

## 1 Contracts an Overview

A contract is an agreement between two parties. The rasion d'être for contracts is two parties with conflicting objectives. Contracts are not limited to economical or legal situations. For example, Rousseau outlines the limitations of freedom of an individual against a governing body and the responsibilities of the governing body against an individual in his book *Social Contract* [3]. Similarly, a vow of honesty taken by students is a contract where students promise not to cheat to gain instructor's respect. Another example of contract is rules of any (sports, card, etc.) game. Contracts are generellay designed to sort out confusions or conflicts that may come up.

Supply chain contracts are a special case of economical contracts. Most supply chain contracts include only two parties. The supplier is in the upstream supply chain and sells materials to the retailer in the downstream supply chain. Although supply chains include more than two parties, this simplification allows for studying *optimal* contracts. In this simple setting, the supplier determines prices and retailer determines order quantities. The supplier's price affects retailer's order quantity: if the prices are low, the retailer may buy a lot. Low supplier price gives the retailer more leverage in determining the market price. If the market demand is price sensitive, the supplier's price affects the market demand which in turn affects the retailer's order quantity. Large retailer orders may induce the supplier to reduce its price. Thus, we complete a cyclical argument that supplier prices affect themselves and this is clearly a chicken and egg dilemma. That is why, even in this simple setting of only two parties, the pricing and ordering mechanisms are not trivial.

Having argued for the nontrivial nature of contracts. Let us clasify pricing contracts from a practical point of view:

- Constant Price Contracts: Price is constant during the contract and cannot be modified under any condition without mutual agreement.
- Dependent Price Contracts: Price is not constant but is calculated using a predetermined formula. The inputs to the formula may change over the time but the formula does not. Inputs such as price indices, interest rates or gross domestic product are often used in the formula. Note that these inputs are clearly defined and easily accessible numbers. Ambiguous inputs can defeat the purpose of the contracts — avoiding confusion and conflicts. This type of contract is preferable when there is uncertainty in the future economic environment but this uncertainty can be captured fairly satisfactorily with several inputs.
  - An important special case of dependent price contracts is contracts with incentives. This type of contract allows for the supplier to retain a certain proportion of the cost savings it achieves due to efficiency increases, learning, innovations, etc. Therefore, it motivates the supplier to cut costs. An infamous example comes from the contracts written between GM and its suppliers where GM forces suppliers to reduce prices at a given annual rate. This policy forced many GM suppliers into economic troubles.
- Alterable Price Contracts: Price may or may not be constant. At fixed time intervals contracting parties come together to redetermine the price. If they can agree on a new price, price changes. Otherwise, they must abide with the associated clauses of the contract. This contract is advisable when there is uncertainty in the future economic environment and this uncertainty can not be captured using surrogate variables (as inputs in dependent price contracts). Then contracting parties are bound to come together first to analyze the resolved economic uncertainty and then to set the prices. The contract evaluation intervals can be long or short. Contracts with short evaluation intervals can be responsive to economic factors but they eat up

valuable human resources. Therefore, there is a tradeoff between the responsiveness and the effort in setting the intervals.

Although our classification is based on only pricing. You may replace the word price with order quantity above to get analogous contracts for ordering. Since practitioners focus more on prices than quantities, we have presented contracts as price contracts. You can also consider price and quantity together and make up hybrid contracts such as Dependent Price Fixed Quantity Contract.

Sometimes contracts are based on production costs. In this case, supplier reveals its production cost to the retailer and asks the retailer for reimbursement for the cost plus a fee. This fee can be fixed, dependent or alterable just like the price in the price based contracts. The tricky part here is whether the buyer trusts in the production price the supplier reveals now and in the future. There must be objective criteria for determining these costs. An example of this contract comes from what happens when you go to a car repair shop to have new spark plugs installed into your car. You would generally get two quotes from the clerk, the first is the cost of the spark plugs and the second is the installation fee. If you accept the quoted price and the fee, you will be getting into a price based contract. If the retailer accepts a cost based product, it pays for all production costs. That leaves minimal incentive to the supplier to improve its production technology to cut down its costs. The US Military is known for giving many cost based contracts to suppliers which make sweet profits at the expense of taxpayers.

