SC Design
Facility Location

Sections 4.1, 4.2

Chapter 5 and 6

Outline

- Frequency decomposition of activities
- A strategic framework for facility location
- Multi-echelon networks
- Analytical methods for location

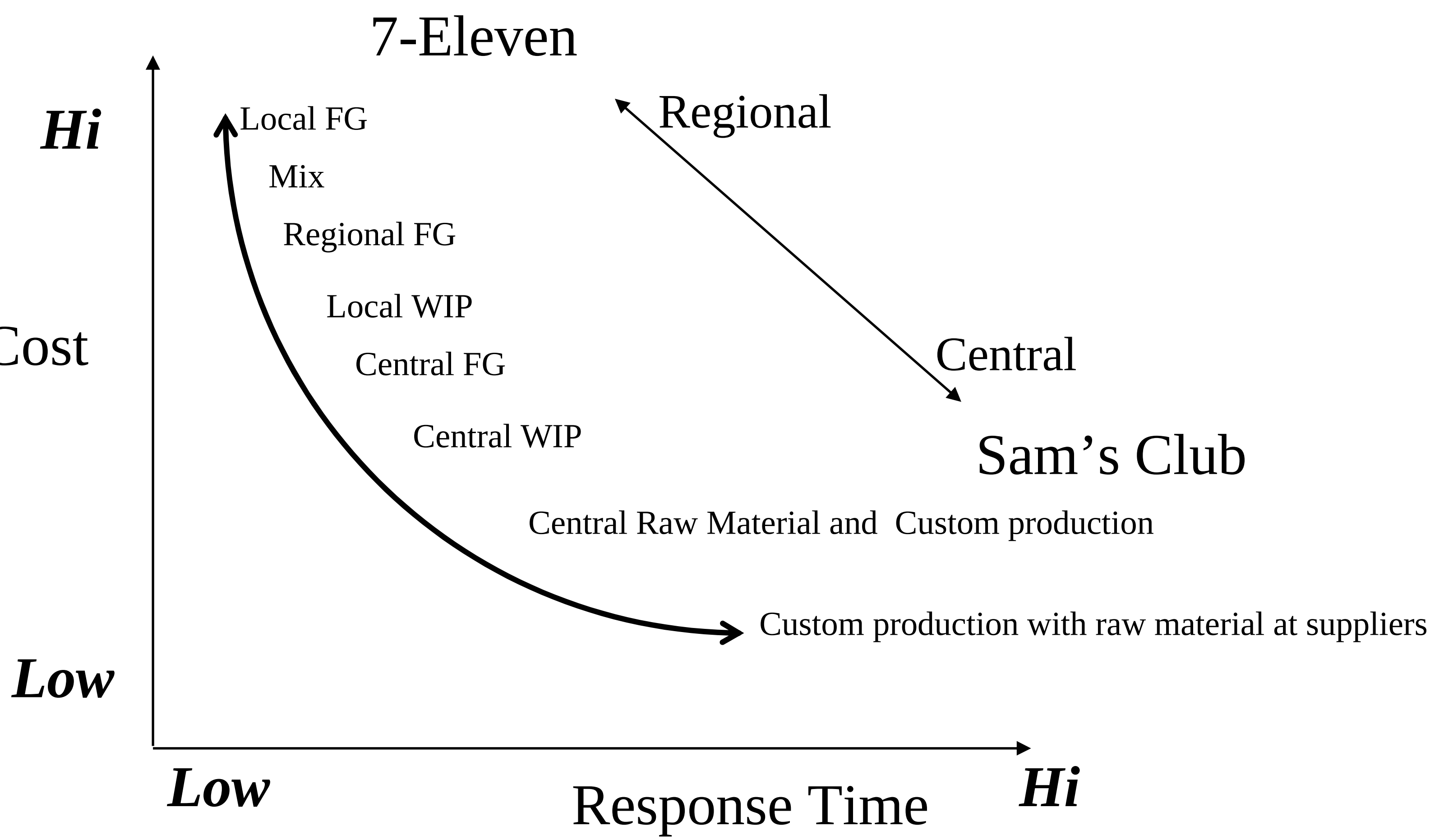
Frequency Decomposition

- SCs are enormous
- It is hard to make all decisions at once
- Integration by smart decomposition
- Frequency decomposition yields several sets of decisions such that each set is integrated within itself

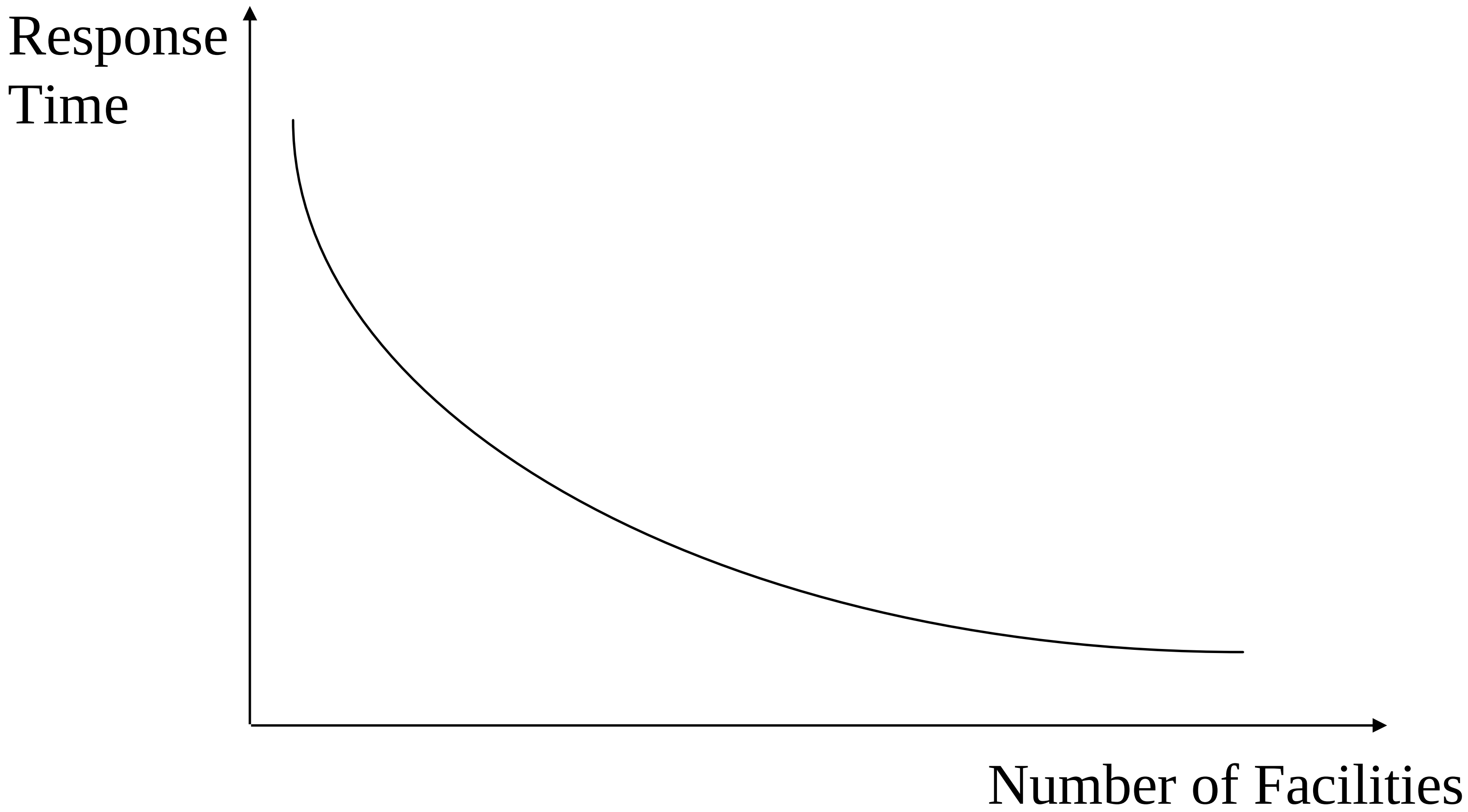
Frequency Decomposition

- Low frequency activity, ~ once a year, high fixed cost
  - Capacity expansion budget
- Moderate frequency activity, ~ once a month
  - Specific machines to purchase
- High frequency activity, ~ once a day, low fixed cost
  - What to produce

