mathrm{E}\, p(T,T) + \mathrm{E} \min\{-p(T,T) + f_C, 0\} - f_F = \mathrm{E} \min\{f_C, p(T,T)\} - f_F$$
$$\leq f_C - f_F.$$



Swaption: Option + Swap

- Buyer of the Swaption has the option to cancel the swap before the first swap becomes due.



- When buying these options, K can be explicit or implicit in the strike prices. To decrease K, call option seller will increase the (ceiling) strike price and similarly put option seller will decrease the (floor) strike price.
- When K is explicit, what value should it take?
  - Call several option sellers before buying an option, try Cargill-ETM.com, CMEGroup.com, theice.com/electricity.jhtml, BP, Shell for oil. Negotiate for prices. You mat not have too much time to figure out prices which change from one hour to another.

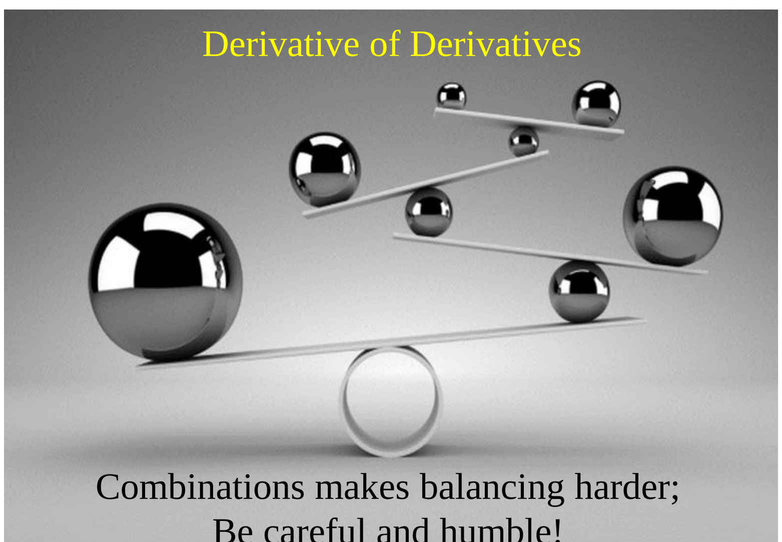
American and Asian Options

- European Options (discussed above) can be exercised only at maturity *T*.
- American options can be exercised any time from now to maturity, i.e., over the interval [0,*T*].
- Asian options are exercised only at the maturity but payoff is not just the price at maturity but typically an arithmetic mean of prices collected from now until maturity.
  - Asian options are relevant in energy because oil, gas, electricity transactions happen over a time period and contracts can include terms on average prices during that period.
  - Averaging prices over a period makes an option path-dependent and significantly complicates its evaluation.
  - Evaluation of Asian options is challenging and cannot be done analytically.
    - You can develop your own numerical method, or
    - Use existing numerical approximations.
      - First check out the Rmetrics package of open source software R: https://www.rmetrics.org. especially the package "foptions".
      - Search online for the manual that is called foptions.pdf , when this line is written.

Derivative Arithmetic - Sarcastically

- Forward = Call – Put, same strike price
- Collar = Put – Call, different strike prices
- Question: What is Forward + Collar, all at the same strike price

- Protective Put = Forward + Put
- Covered Call = Forward − Call
- Question: What is Protective Put – Covered Call?
  - Under matching strike prices, Protective Put – Covered Call = Put – Call = Collar
- Swaption: Option + Swap



Hedging of Oil Companies vs. Oil Services Companies

- Oil (producing) companies buy put options to sell their oil at a certain floor price even when the spot price of oil drops below that certain price. This strategy hedges the risk of dropping oil price and the revenue of the oil producing company.

- Oil Services companies do not have oil to sell so they cannot buy the same put options. When the price of the oil drops, oil services companies suffer more than oil companies.

- Oil service companies are more exposed to oil price fluctuations than oil companies.
  - Land drillers [oil service companies] are the end of the whip. They move first and most violently, as the owners of the expensive, depreciating capital at the genesis of new activity.

    - Credit Suisse analyst James Wicklund quoted in Barron's. Jan 7, 2014.

- July 1 - Dec 31, 2014,
  - Change in the Oil price: 100.78 → 53.27: 47% drop
  - Change in the Oil Service Companies Index (OSX): 310.72 → 210.86: 32% drop
  - Change in the Oil Companies Index (XOI): 1,687.12 → 1,348.13: 20% drop

- Oil companies are hedged better against price drops than oil service companies.
- What should oil service companies do? Buy airline stock.

Trading Weather Derivative Example

CME offers US Monthly Weather HDD Future contracts where each unit (1 degree for 1 day) worths $20: see http://www.cmegroup.com/trading/weather/temperature/us-monthly-weather-heating.html.