Contracts are vastly used among businesses for two advantages they provide to the involved parties:

- **Uncertainty Reduction:** Once the contract is in place, the supplier knows the order quantity and the retailer knows the supplier's price so there is less uncertainty for each party to deal with. Therefore, more (cost or profit) efficiency can be sought while planning at the expense of flexibility — remember the need for flexibility goes down with more certainty.
- **Relationship Leveraging:** Contracts start up relationships between companies. Then companies can drive their contractual partners towards a more mutually preferable stand on policies. A concrete example is simple information exchange between companies. Such an exchange can lead to innovations and improvements at both the supplier and the retailer. Another example is blocking the supplier's proprietary technology. In this case the retailer blocks the supplier from selling to the retailer's competitors. Especially when the supplier owns a proprietary technology, the contract becomes a tool for the retailer to compete with its competitors.

Now let us look at the disadvantages of contracts, first for suppliers and then for retailers:

- **Supplier's Disadvantages**
  - Supplier may be blocked from other profitable businesses. This may be because of a clause in the contract forbidding the supplier to sell to the retailer's competitors. It also may happen when the supplier uses up most of the retailer's capacity and the retailer cannot accept profitable orders for lack of capacity.
  - Retailer may be too harsh on the supplier, asking for price/cost reductions, high service.
- **Retailer's Disadvantages**
  - Retailer can block the supplier from buying retailer's competitors. If retailer's competitors are providing better deals, then the supplier is basically stuck with a suboptimal choice during the contract's term.
  - Retailer may be too complacent after the contract and stop improving its service, cost, quality since it takes the retailer, the customer, granted.

We close this section noting that good contracts can be written only after a complete forethought of all possible future cases. Otherwise, there will remain loopholes making one party vulnerable for exploitation by another.

## 2 Contracts a Technical View

### 2.1 Buyback Contracts

Consider a supplier(S) and a retailer(R) subject to random demand  $D$  with pdf  $f$ , cdf  $F$  and  $\bar{F} := 1 - F$ . Let us define the following cost parameters:

- $c$ : Supplier's cost of supplying 1 unit. If supplier manufactures this unit,  $c$  is the marginal manufacturing cost. If the supplier simply buys the unit,  $c$  is the purchasing price for the supplier.
- $w$ : Wholesale price that supplier charges to retailer.
- $b$ : Supplier's buyback price for any unit unsold at the retailer.
- $p$ : Market price that retailer charges to customers.

Naturally  $c \leq w \leq p$  and  $b \leq w$  (why?). We assume that  $c$ ,  $w$  and  $p$  are constant. However, the supplier can determine  $b$ . We will study finding optimal  $b$  to coordinate the supply chain. In a coordinated (integrated) supply chain, there is a mechanism (e.g. a buyback contract) which pushes independent decision makers (e.g. supplier and retailer) to make unselfish decisions. If the chain is not coordinated, each decision maker selfishly optimizes its own objective. The coordination mechanism slightly modifies each decision maker's objective so that these modified objectives and the total objective of the supply chain yield the same optimal solution.

To make ideas concrete, let  $\Pi^S(b|y)$  be supplier's profit when the buyback price is  $b$  and given that the retailer orders  $y$ . Since  $b$  is controlled by the supplier whereas  $y$  is dictated by the retailer, we use the notation  $(b|y)$ . Similarly let  $\Pi^R(y|b)$  be supplier's profit when it orders  $y$  and given that the buyback price is  $b$ . Notice the switch in notation from  $(b|y)$  to  $(y|b)$  once we change the point of view from the supplier to the retailer. Let  $\Pi^C(y, D)$  be the coordinated supply chain profit when the chain orders  $y$  and demand is  $D$ , this profit does not depend on  $b$  or  $w$  because they are internal (transfer) prices in a coordinated supply chain. Since  $\Pi^C(y, D) = p(y \wedge D) - cy$ ,  $\Pi^C(y, D)$  is a random function, taking its expected value:

$$\begin{aligned} \Pi^C(y) &:= E(\Pi^C(y, D)) = p \int_0^\infty (y \wedge D) f(D) dD - cy = p \int_0^\infty \int_{x=0}^{y \wedge D} dx f(D) dD - cy \\ &= p \int_0^y \int_x^\infty f(D) dD dx - cy = p \int_0^y \bar{F}(D) dD - cy. \end{aligned}$$

where  $\wedge$  denotes the minimum of two numbers. We have manipulated the integrals by changing their order of integration, in general such an operation yields

$$\int_0^\infty (y \wedge D) f(D) dD = \int_0^y \bar{F}(D) dD. \quad (1)$$

In words, this quantity is expected number of units sold to the market if  $y$  units are ordered. Let  $y_C^*$  be the optimal order size maximizing  $\Pi^C(y)$ . From newsvendor problem we already know that  $y_C^* = \bar{F}^{-1}(c/p)$ .

Let us consider supplier's profit. Supplier makes at least  $w - b$  for each unit sold to the retailer and in addition  $b$  if the unit is sold by the retailer to customers. Supplier incurs a cost of  $c$  for each unit the retailer orders. Putting all these revenues and costs together:

$$\Pi^S(b|y) = (w - b)y + b \int_0^y \bar{F}(D) dD - cy.$$

Now let us look at the retailer. Retailer first pays  $wy$  to the supplier for its order.  $y - \int_0^y \bar{F}(D) dD$  can not be sold, and are returned to the supplier at a price of  $b$ .  $\int_0^y \bar{F}(D) dD$  units are sold at a price of  $p$  each. Putting all these revenues and costs together:

$$\Pi^R(y|b) = -wy + b \left( y - \int_0^y \bar{F}(D) dD \right) + p \int_0^y \bar{F}(D) dD = -(w - b)y + (p - b) \int_0^y \bar{F}(D) dD.$$

Let  $y_R^*(b)$  be the optimal order size maximizing  $\Pi^R(y|b)$ . From newsvendor problem we already know that

$$y_R^*(b) = \bar{F}^{-1} \left( \frac{w-b}{p-b} \right).$$

The main question is how the supplier can coordinate the supply chain. The only leverage the supplier has is the buyback price, supplier must find a buyback price  $b$  to achieve supply chain coordination:

$$\Pi^C(y_C^*) = \Pi^S(b|y_R^*(b)) + \Pi^R(y_R^*(b)|b).$$

Since  $\Pi^C(y) = \Pi^S(b|y) + \Pi^R(y|b)$ , any value of  $b$  that makes the retailer order  $y_C^*$  coordinates the supply chain. in other words, we are looking for the  $b$  value such that

$$\bar{F}^{-1} \left( \frac{c}{p} \right) = y_C^* = y_R^*(b) = \bar{F}^{-1} \left( \frac{w-b}{p-b} \right), \quad \text{or} \quad \frac{c}{p} = \frac{w-b}{p-b}$$

Solving for  $b$  that coordinates the supply chain,

$$b^C := \frac{w-c}{1-c/p}. \tag{2}$$

This value of  $b$  makes sure that although the retailer and the supplier makes independent decisions, their decisions maximize the total supply chain profit. It can be shown that  $b^C \leq w$  (how?), so (2) does not lead to counterintuitive results. For numerical examples see Table 9.7 of Chopra and compute optimal  $b$  values using the formula above. Fortunately, the value of  $b^C$  does not depend on the demand distribution so it is numerically trivial to find it. For extensions see [2] where the buyback price depends on the demand distribution and [1] where supplier's cost is nonlinear.

Let us see how supplier and retailer split the integrated supply chain profits. If the supplier uses  $b^C$ , then the retailer profit is:

$$\Pi^R(y|b^C) = -(w-b^C)y + (p-b^C) \int_0^y \bar{F}(D)dD \stackrel{\text{by(2)}}{=} -\frac{p-w}{p-c}cy + \frac{p-w}{p-c}p \int_0^y \bar{F}(D)dD = \frac{p-w}{p-c}\Pi^C(y)$$

Then the retailer gets the  $(p-w)/(p-c)$  proportion of the coordinated supply chain profits. The remaining profits are  $1 - (p-w)/(p-c) = (w-c)/(p-c)$  portion of the coordinated profits and are left to the supplier. These profit ratios depend on neither demand distribution  $f$  nor retailer order quantity  $y$ . Note that whether the supplier or the retailer recoups more profit in the coordinated supply chain depends where  $w$  falls in the interval  $[c, p]$ . If the wholesale price  $w$  is close to the market price  $p$ , then the supplier gets most of the profits and vice versa.