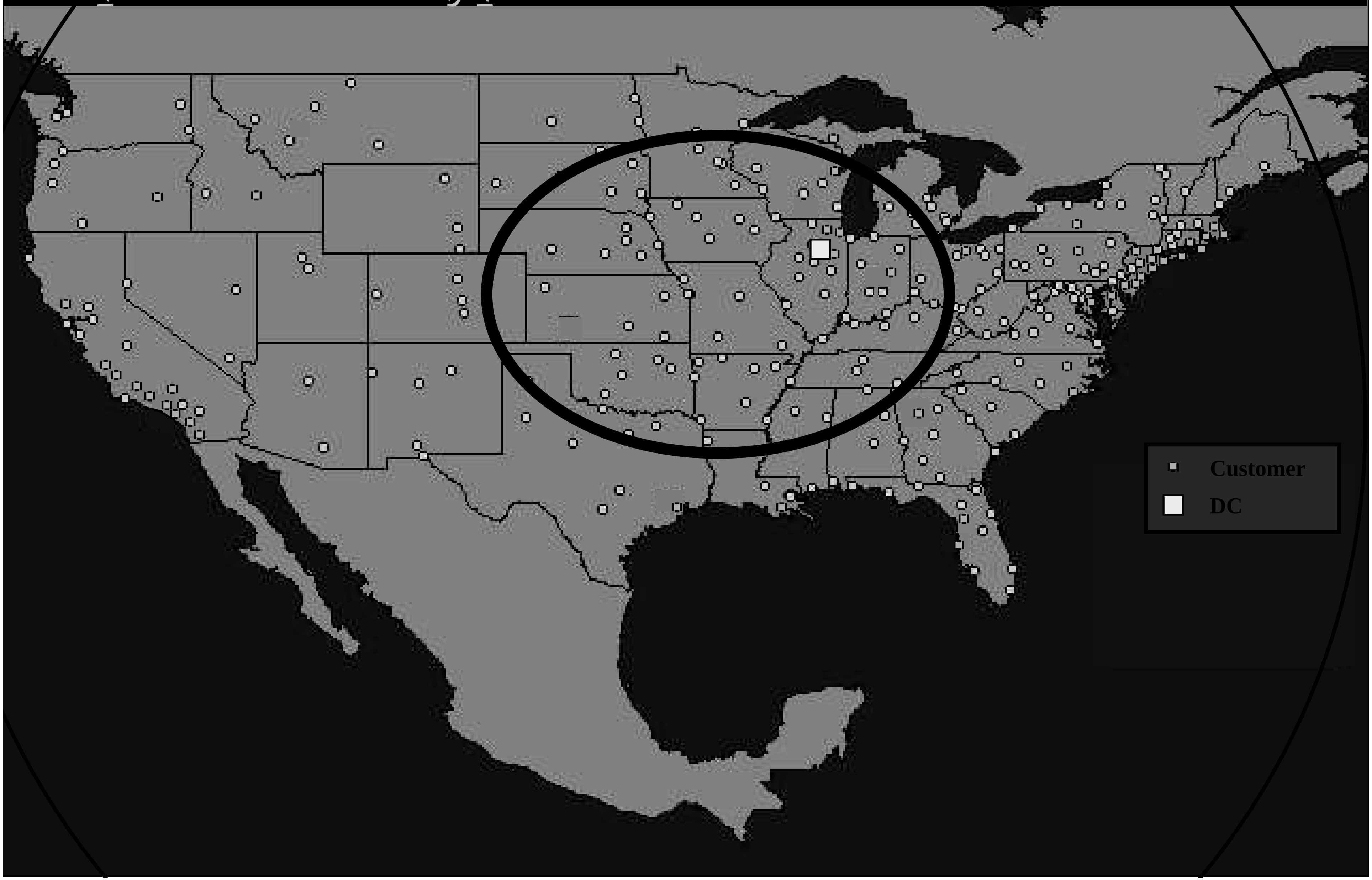
The Cost-Response Time Frontier



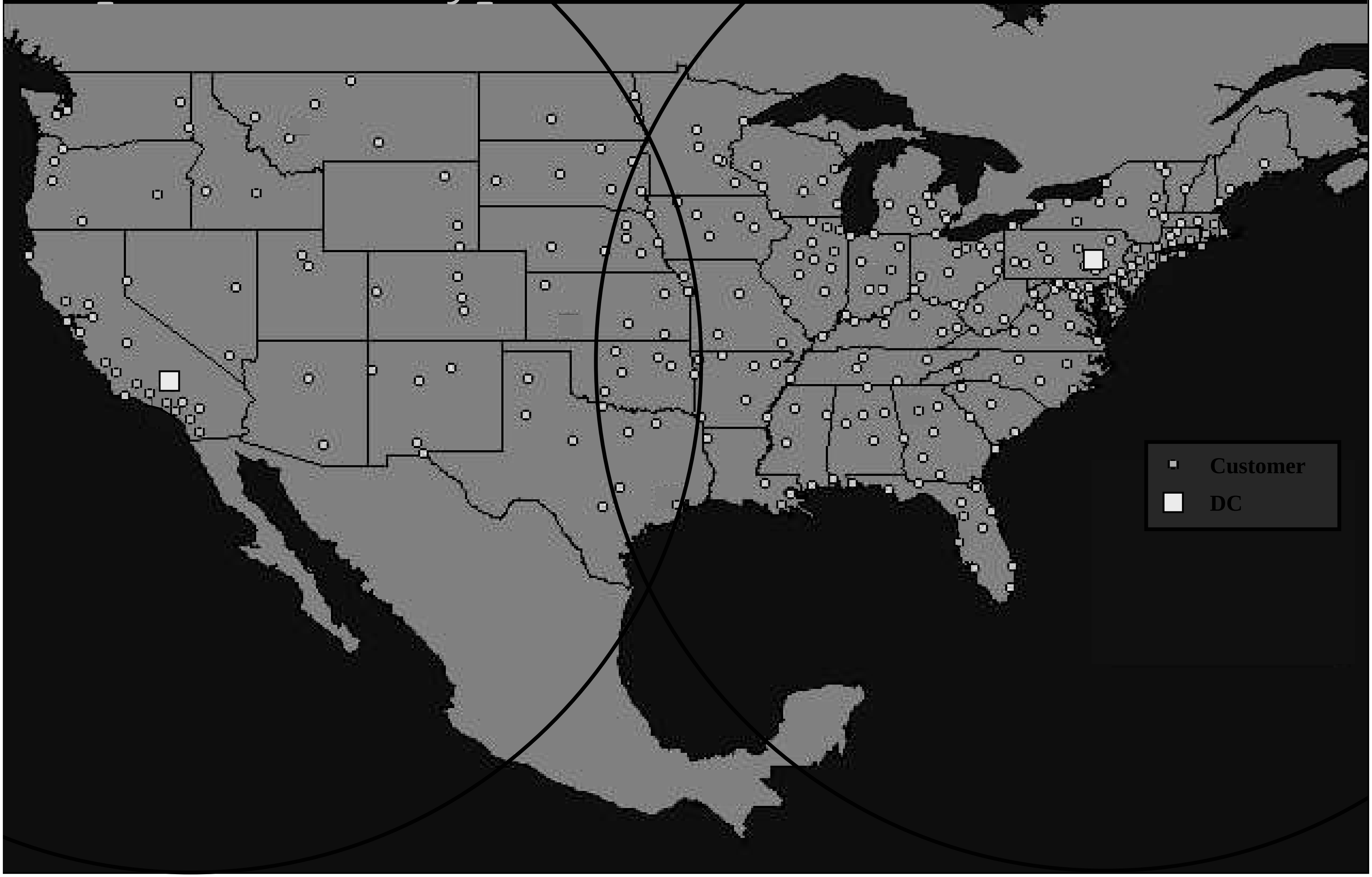
Service and Number of Facilities



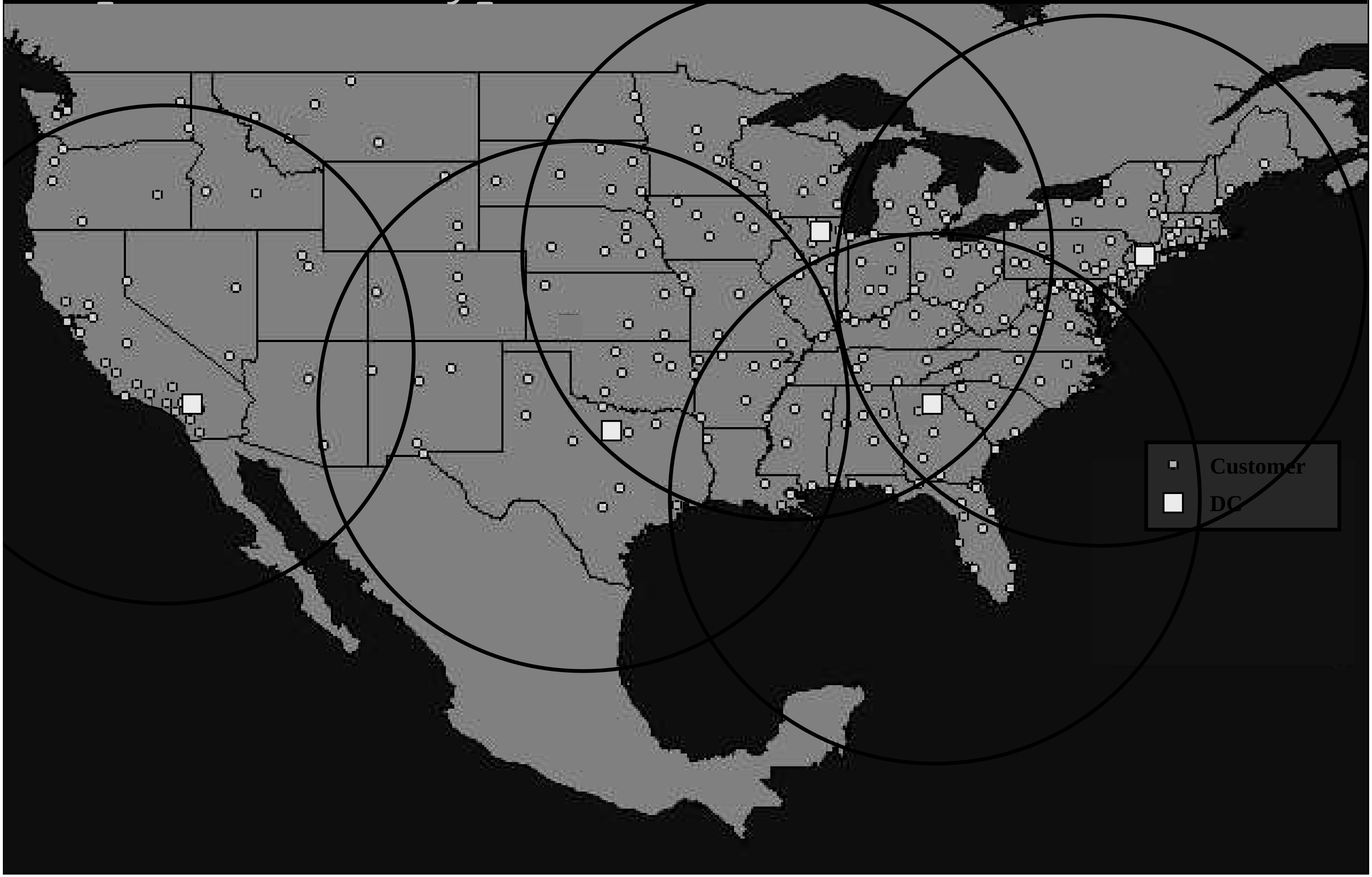
Where inventory needs to be for a one week order response time - typical results --> 1 DC



