

Important continuous distributions

1. Exponential distribution

Density	$f(x) = \lambda e^{-\lambda x}$
CDF	$F(x) = 1 - e^{-\lambda x}$
Failure rate	$h(t) = \lambda$
Parameter	λ
Possible values	$x > 0$

2. Gamma (Erlang) distribution

X = a sum of r independent $Exp(\lambda)$ random variables.

$$\text{Density} \left| f(x) = \frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x} \right.$$

For integer r , $\Gamma(r) = (r - 1)!$

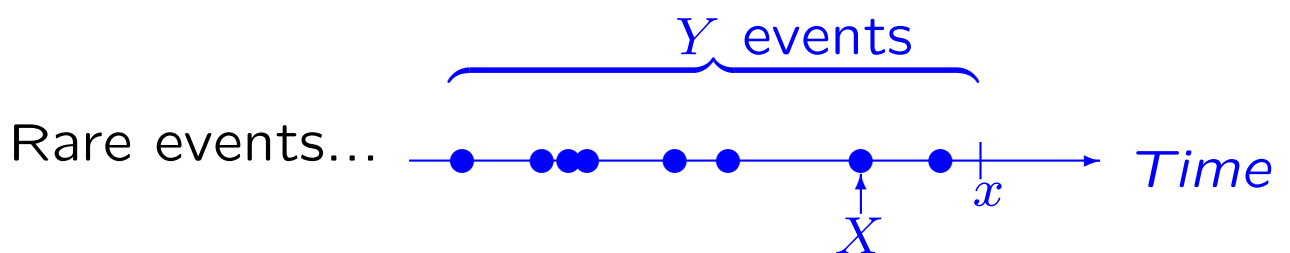
$$\text{CDF} \left| F(x) = \int_0^x f(t) dt \right.$$

Parameters λ, r

Possible values $x > 0$

A sum of independent $Exp(\lambda)$ random variables with different λ has *Hypoexponential distribution*

Gamma-Poisson formula



X = time of the r -th event

$X \sim \text{Gamma}(r, \lambda)$

Y = number of events by the time x

$Y \sim \text{Poisson}(\lambda x)$

$$P(X < x) = P(Y \geq r)$$

$$P(X > x) = P(Y < r)$$

3. Uniform distribution

Density	$f(x) = \frac{1}{b-a}$ (constant)
Parameters	a, b
Possible values	$a < x < b$

Uniform property:

$$P(x < X < x + h) = \frac{h}{b-a}$$

for all x such that $[x, x + h] \subset [a, b]$

Standard Uniform distribution: $a = 0, b = 1$.

$$f(x) = 1 \quad \text{for} \quad 0 < x < 1$$

4. Normal (Gaussian) distribution

Density	$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/2\sigma^2}$
CDF	Table C3 for Standard Normal
Parameters	μ, σ
Possible values	$-\infty < x < +\infty$

Standard Normal distribution: $\mu = 0, \sigma = 1.$

Normal approximation to Binomial distribution

$$\text{Binomial}(n, p) \approx \text{Normal}(\mu, \sigma)$$

$$\text{if} \left\{ \begin{array}{l} n \text{ is large} \\ p \text{ is neither small nor large} \\ \mu = np \\ \sigma = \sqrt{np(1-p)} \end{array} \right.$$

Continuity correction:

For a discrete (Binomial) X ,

$$P(X < 10) = P(X < 9.5)$$

$$P(X \leq 10) = P(X < 10.5)$$