If the supplier is very greedy and wants to recoup all the supply chain profits, its should set whole sale price as large as possible  $w = p$  which implies that  $b^C = p$ . In this case the supplier gets 100% of supply chain profits. Retailer buys at the whole sale price of  $p$  and sells at the same price hence making no profit. Of course these observations are valid only when supplier has the monopoly over the supply market. Otherwise, the retailer will stop buying from a greedy supplier charging  $w = p$ .

What if the retailer is very greedy and wants all the profit of the supply chain. Retailer may choose to increase  $p$  to bring  $(p-w)/(p-c)$  towards 1. If the supplier does not react to this and the market demand is insensitive to price, the retailer will get a bigger portion of the supply chain profits. If the supplier reacts with  $w = p$ , once more the supplier gets all the profit. It is clear that the supplier has more power over the retailer than the other way around. This is no surprise given that the supplier has the monopoly.

## 2.2 Quantity Flexibility Contracts

Now we will study quantity flexibility contracts. In the current setting, the retailer first gives a sign about its demand to the supplier. The supplier arranges its production according to this signal. Then demand is realized at the retailer, i.e. demand is known, and the retailer exactly buys as much as the demand subject to supply availability. The early signal to the supplier gives a partial but advance information to the supplier. We will see how this advance information along with flexible order quantities can be used to coordinate the supply chain.

Suppose that we have the cost structure defined in the previous section except that the supplier does not buy back any of the remaining inventory at the retailer's side. Instead both retailer and the supplier can salvage their remaining inventory at a price of  $u$ ,  $u \leq c$ . As in the last section, we can go over the newsvendor model to figure out that the coordinated profit:

$$\Pi^C(y) = pE(\text{Sales}) - cy + uE(\text{Salvage})$$

where

$$E(\text{Sales}) = \int_0^y Df(D)dD + y(1 - F(y)) \text{ and } E(\text{Salvage}) = \int_0^y (y - D)f(D)dD.$$

Simplifying the profit we obtain:

$$\Pi^C(y) = (p - u) \left( \int_0^y Df(D)dD - yF(y) \right) + (p - c)y$$

then the coordinated lot size would be  $y_C^* = F^{-1}((p - c)/(p - u))$ .

Let us see the mechanism of the quantity flexibility contract to achieve this coordinated solution:

1. Retailer knows the demand distribution  $F$  and makes a forecast  $q$  for its order quantity. Retailer passes  $q$  to supplier.
2. Supplier guarantees to supply at least  $q(1 + \alpha)$  and retailer guarantees to buy at least  $q(1 - \beta)$  where  $0 \leq \beta \leq 1$  and  $0 \leq \alpha$ . Supplier produces  $Q \geq q(1 + \alpha)$ .
3. Retailer sees the demand and buys exactly the demand from the supplier as long as the supplier has enough inventory to meet the demand.

We assume that all cost parameters are given and constant except for  $w$ . We also assume that  $\alpha$  and  $\beta$  are given. We want to find the value of  $w$  that will coordinate the chain for each  $(\alpha, \beta)$ . Note  $(\alpha, \beta)$  are contract parameters measuring the flexibility of order quantity. Large  $\alpha$  and  $\beta$  (remember  $\beta \leq 1$ ) indicates quantity flexibility for the retailer. In the special case of  $(\alpha = \infty, \beta = 1)$ , retailer is guaranteed all of its supply requests so overage/underage risk is passed entirely to the supplier. In the other extreme  $(\alpha = 0, \beta = 0)$ , supplier produces only  $q$  and sells all to the retailer so overage/underage risk remains at the retailer. Of course the party that is subject to more risk deserves to ask for financial compensation. Operationally this implies that if the retailer carries more risk than the supplier,  $w$  should be small and vice versa. It must be clear by now that contract parameters  $(\alpha, \beta)$  define order quantity flexibility for the retailer who may accept to pay large wholesale prices for flexibility: therein lies the tradeoff.