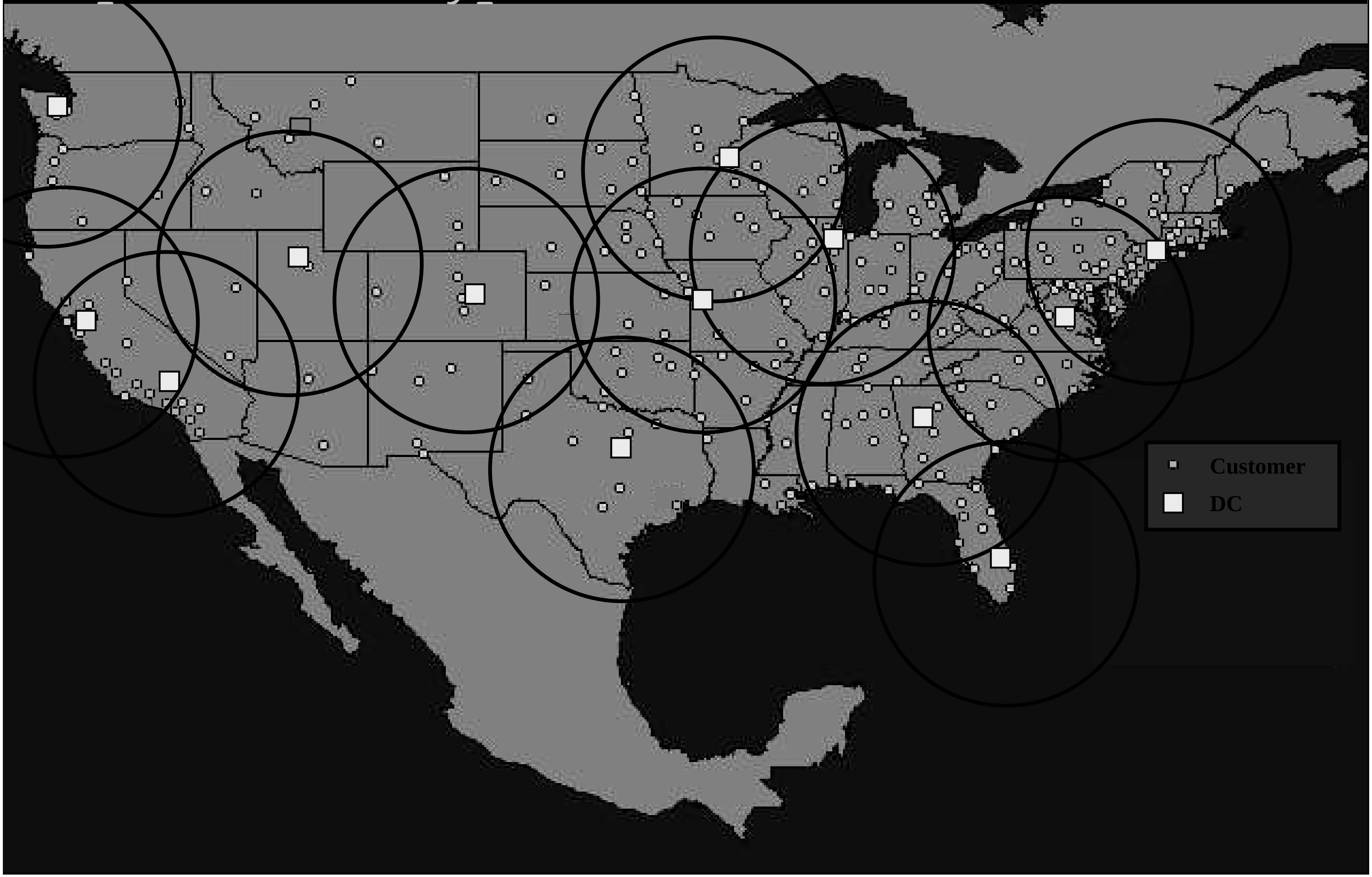
Where inventory needs to be for a 5 day order response time - typical results --> 2 DCs



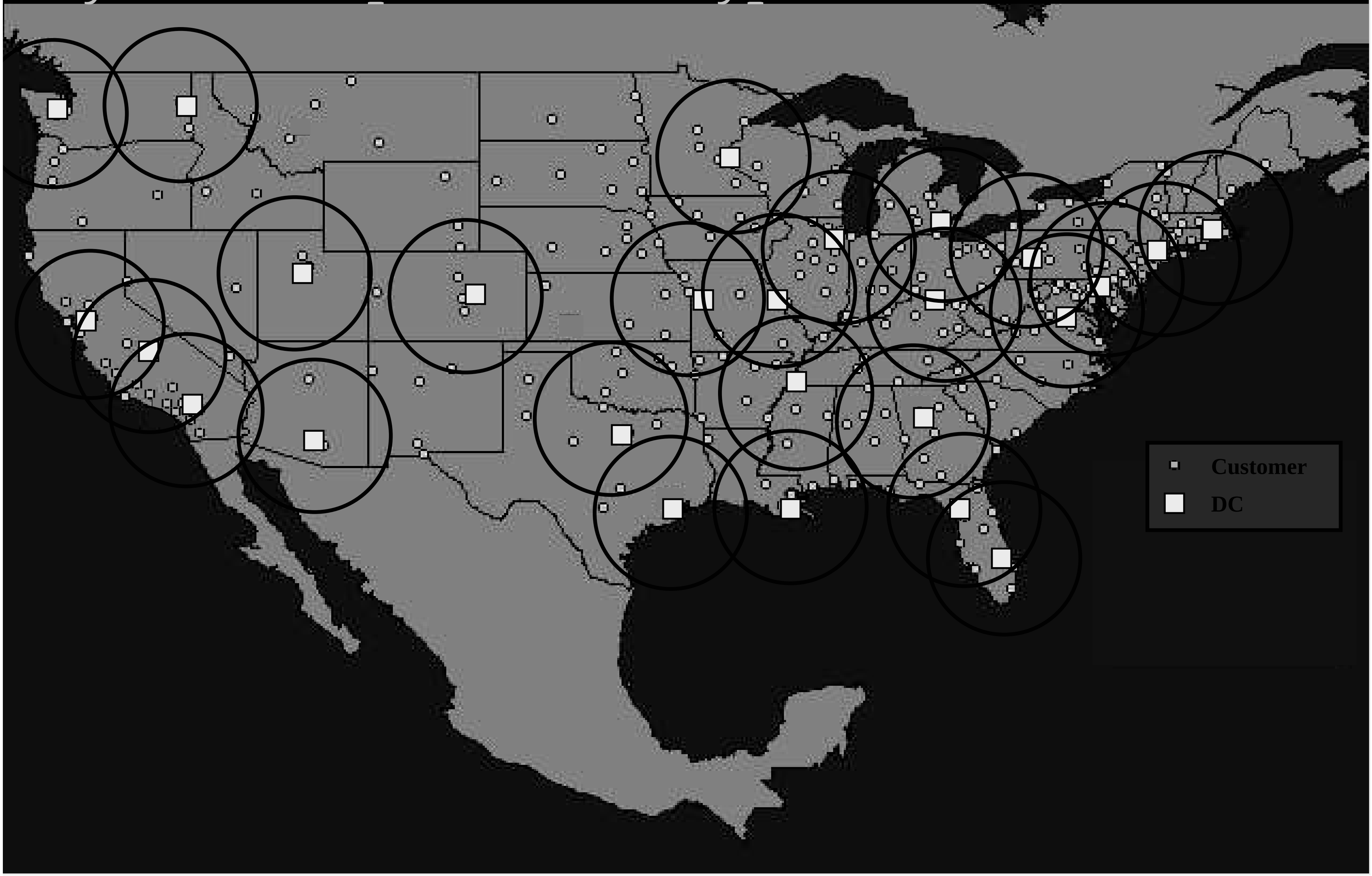
Where inventory needs to be for a 3 day order response time - typical results --> 5 DCs



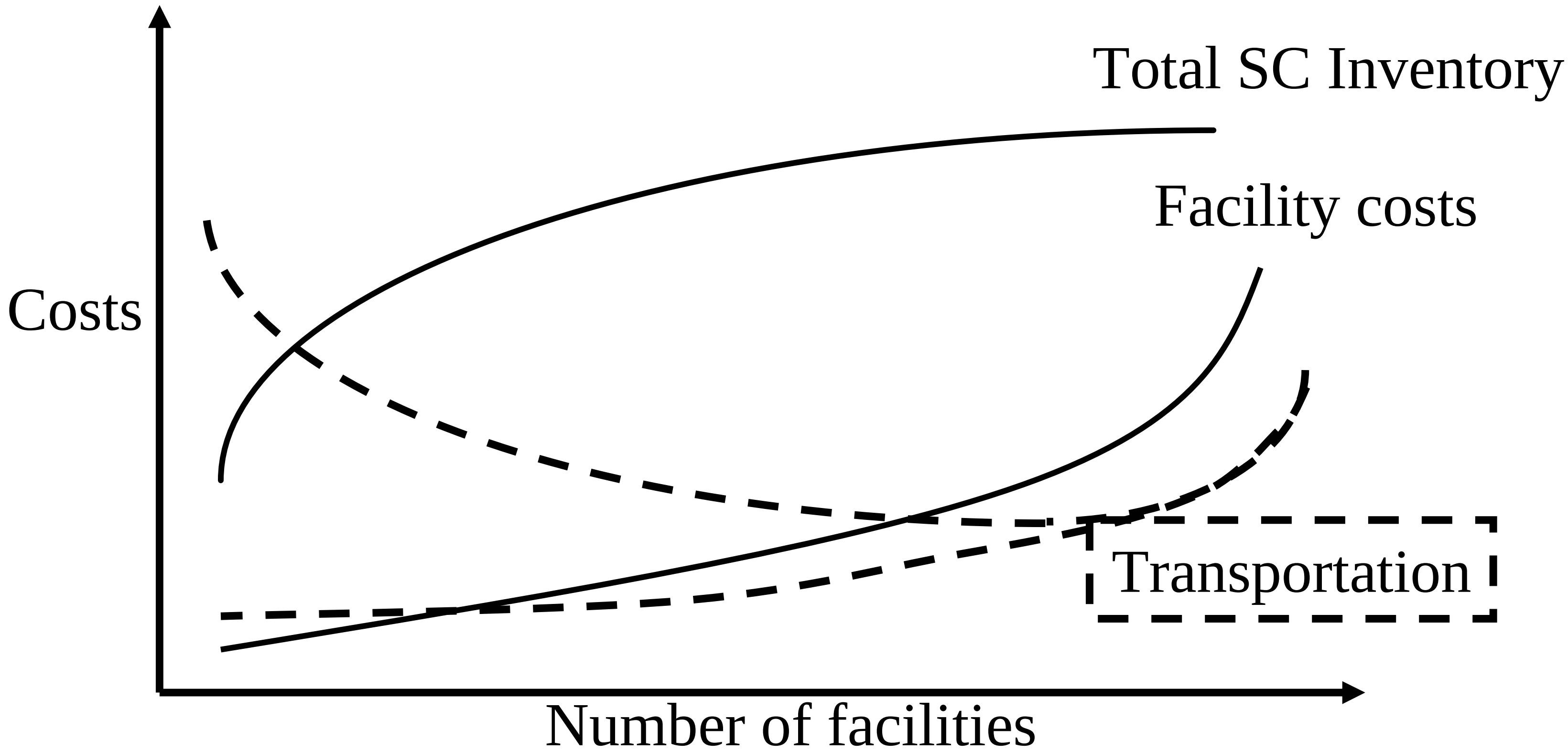
Where inventory needs to be for a next day order response time - typical results --> 13 DCs



Where inventory needs to be for a same day / next day order response time - typical results --> 26 DCs