- HDD on a particular day = max {65-average daily temperature}
- Similarly, CDD (cooling degree days) on a particular day = max {average daily temperature-65}
- HDD in Dallas in Jan: sum over 31 Jan days [max {65-average daily temperature}]

- Suppose the Dallas HDD index for March is 50.
- Suppose Six Flags buys 80 units by paying $80,000 (=80*50*20). Consider HDD index as an asset.
- If March turns out to be cold and HDD becomes 150 (as heating is required),
  - Six Flag's HDD units will worth $240,000 (=80*150*20).
  - Profit from HDD units=$160,000.
- If March turns out to be hot and HDD becomes 0,
  - Six Flag's HDD units will worth $0 (=80*0*20).
  - Loss from HDD units=$80,000.
- **Cold Weather in March**= HDD is high so Six Flags ticket revenue drops by $4,000 per day
- **Hot Weather in March**= HDD is low so Six Flags ticket revenue increases by $4,000 per day.
- Profit from HDD units + Change in Ticket revenue
  - 160,000 −30*4,000 = 40,000 if March is cold
  - −80,000 +30*4,000 = 40,000 if March is hot
- Although Six Flag's earnings can wash away its losses on average (40,000=(160,000-80,000)/2), what the HDD contract offers is not an increase in profits but a stability of profits (40,000 and 40,000 hot or cold):
  - Net profits without contract = −120,000 and 120,000 only from ticket sales
  - Net profits with contract= 40,000 and 40,000 from ticket sales as well as the contract.

Weather derivatives decrease risk exposure

Trading Weather Derivative Example Average Payoff and Risk

- In the last example, it appears that contract always delivers “more” net profits than otherwise.
  - This is correct on average if cold and hot March are equally likely as
    (0.5)40,000+(0.5)40,000=40,000 > 0 = 0.5(120,000)+0.5(-120,000).
  - This is incorrect on average if probability for cold March is less than 1/3, say 1/4, as
    (1/3)40,000+(2/3)40,000 = 40,000 = (2/3)(120,000)+(1/3)(-120,000) and
    (3/4)40,000+(1/4)40,000 = 40,000 < 60,000 = (3/4)(120,000)+(1/4)(-120,000).
- If the cold March and hot March have probabilities of 1/3 and 2/3, same expected (average) payoffs are obtained with or without the contract but the contract still reduces the variability of the payoff from {-120,000,120,000} to 40,000.
  - To make things more concrete, think of the variance of payoffs:
    $(1/3)(40{,}000-40{,}000)^2+(2/3)(40{,}000-40{,}000)^2=0 < (38.4/3)*10^9=(2/3)(120{,}000-40{,}000)^2+(1/3)(-120{,}000-40{,}000)^2$.
    Contract reduced the variance down to zero in this specific example.

Summary

- Forwards, Futures and Swaps
- Options: Call, Put, Collar, Covered Call, Covered Put, Swaption
- Hedging
- Weather Derivatives

For $q_f \le q_p \le q_c$

Contract	Benefit in Future	Alternatively Buy	Alternatively Sell
Forward (q)	$\mathrm{E}(P) - q$	-	-
Call (q)	$\mathrm{E}(\max\{P - q, 0\})$	-	-
Put (q)	$\mathrm{E}(\max\{q - P, 0\})$	-	-
Combinations			
Forward (q)		Call (q)	Put (q)
Collar (q_p, q_c)	$\mathrm{E}(\max\{q_p - P, 0\}) - \mathrm{E}(\max\{P - q_c, 0\})$	Put (q_p)	Call (q_c)
Collar (q, q)	$\mathrm{E}(\max\{q - P, 0\}) - \mathrm{E}(\max\{P - q, 0\}) = q - \mathrm{E}(P)$	-	Forward (q)
Protective Put (q_f, q_p)	$\mathrm{E}(P) - q_f + \mathrm{E}(\max\{q_p - P, 0\}) = \mathrm{E}(\max\{P, q_p\}) - q_f$	Forward (q_f), Put (q_p)	-
Covered Call (q_f, q_c)	$\mathrm{E}(P) - q_f - \mathrm{E}(\max\{P - q_c, 0\}) = \mathrm{E}(\min\{P, q_c\}) - q_f$	Forward (q_f)	Call (q_c)
Constant price (q)	$\mathrm{E}(\max\{P - q, 0\}) + \mathrm{E}(\max\{q - P, 0\})$	Call (q), Put (q)	-

Summary of the Course

- **Resources**
  1. Hydrocarbon Geology
  2. Oil E&P
  3. Gas
  4. Coal
- **Technology**
  5. Enhanced Oil Recovery
  6. Nuclear
  7. Wind
  8. Solar
  9. Hydro et al.
- **Trans** (-formation & -portation)
  10. Refineries
  11. Electricity Generation
  12. Pipelines
  13. Electric Grid
- **Markets and Risk**
  14. Markets
  15. Market Models
  16. Derivatives

My teaching is over
Our learning is forever
Towards better
Do not give up ever

Aside: Continuous Compounding

- If my \$1investment earns an interest of r per year, what is my interest+investment at the end of the year?

  Answer: (1+r)
- If I earn an interest of r/2 per six months, what is my interest+ investment at the end of the year?

  Answer: $(1+r/2)^2$
- If I earn an interest of (r/m) per (12/m) months, what is my interest+investment?

  Answer: $(1+r/m)^m$
- Think of continuous compounding as the special case of discrete-time compounding when m approaches infinity.
- What if I earn an interest of (r/infinity) per (12/infinity) months?

$$\text{Answer: } \lim_{m\to\infty}\left(1+\frac{r}{m}\right)^m = e^r \quad \text{where } e=\sum_{n=0}^{\infty}\frac{1}{n!}=\frac{1}{1}+\frac{1}{1}+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\frac{1}{120}$$

[Present value of future value *V* at time *T*] = exp(-*rT*)*V*.