In the absence of a contract, the supplier performs a newsvendor analysis with its own objective function (where underage and overage costs are  $w - c$  and  $c - u$ ) and chooses its order quantity as  $y_S^* = F^{-1}((w - c)/(w - u))$ . Since  $w \leq p$  we have the next proposition.

**Proposition 1**  $y_S^* \leq y_C^*$  which implies underproduction/ordering without coordination.

The challenge is to choose  $w$  so that supplier's order quantity becomes the coordinated chain's order quantity.

In order to analyze the contract mechanism, we will move in counterchronological order. The 3rd step is trivial, there is no decision making here. In the 2nd step, the supplier must determine its production/order quantity  $Q$  after seeing retailer's signal  $q$ , supplier will seek to maximize its profit:

$$\Pi^S(Q|q) = -cQ + wE(\text{SalesToRetailer}) + uE(\text{Supplier'sSalvage})$$

where

$$E(\text{SalesToRetailer}) = q(1 - \beta)F(q(1 - \beta)) + \int_{q(1-\beta)}^Q Df(D)dD + Q(1 - F(Q))$$

$$E(\text{Supplier'sSalvage}) = (Q - q(1 - \beta))F(q(1 - \beta)) + \int_{q(1-\beta)}^Q (Q - D)f(D)dD$$

This is another newsvendor type problem with additional constraint that  $Q \geq q(1 + \alpha)$ . The optimal order quantity for the supplier with the contract is  $y_{S,C}^* = F^{-1}((w - c)/(w - u) \vee q(1 + \alpha))$  (this is due to maximizing the concave profit function over a limited range) where  $\vee$  denotes maximum of two numbers. In other words,  $y_{S,C}^* = y_S^* \vee q(1 + \alpha)$ . If the retailer gives a small signal, i.e.,  $q \leq y_S^*/(1 + \alpha)$ , the supplier will order its independently determined lot size, i.e.  $y_{S,C}^* = y_S^*$ . In this case, supplier believes that it can sell more than  $q(1 - \beta)$  with or without the contract, so the supplier does not gain anything by participating in the contract. Let us assume that  $q$  is large enough that the supplier participates, i.e.,  $q \geq y_S^*/(1 + \alpha)$ . This leads to the next proposition.

**Proposition 2** *When the supplier participates in the contract, it orders  $Q = q(1 + \alpha)$ .*

Let us now consider the 1st step. This time retailer determines its signal. A large signal will force it to buy a large quantity while a small signal may cause the supplier to produce little. In trading off these concerns, the retailer maximizes its profit:

$$\Pi^R(q) = -wE(\text{PurchaseFromSupplier}) + pE(\text{SalesToMarket}) + uE(\text{Retailer'sSalvage})$$

where

$$E(\text{PurchaseFromSupplier}) = q(1 - \beta)F(q(1 - \beta)) + \int_{q(1-\beta)}^Q Df(D)dD + Q(1 - F(Q))$$

$$E(\text{SalesToMarket}) = \int_0^Q Df(D)dD + Q(1 - F(Q))$$

$$E(\text{Retailer'sSalvage}) = \int_0^{q(1-\beta)} (q(1 - \beta) - D)f(D)dD$$

$$Q = q(1 + \alpha)$$

Naturally  $E(\text{PurchaseFromSupplier}) = E(\text{SalesToMarket}) + E(\text{Retailer'sSalvage})$ . Since we assume supplier's participation we set  $Q = q(1 + \alpha)$ , the profit depends solely on retailer's signal  $q$ . To maximize the supplier profit, we take its derivative and set it equal to zero:

$$(1 + \alpha)(p - w)(1 - F(q(1 + \alpha))) - (1 - \beta)(w - u)F(q(1 - \beta)) = 0$$