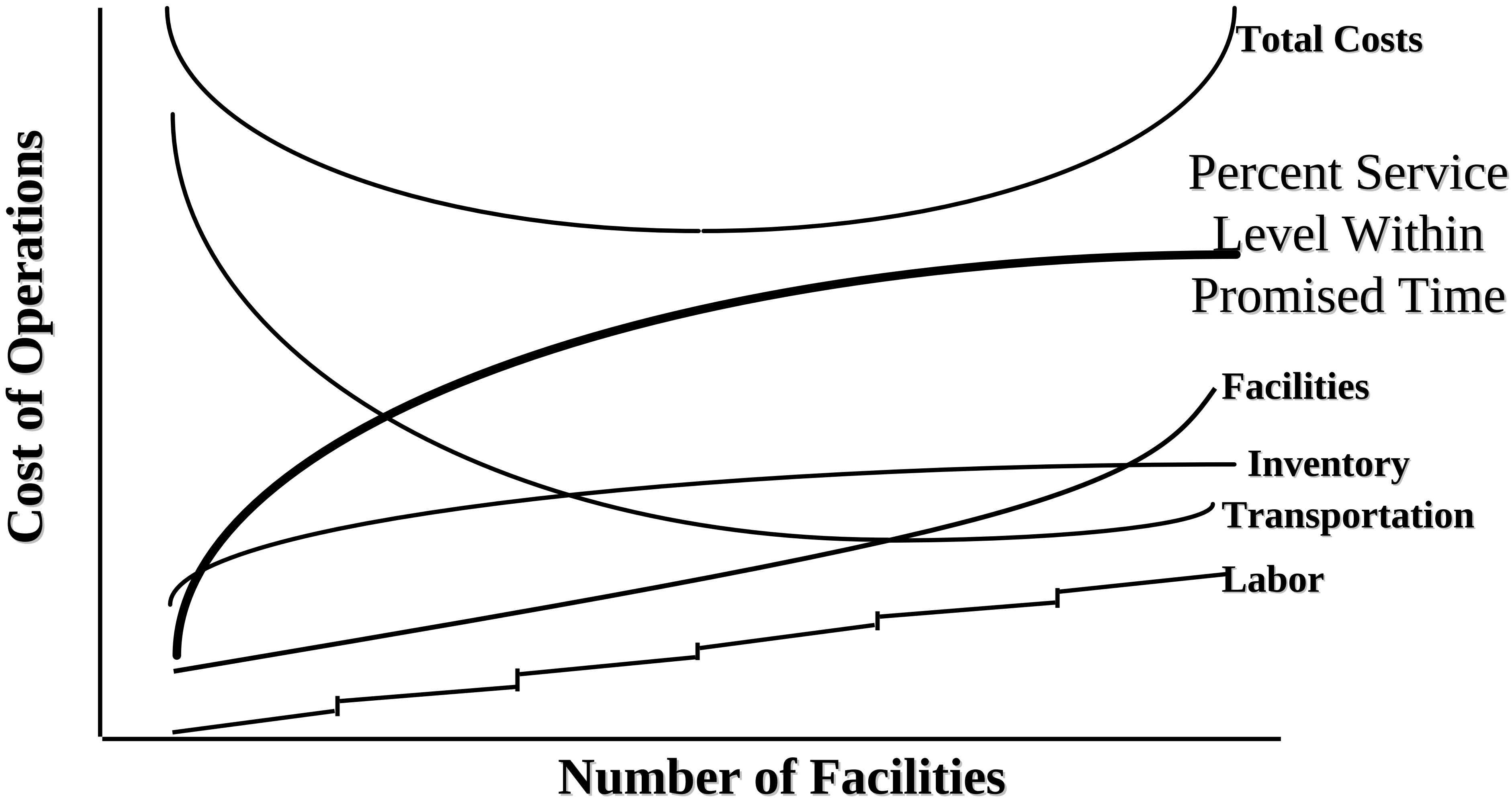


Costs and Number of Facilities



No economies of scale in shipment size
Economies of scale in inbound shipping

Cost Build-up as a function of facilities

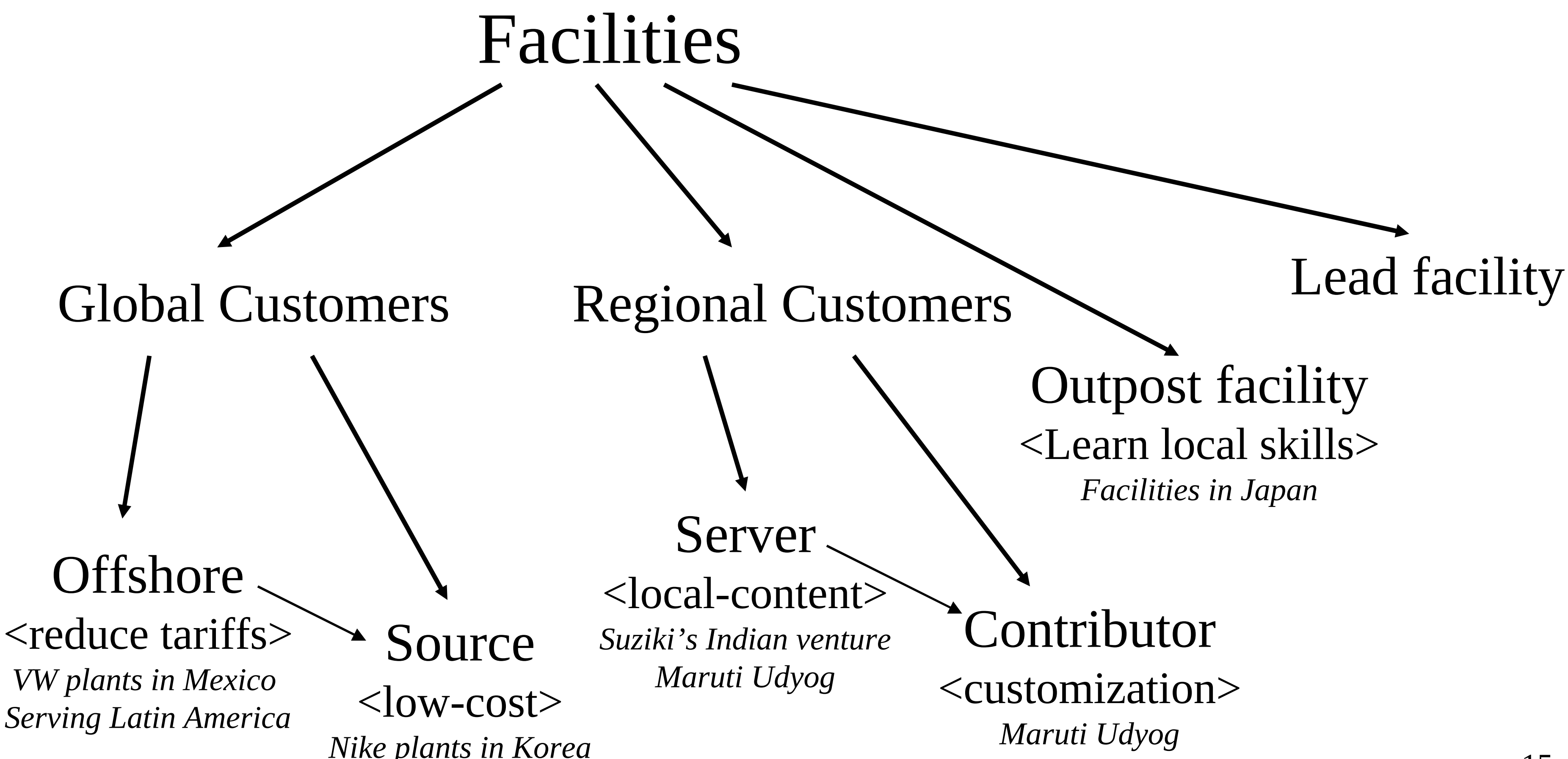


Network Design Decisions

- Facility function: Plant, DC, Warehouse
  - Where to locate functions, e.g. packaging
- Facility location
- Capacity allocation
- Market and supply allocation
  - Who serves whom

Factors Influencing Network Design Decisions

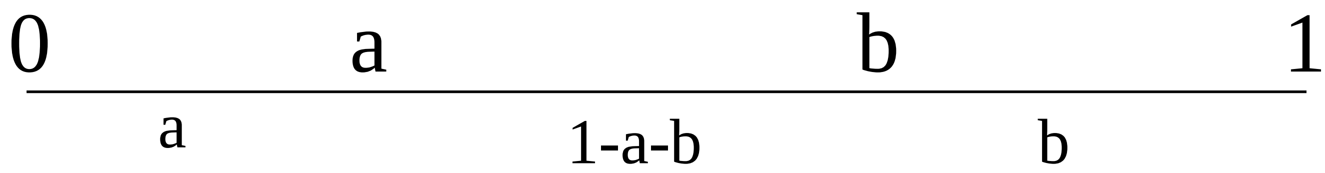
- Strategic



Factors Influencing Network Design Decisions

- Technological,
  - availability and economies of scale (fixed operational costs)
- Macroeconomic,
  - Tariffs, exchange rate volatility, economic volatility
- Political, stability
- Infrastructure, electricity, phone lines, suppliers
- Competitive
  - Negative externalities, see the next slide
  - Positive externalities
    - Nissan in India
    - Toyota City
    - Shopping Malls
    - Telecom corridor
- Logistics and facility costs

Negative externality:
Market Splitting by Hotelling's Model



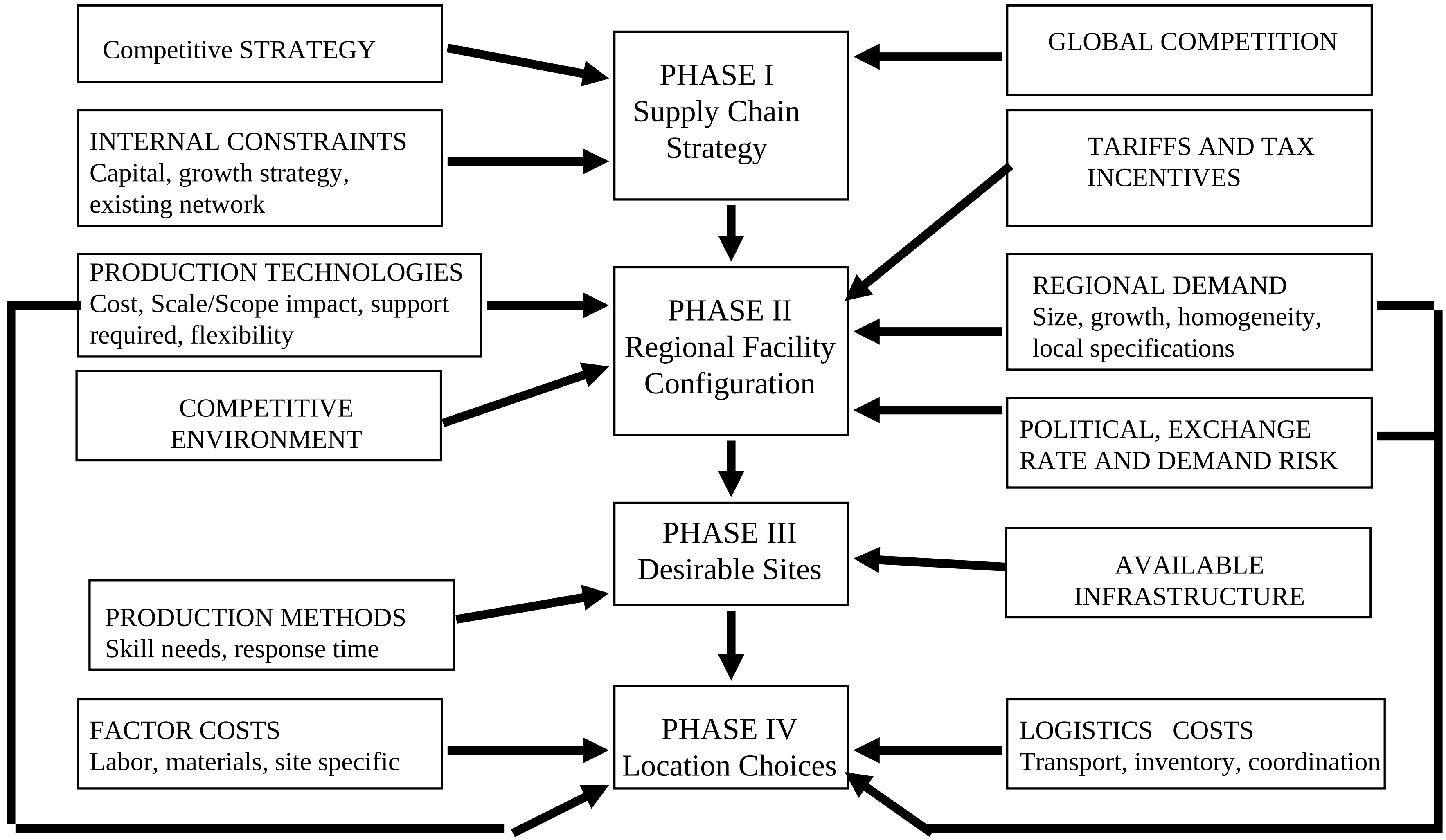
Suppose customers (preferences) are uniformly distributed over [0,1]

How much does firm at a get, how about firm at b?

