Modeling Random Phenomena and Making Decisions under Uncertainty

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Uncertainty in computer environment

- Arrival of jobs
- Execution time
- Memory requirement
- Failure of components
- Exposure to viruses
- Errors in codes
- Etc., etc., etc.

Random phenomena elsewhere

- Economy:
  - stock prices
  - number of jobs
  - price of oil
- Environment
  - temperature
  - pollution
  - natural disasters

- Going to UTD
  - number of green lights
  - available parking

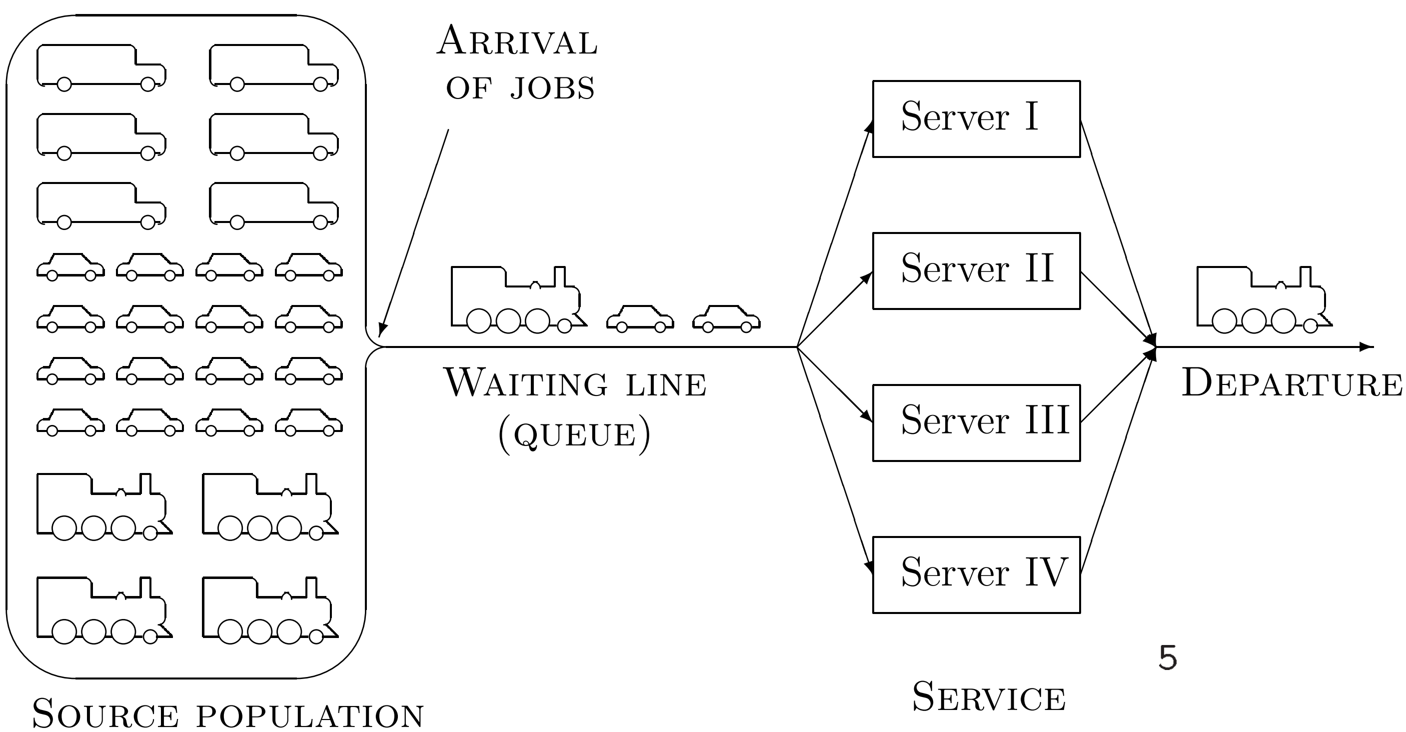
- This class
  - quiz problems
  - time spent on each topic
  - grades

This course: **Quantify uncertainty, model uncertainty, make decisions**

What we'll learn

- Uncertainty; probabilities of events
- Random variables
- Monte Carlo simulation
- Stochastic processes

- Application: queuing systems



- Statistical inference
- Estimation
- Testing hypothesis

Probability

$=$ chance (common sense)

$=$ odds (gambling)

$=$ long-term proportion (relative frequency)

$=$ likelihood (forecasting)

$=$ finite measure (mathematics)

Probability
= function of an event = $P(E)$

Domain: events

Event = combination of *outcomes*

Consider an experiment, results are outcomes,

Sample space S = {all possible outcomes}

Event E = a set of outcomes

= subset of S ($E \subset S$)

Range: $[0, 1]$

"Every event" E has probability $P(E)$,

$$0 \leq P(E) \leq 1$$

If

$$E = \{O_1, ..., O_n\} = \{\text{outcomes}\}$$

then

$$P(E) = \sum_{k=1}^{n} P(O_k) = P(O_1) + \cdots + P(O_n)$$

An empty event $\emptyset$ is an event,

$$P(\emptyset) = 0$$

Also, $P(S) = 1$.

Set operations

$E_1, E_2, ..., E_n$ - events (sets of outcomes)

Union of $E_1, ..., E_n$ is an event

- consists of all outcomes of $E_1, ..., E_n$
- occurs if any of $E_1, ..., E_n$ occurs

$$\{E_1 \cup ... \cup E_n\} = \{E_1 \text{ or } ... \text{ or } E_n\}$$

Intersection of $E_1, ..., E_n$ is an event

- consists of common outcomes of $E_1, ..., E_n$
- occurs if each $E_1, ..., E_n$ occurs

$$\{E_1 \cap ... \cap E_n\} = \{E_1 \text{ and } ... \text{ and } E_n\}$$

Complement of E is an event

- consists of outcomes that are not in E
- occurs if E does not occur

$$\bar{E} = \{ \text{ not } E\}$$

Disjoint, or mutually exclusive events

- cannot occur together
- $A \cap B = \emptyset$

Exhaustive events

- their union is Sample Space
- at least one occurs for sure
- $A \cup B \cup C = S$

Basic rules of Probability

- $0 \leq P(E) \leq 1$ for all E

- $P(\emptyset) = 0$ and $P(S) = 1$

- Disjoint events:

  $P(E_1 \cup ... \cup E_n) = P(E_1) + ... + P(E_n)$

- Any events:

  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- Complement rule

  $P(E) + P(\bar{E}) = 1$
  (disjoint and exhaustive)

  $\Rightarrow \qquad P(\bar{E}) = 1 - P(E)$

Question:
What should I compute: $P(E)$ or $P(\bar{E})$?

Independence

- Independent events:

$$P(A \cap B) = P(A)P(B)$$

$$P(E_1 \cap ... \cap E_n) = P(E_1) \times ... \times P(E_n)$$

Questions:
Can disjoint events be independent?
Can exhaustive events be independent?