After letting  $\mathfrak{S} = (1 + \alpha)/(1 - \beta)$  we can write this optimality condition in terms of  $Q = q(1 + \alpha)$  as

$$F\left(\frac{Q}{\mathfrak{S}}\right) = \mathfrak{S}\left(\frac{p - w}{w - u}\right)(1 - F(Q)) \quad (3)$$

Before we rush to the solution of this equation, let us examine  $\mathfrak{S}$ .  $\mathfrak{S}$  is greater than 1 and it increases with both  $\alpha$  and  $\beta$ . We can envision  $\mathfrak{S}$  as a measure of retailer's flexibility. If  $\mathfrak{S} = \infty$  (i.e.  $(\alpha = \infty, \beta = 1)$ ), the retailer has infinite flexibility: it can give a small signal and have almost infinite supply available although it is allowed

to buy almost nothing from the supplier. Also note that (3) is a function of only  $\mathfrak{S}$ : what matters is not absolute values of  $\alpha$  or  $\beta$  but their ratio. Namely coordination can be achieved irrespective of whether supplier or the retailer provides the flexibility, as long as there is some flexibility in the system. We have purposefully switched to  $Q$  notation as opposed to  $q$  in (3), because after all we want to see what value of  $w$  will make the solution of (3) be equal to the coordinated solution  $y_C^*$ .

For supply chain coordination, we want the solution of (3) be  $Q = y_C^* = F^{-1}((p - c)/(p - u))$ . There exists a single wholesale price  $w$  which achieves this coordination. Coordinating wholesale price is given as a proposition, the verification of this price is straightforward.

**Proposition 3** *The supplier can coordinate a quantity flexibility contract with flexibility  $\mathfrak{S}$  by setting its wholesale price at the specific value:*

$$w = u + \frac{c - u}{\frac{1}{\mathfrak{S}}F\left(\frac{1}{\mathfrak{S}}F^{-1}\left(\frac{p-c}{p-u}\right)\right) + \frac{c-u}{p-u}}$$

Let us revisit the connection between flexibility and the wholesale price. If the retailer wants infinite flexibility ( $\mathfrak{S} = \infty$ ) and pass all the risk to the supplier, it has to accept a wholesale price of  $w = p$ . However, at this wholesale price, retailer makes no profit. If the retailer is bold and wants minimal flexibility ( $\mathfrak{S} = 1$ ) and hence assumes all the risk, it can buy from the supplier at supplier's production cost,  $w = c$ . This time supplier assumes no risk but makes no profit. Since the coordinating wholesale price  $w$  is continuous in flexibility  $\mathfrak{S}$ , any value of  $w \in [c, p]$  can be obtained as a coordinating price as flexibility varies.

There are also Revenue and Profit Sharing Contracts which we do not discuss here (for now) in the interest of time.

### 3 Exercises

1. As a retailer, would you prefer a cost based or price based contract in each of the following situations, please briefly explain:
  - (a) Supplier is in a high-tech industry and its technology is likely to evolve.
  - (b) Cost of the components used in supplier's production are uncertain.
  - (c) Supplier desires a long term contract only.
  - (d) This is your first contract with the supplier so your trust for the supplier is limited.
2. Consider the buyback price  $b$  that coordinates the supply chain. Study how  $b$  would change as each of problem parameters  $c, w, p$  increase. Then determine:
  - (a) Should the supplier decrease the buyback price if its costs go up?
  - (b) Should the supplier increase its buyback price if it increases its wholesale price?
  - (c) Should the supplier decrease its buyback price if market prices go up?
3. Prove Proposition 3.
4. Repeat the arguments for flexible quantity contracts assuming that retailer pays an additional cost of goodwill  $s$  per unmet demand. Show that the coordinating wholesale price becomes:

$$w = u + \frac{c - u}{\frac{1}{\mathfrak{S}}F\left(\frac{1}{\mathfrak{S}}F^{-1}\left(\frac{p+s-c}{p+s-u}\right)\right) + \frac{c-u}{p+s-u}}$$

Give a verbal justification for this expression by comparing it with that of Proposition 3. See [4] for more details.

## References

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