If a locates first, where should b locate?

If a estimates how b will locate in response to a's location, where should a locate?

A Framework for Global Site Location



Analytical Models for SC Design

- Objective functions
  - » Private sector vs. Public sector. Equity?
- Demand allocation
  - » Distance vs. Price vs. Quality
    - Recall Hotelling
- Demand pattern over a geography
  - » Discrete vs. Continuous
- Feasibility check
  - » Ante vs. Post
- Distances
  - » Euclidean vs. Rectilinear
  - » Triangular inequality

Network Optimization Models

- Allocating demand to production facilities
- Locating facilities and allocating capacity

Key Costs:

- Fixed facility cost
- Transportation cost
- Production cost
- Inventory cost
- Coordination cost

Which plants to establish? How to configure the network?

Demand Allocation Model: Transportation Problem

Which market is served by which plant?

Which supply sources are used by a plant?

Given m demand points, j=1..m

with demands D_j

Given n supply points, i=1..n

with capacity K_i

Send supplies from supply points to demand points

x_{ij} = Quantity shipped from plant site i to customer j

Each unit of shipment from supply point I to demand point j costs c_{ij}

$$Min \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij}$$

$$s.t.$$

$$\sum_{i=1}^{n} x_{ij} = D_j$$

$$\sum_{j=1}^{m} x_{ij} \le K_i$$

$$x_{ij} \ge 0$$

<See Excel File>

Plant Location with Multiple Sourcing

Which market is served by which plant?

Which supply sources are used by a plant?

None of the plants are open, a cost of f_i is paid to open plant i

At most k plants will be opened

y_i = 1 if plant is located at site i, 0 otherwise

x_{ij} = Quantity shipped from plant site i to customer j

How does cost change as k increases?

$$Min \sum_{i=1}^{n} f_i y_i + \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij}$$

$$s.t.$$

$$\sum_{i=1}^{n} x_{ij} = D_j$$

$$\sum_{j=1}^{m} x_{ij} \leq K_i y_i$$

$$\sum_{i=1}^{m} y_i \leq k$$

$$y_i \in \{0,1\}$$

Plant Location with Single Sourcing

Which market is served by which plant?

Which supply sources are used by a plant?

None of the plants are open, a cost of f_i is paid to open plant i

At most k plants will be opened

y_i = 1 if plant is located at site i, 0 otherwise

x_{ij} = 1 if market j is supplied by factory i, 0 otherwise

How does cost change as k increases?

$$Min \sum_{i=1}^{n} f_i y_i + \sum_{i=1}^{n} \sum_{j=1}^{m} D_j c_{ij} x_{ij}$$

$$s.t.$$

$$\sum_{i=1}^{n} x_{ij} = 1$$

$$\sum_{j=1}^{m} D_j x_{ij} \leq K_i y_i$$

$$y_i \in \{0,1\}$$

Gravity Methods for Location

Ton Mile-Center Solution

Given n delivery locations, i=1..n,

x_i, y_i : Coordinates of delivery location i

d_i : Distance to delivery location i

F_i : Annual tonnage to delivery location i

$$d_i = \sqrt{(x - x_i)^2 + (y - y_i)^2}$$

$$\underset{x,y}{\text{Min}} \sum_{i=1}^{n} F_i \sqrt{(x_i - x)^2 + (y_i - y)^2}$$

Locate a warehouse at (x,y)

$$x = \frac{\sum_{i=1}^{n} \frac{x_i F_i}{d_i}}{\sum_{i=1}^{n} \frac{F_i}{d_i}} \qquad y = \frac{\sum_{i=1}^{n} \frac{y_i F_i}{d_i}}{\sum_{i=1}^{n} \frac{F_i}{d_i}}$$

<Show Excel File>

Gravity Methods for Location

Change the distance

$$d_i = (x - x_i)^2 + (y - y_i)^2$$

$$\underset{x,y}{Min} \sum F_i [(x_i - x)^2 + (y_i - y)^2]$$

Given n delivery locations, i=1..n,

x_i, y_i : Coordinates of delivery location i

d_i : Distance to delivery location i

F_i : Annual tonnage to delivery location i

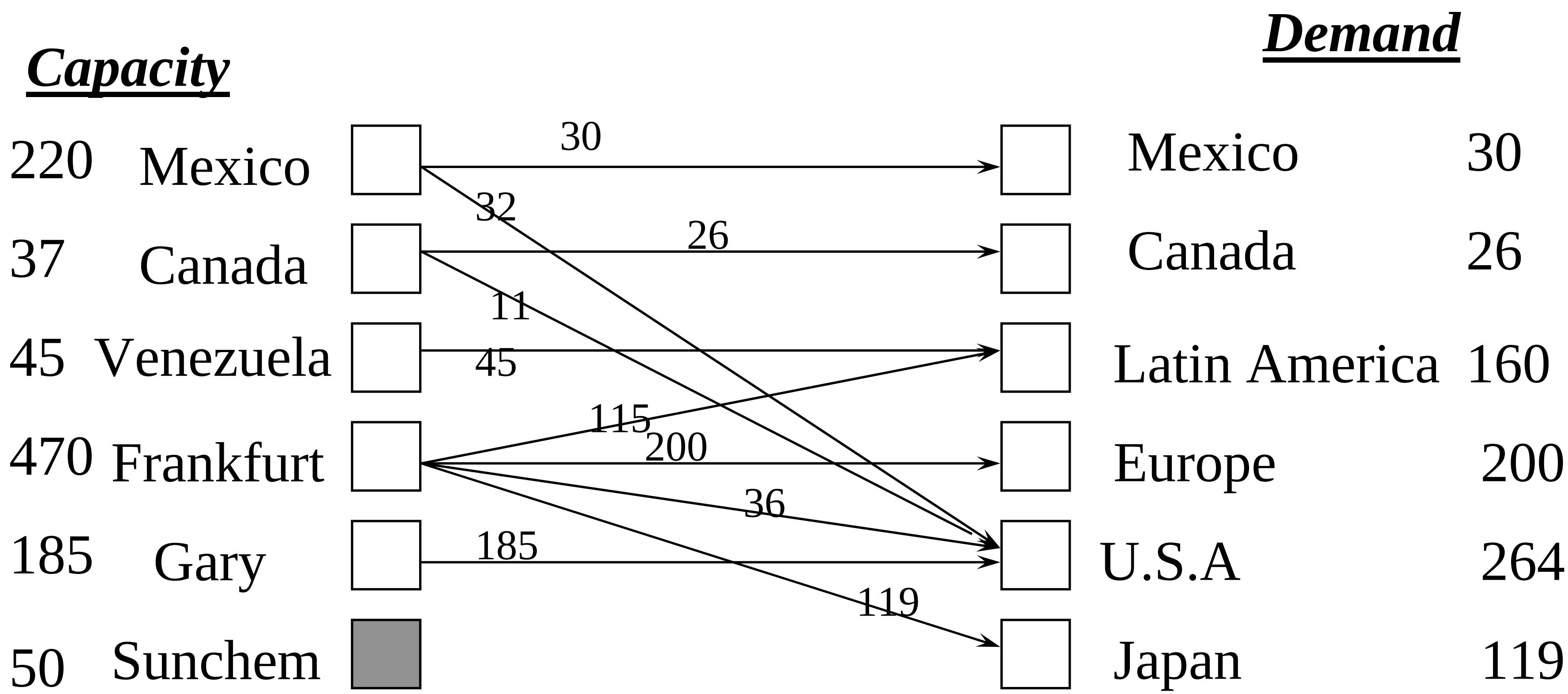
$$x = \frac{\sum_{i=1}^{n} x_i F_i}{\sum_{i=1}^{n} F_i} \qquad y = \frac{\sum_{i=1}^{n} y_i F_i}{\sum_{i=1}^{n} F_i}$$

Locate a warehouse at (x,y)

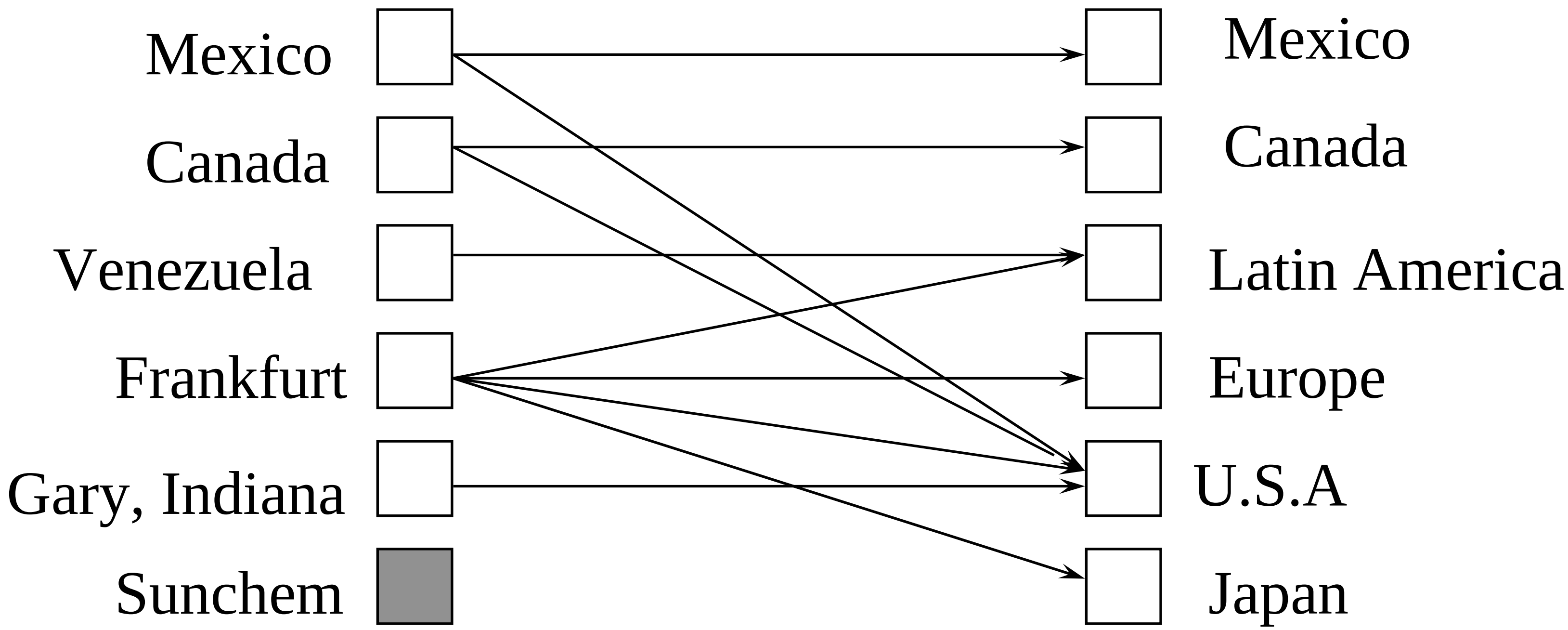
Applichem Demand Allocation

To / From	Mexico	Canada	Venezuela	Frankfurt	Gary	Sunchem	Capacity
Mexico	$ 81	$ 92	$ 136	$ 101	$ 96	$ 101	220
Canada	$ 147	$ 78	$ 135	$ 98	$ 88	$ 97	37
Venezuela	$ 172	$ 106	$ 96	$ 120	$ 111	$ 117	45
Frankfurt	$ 115	$ 71	$ 110	$ 59	$ 74	$ 77	470
Gary, Indiana	$ 143	$ 77	$ 134	$ 91	$ 71	$ 90	185
Sunchem	$ 222	$ 129	$ 205	$ 145	$ 136	$ 116	50
Demand	30	26	160	200	264	119	

Applichem Demand Allocation (1982)

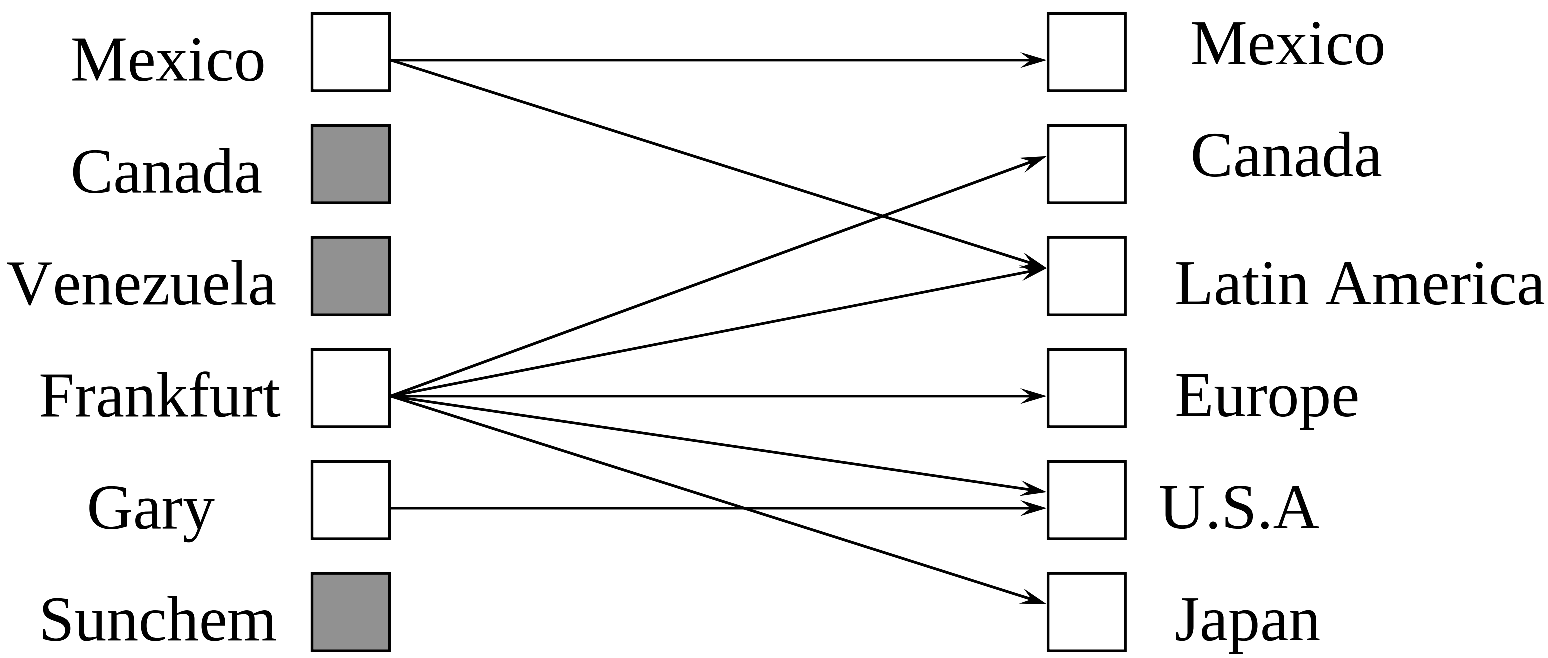


Applichem Production Network 1982 (with duties)



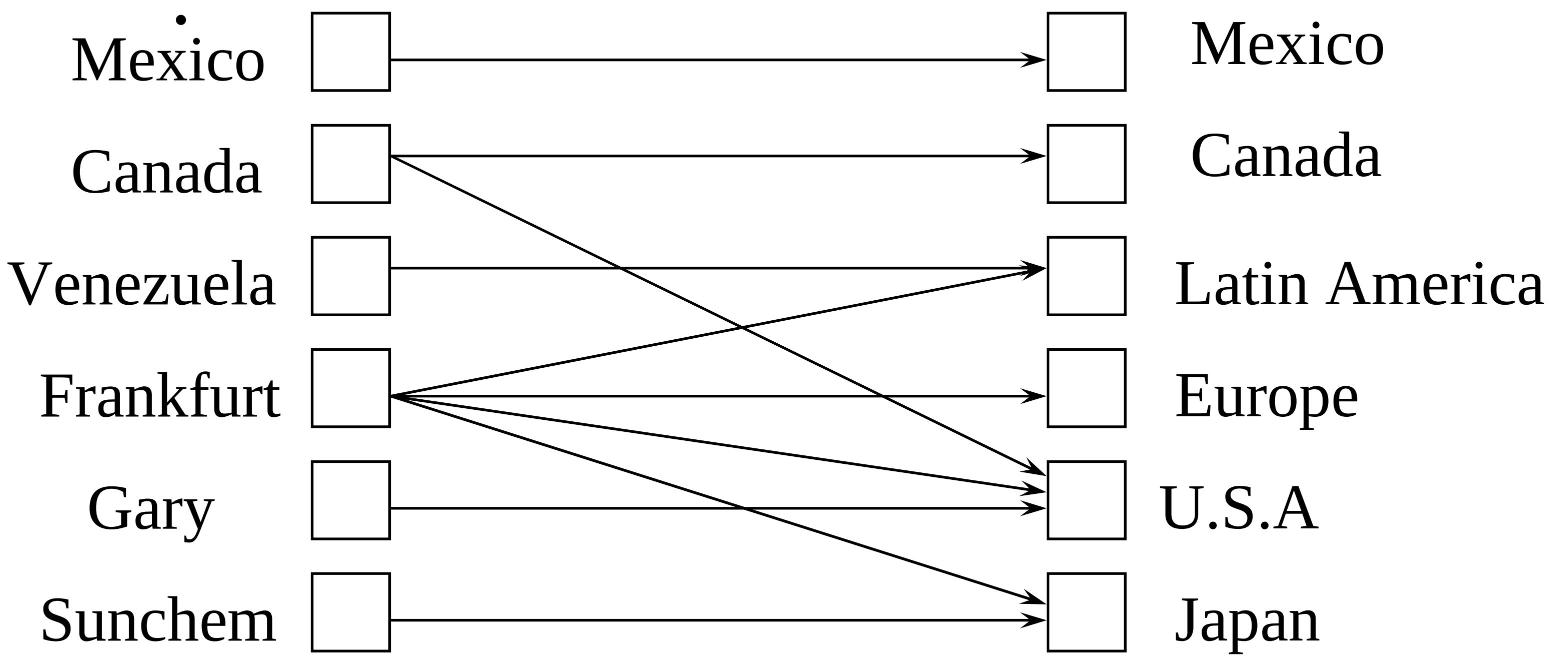
Annual Cost = $72,916,400

Applichem Production Network 1982 (without duties)



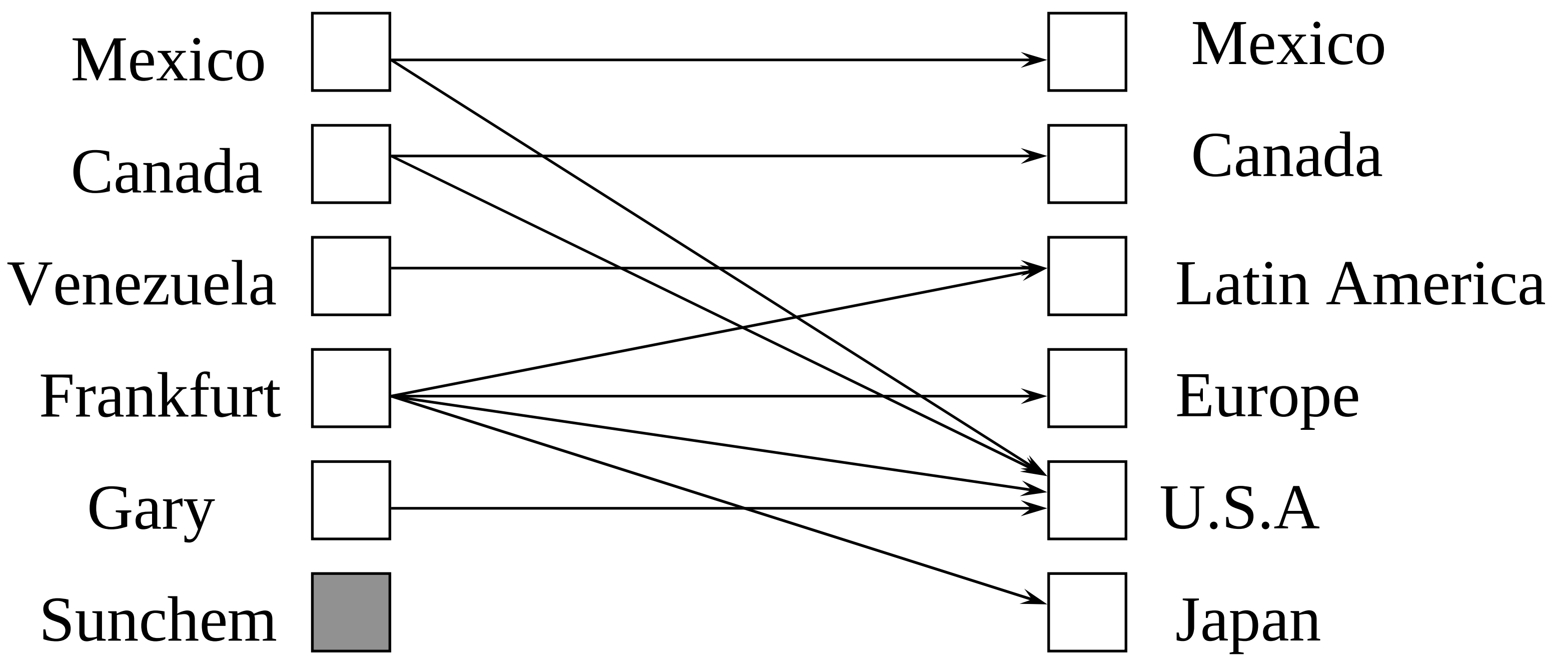
Annual Cost = 66,328,100

1981 Network



Annual Cost = $79,598,500

1981 Network (Sunchem Closed)



Annual Cost = $82,246,800

Cash Flows From Sunchem Plant

Year	1977	1978	1979	1980	1981	1982
Optimal ($ Million)	60.562	68.889	75.999	79.887	79.598	72.916
Sunchem Closed	60.721	68.889	77.503	80.999	82.247	72.916
Difference	0.159	0.000	1.504	1.112	2.649	0.000

Value of Adding 0.1 M Pounds Capacity (1982)

Shadow (dual) prices from LP tells you where to invest.

Location	Shadow price
Mexico	$0
Canada	$8,300
Venezuela	$36,900
Frankfurt	$22,300
Gary	$25,200
Sunchem	$0

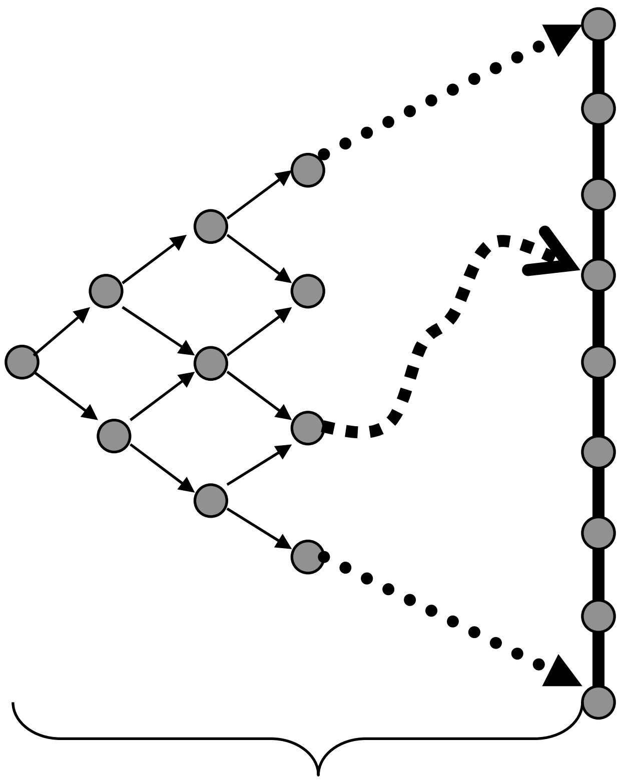
Should be evaluated as an option and priced accordingly.

Chapter 6

Network Design in an Uncertain Environment

A tree representation of uncertainty

- One way to represent Uncertainty is binomial tree
- Up by 1 down by -1 move with equal probability



T steps

$$Normal(0, T\sigma^2)$$

$$\sigma^2 = (1)^2(0.5) + (-1)^2(0.5) = 1$$

<Show Applet>

Decision tree

- One column of nodes for each time period
- Each node corresponds to a future state
  - What is in a state?
    - Price, demand, inflation, exchange rate, your OPRE 6366 grade
- Each path corresponds to an evolution of the states into the future
- Transition from one node to another determined by probabilities
- Pieces of optimal paths must be optimal
  - Find shorter and optimal paths starting from period T and work backwards in time to period 0.

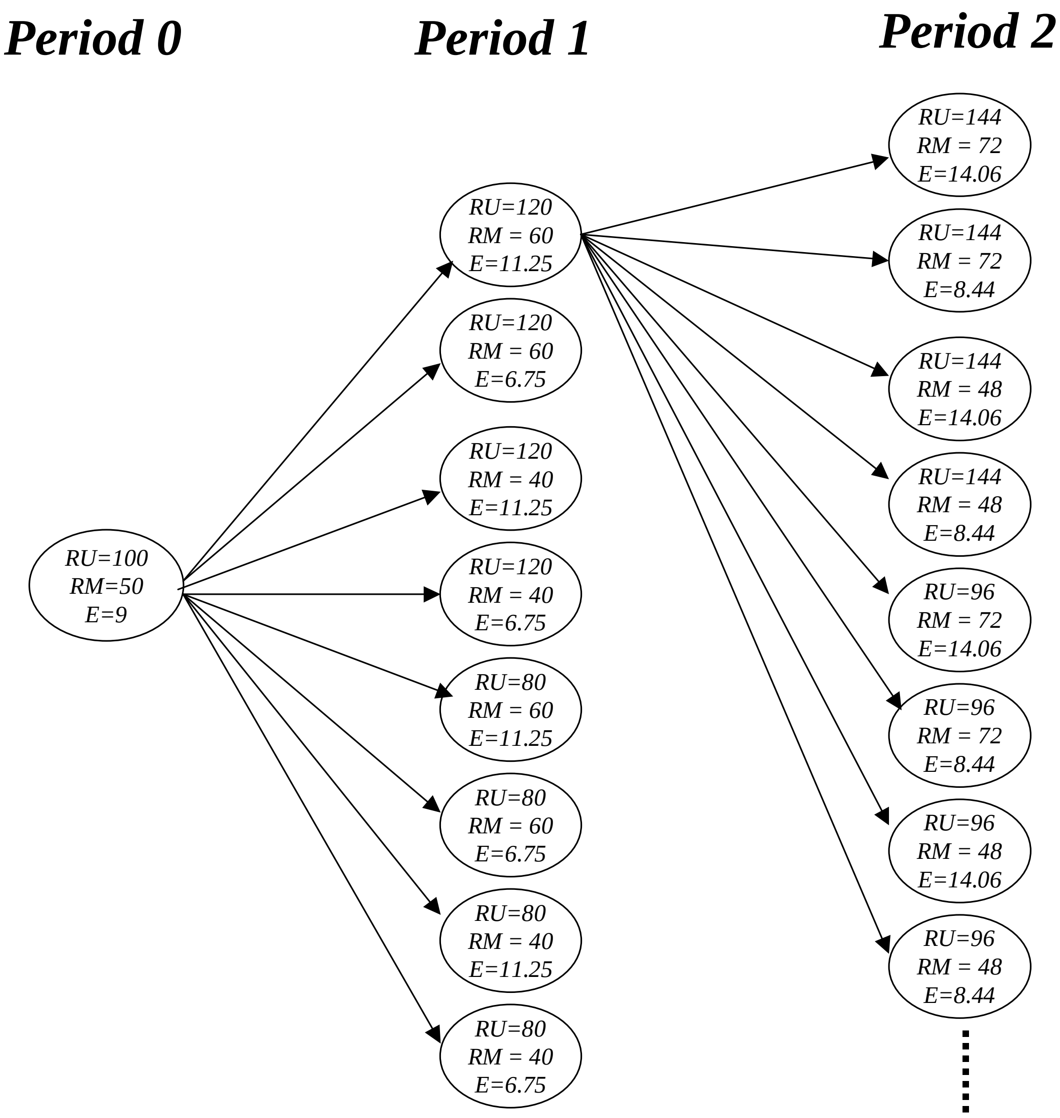
Evaluating Facility Investments: AM Tires. Section 6.5 of Chopra.

Plant	Dedicated Plant		Flexible Plant	
	Fixed Cost	Variable Cost	Fixed Cost	Variable Cost
US 100,000	$1 M /year.	$15 /tire	$1.1 M /year	$15 /tire
Mexico 50,000	4 M pesos / year	110 pesos /tire	4.4 M pesos /year	110 pesos /tire

U.S. Demand = 100,000; Mexico demand = 50,000. 1US$ = 9 pesos

Demand goes up or down by 20 percent with probability 0.5 and exchange rate goes up or down by 25 per cent with probability 0.5.

AM Tires



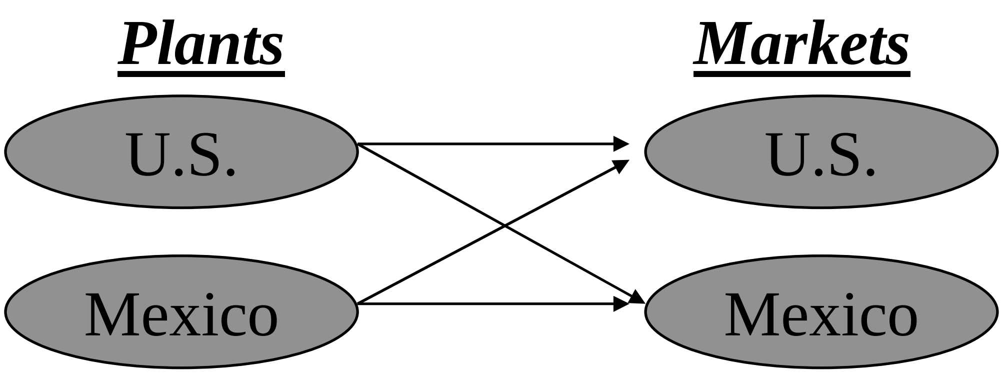
Period 0
Period 1
Period 2
RU=100
RM=50
E=9
RU=120
RM = 60
E=11.25
RU=120
RM = 60
E=6.75
RU=120
RM = 40
E=11.25
RU=120
RM = 40
E=6.75
RU=80
RM = 60
E=11.25
RU=80
RM = 60
E=6.75
RU=80
RM = 40
E=11.25
RU=80
RM = 40
E=6.75
RU=144
RM = 72
E=14.06
RU=144
RM = 72
E=8.44
RU=144
RM = 48
E=14.06
RU=144
RM = 48
E=8.44
RU=96
RM = 72
E=14.06
RU=96
RM = 72
E=8.44
RU=96
RM = 48
E=14.06
RU=96
RM = 48
E=8.44

AM Tires

Four possible capacity scenarios:

- Both dedicated
- Both flexible
- U.S. flexible, Mexico dedicated
- U.S. dedicated, Mexico flexible

For each node solve the demand allocation model.



AM Tires: Demand Allocation for RU = 144; RM = 72, E = 14.06

Source *i*	**Destination** *j*	**Variable cost**	**Shipping cost**	***E***	**Sale price**	**Margin($)** m_{ij}
U.S.	U.S.	$15	0	14.06	$30	$15
U.S.	Mexico	$15	$1	14.06	240 pesos	$1.1
Mexico	U.S.	110 pesos	$1	14.06	$30	$21.2
Mexico	Mexico	110 pesos	0	14.06	240 pesos	$9.2

$$\text{Max} \sum_{i=1}^{n} \sum_{j=1}^{m} m_{ij} x_{ij} \quad \text{such that}$$

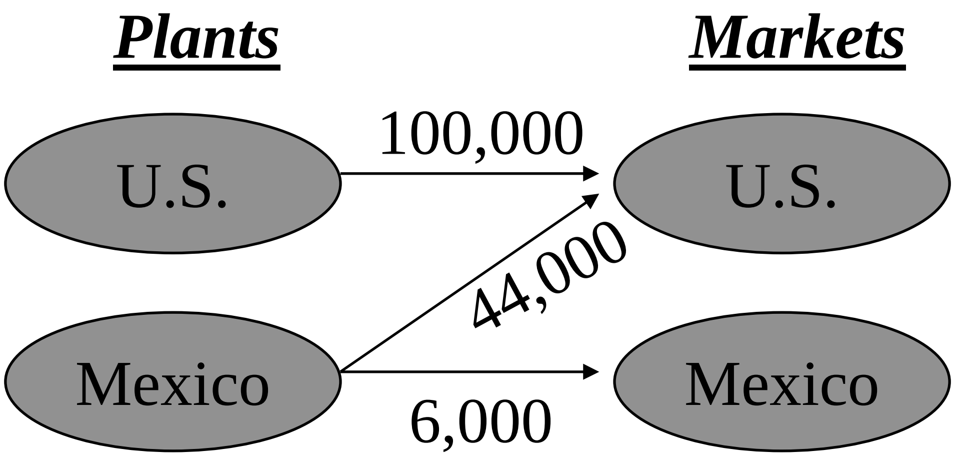
$$\sum_{i=1}^{n} x_{ij} \le D_j$$

$$\sum_{j=1}^{m} x_{ij} \le K_i$$

$$x_{ij} \ge 0$$

Compare this formulation to the Transportation problem.

AM Tires: Demand Allocation for RU = 144; RM = 72, E = 14.06



Profit =Revenue-Cost

The computations in the book go beyond my understanding now.
Profit without fixed costs=Objective value of optimization=$1,138,000
The number associated with Node (RU=144,RM=72,E=14.06) must be $1,138,000.
Fixed costs should not be deducted now. They are incurred in year 0
so must be deducted in year 0.

Facility Decision at AM Tires

Make profit computations for the first year nodes one by one:
Compute the profit for a node and add to that
(0.9)(1/8)(*Sum of the profits of all 8 nodes connected to the current one*)

Plant Configuration		NPV
United States	Mexico	
Dedicated	Dedicated	$1,629,319
Flexible	Dedicated	$1,514,322
Dedicated	Flexible	$1,722,447
Flexible	Flexible	$1,529,758

Capacity Investment Strategies

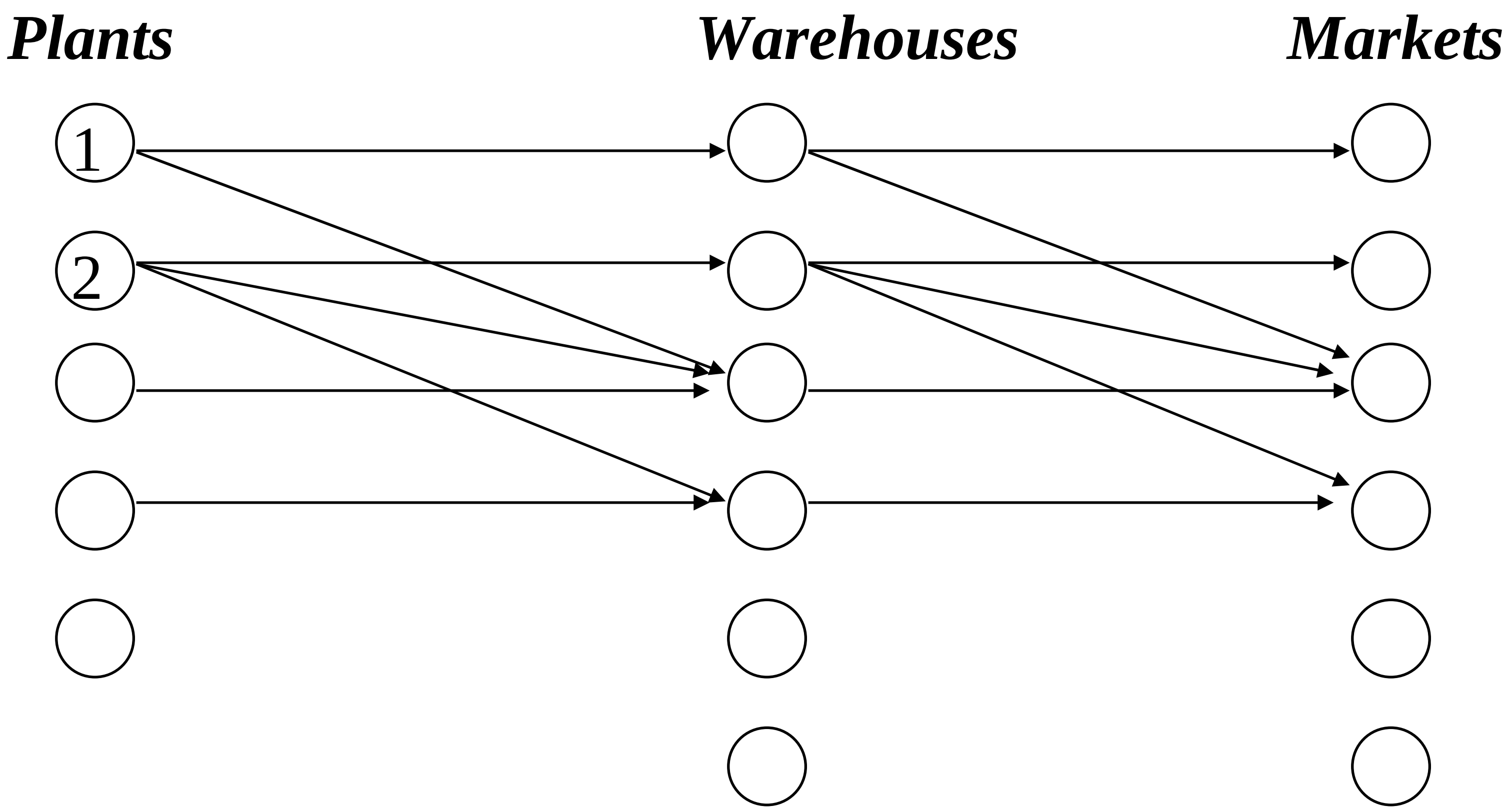
- Single sourcing
- Hedging Strategy
  - Risk management?
  - Match revenue and cost exposure
- Flexible Strategy
  - Excess total capacity in multiple plants
  - Flexible technologies

- More will be said in aggregate planning chapter

Summary

- Frequency decomposition
- Factors influencing facility decisions
- A strategic framework for facility location
- Gravity methods for location
- Network-LP-IP optimization models
- Value capacity as a real option

Location Allocation Decisions



Which plants to establish? Which warehouses to establish? How to configure the network?

p-Median Model

Inputs:
A set of feasible plant locations, indexed by j
A set of markets, indexed by i

D_i demand of market i
No capacity limitations for plants
At most p plants are to be opened

d_{ij} distance between market i and plant j

y_j = 1 if plant is located at site j,
0 otherwise
x_{ij} = 1 if market i is supplied from plant site j,
0 otherwise

$$Min \sum_i D_i \sum_j d_{ij} x_{ij}$$

$$s.t.$$

$$\sum_j y_j = p$$

$$x_{i,j} \leq y_j \quad \text{for all i, j}$$

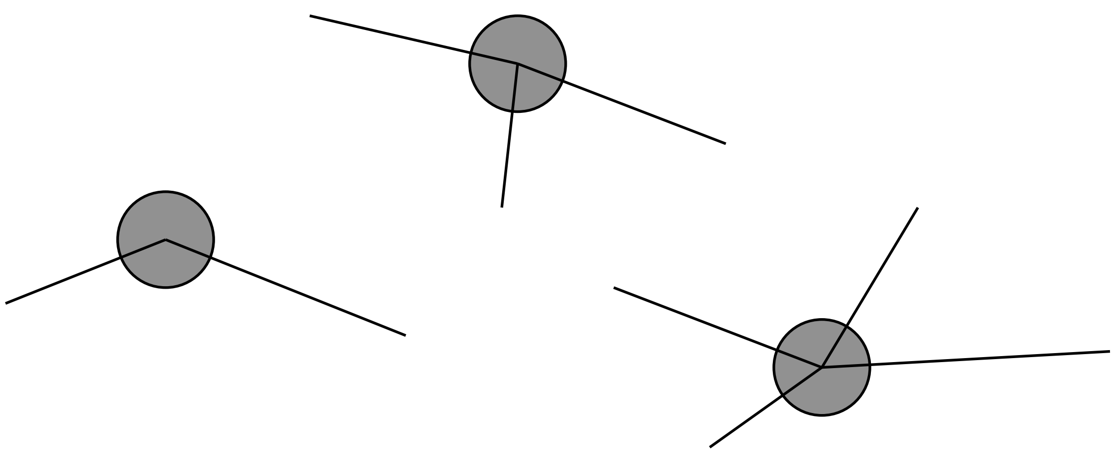
$$\sum_j x_{ij} \leq 1 \quad \text{for all i}$$

$$x_{ij}, y_j \in \{0,1\} \quad \text{for all i, j}$$

p-Center Model

Replace the objective function in p-Median problem with
Min Max $\{d_{ij}x_{ij}$: i is a market assigned to plant j$\}$

We are minimizing maximum distance between a market and a plant
Or say minimizing maximum distance between fire stations and all the houses served by those fire stations. An example with p=3 stations and 9 houses:



p-Covering Model

x_i = 1 if demand point i is covered, 0 otherwise

y_j = 1 if facility j is opened, 0 otherwise

N_i facilities associated with demand point i

If j is in N_i, j can serve i

Can you read constraint (*) in English?

$$Max \sum_i D_i x_i$$

$$s.t.$$

$$\sum_{j \in N_i} y_j \geq x_i \quad \text{for all i} \quad (*)$$

$$\sum_j y_j = p$$

$$x_i, y_j \in \{0,1\} \quad \text{for all i, j}$$

Other Models

- p-Choice Models
  - Criteria to choose the server: distance, price?
- Models with multiple decision makers
  - Franchise model