Fill Rate of Single-Stage General Periodic Review Inventory Systems

Jiang Zhang‡ · Jun Zhang†[1]

‡Department of Management, Marketing, and Decision Sciences, School of Business, Adelphi University, Garden City, New York 11530, USA

†School of Management, The University of Texas at Dallas, Richardson, TX 75083, USA

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Abstract

This paper develops an exact formula for the fill rate of a single-stage inventory system that uses a general periodic review base-stock policy. For normal demand, we present a fill-rate expression that uses the standard normal PDF and CDF, and develop two approximations for the fill rate.

Keywords: inventory; periodic review; base-stock policy; fill rate

1 Introduction

We consider a single-stage inventory system supplying external demands and receiving stocks from an ample supply. Time is divided into periods of fixed length; a period can be a day or a week depending on the situation. In each period, demands arrive and are either filled or backlogged. The system operates under a general periodic-review base-stock policy. Under this policy, the inventory position (defined as amount on-hand + on-order − backlog) is reviewed once every R periods. If the inventory position is below a base-stock level τ, an order is placed to raise the inventory position back to τ. An order placed at the beginning of a review period t is delivered at the beginning of period $t + L$ (L and R are positive integers). Random demand occurs after the order placed L periods ago arrives in a period. The demands in different periods are assumed to be independent and identically distributed. All excess demands are backlogged. The inventory system we study in this paper is widely used in practice and is discussed under the names "Fixed-Time Period Models" [6], "Periodic Review (P) Systems" [9], and so on in popular introductory operations management textbooks. When $R = 1$, the general periodic review policy becomes the well studied periodic-review base-stock policy in the research literature.

This paper studies the system's fill rate, the long-run average fraction of demand satisfied immediately using on-hand stocks. We develop a formula for the exact fill rate for general demand

[1]Corresponding author. Tel: +1 972 883 6288; fax: +1 972-883-2089. Email address: jun.zhang@utdallas.edu.

distributions. This result allows us to establish sensitivity results on the fill rate with respect to the lead time L and the review cycle R. In particular, we show that the fill rate decreases in L and R. Further, for a fixed vulnerable period $L + R$, we demonstrate that the fill rate decreases in L and increases in R. When demand is normally distributed, we develop an exact fill rate expression using only the standard normal distribution function and density function. Our exact formula for normal demand can be easily implemented in commercial spreadsheet software using built-in functions (i.e., no macros). We also develop two approximate formulas for normally distributed demand. The first approximate formula is based on basic properties of normal distributions and the second formula is based on a logistic distribution approximation. Our approximation using the logistic distribution only involves *elementary functions* and the corresponding fill rate can be computed using a non-programmable calculator. Our exact and approximation formulas for the fill rate can be used to quickly determine base-stock levels to achieve a specified target service level.

One may think that a general periodic review system can be converted into the standard periodic review system by redefining the period-demand to be the demand over R periods. However, this mapping will not work when L is not a multiple of R. To see this, consider the general periodic review system with $L = 2$ and $R = 3$, normal demand with a mean 2000 and a standard deviation 200, and $\tau = 8658$. The fill rate for such a general periodic review system is 77.63%. In order to map the general periodic review system into the standard periodic review system, we redefine the time units such that one period in the *standard* periodic review system is equal to three periods in the *general* periodic review system. Under this definition, the period-demand mean is $2000 \times 3 = 6000$ and the period-demand standard deviation is $200\sqrt{3} = 346.4$. Because $L < 3$, the lead time can be rounded to zero or one in the standard periodic review system. If L is rounded to zero, the fill rate would be 1.0; if L is rounded to one, the fill rate would be 44.3%. Therefore, it is necessary to develop the fill rate formulas for the general periodic review system.

There is a literature on formulas for the fill rate under different inventory replenishment policies. Most of it concerns the fill rate with a demand process consisting of independent and identically distributed normal random variables; see, for example, Johnson et al. [8] for a review. Sobel [15] develops the expression for the fill rate of general demand distributions under the periodic-review base-stock policy. Our formula extends his work to the general periodic review policy (in the meantime, we point one inconsistency in his development). The policy we consider in this paper is similar to the (R, T) policy studied by Rao [11]. However, time is continuous in his model and he does not evaluate the fill rate. For a fill-rate constrained continuos review inventory problem, Agrawal and Seshadri [1] develop bounds for the order quantity and reorder point that are independent of the lead time demand distribution. Their analysis is based on an exact expression of the fill rate for the continuous review system. Boyaci and Gallego [5], and Shang and Song [14] study continuous-review service-constrained serial inventory systems where the lead time demand for the end product is Poisson distributed. In their models, the fill rate coincides with the Type-1 service level (which is relatively easier to evaluate) under the Poisson demand assumption. Different from their work, we evaluate the fill rate for general demand distributions.

The rest of this paper is organized as follows. Section 2 introduces the general review inventory model and provides fill rate formulas for general demand distributions. §3 presents an exact formula and two approximations for normally distributed demand. We conclude the paper in §4.

2 Exact Fill Rate for General Demand Distributions

We first introduce some notation. At the beginning of period t, let x_t denote inventory level, that is, x_t is the on-hand physical inventory if $x_t \geq 0$, and $-x_t$ is the amount of backlogged demand if $x_t < 0$. Let o_t be the number of items purchased from an outside supplier in period t. Let D_t be the demand in period t, and let $D_1, D_2, \cdots$ be independent, identically distributed, and nonnegative random variables with distribution function G and finite mean μ. To avoid trivialities, it is assumed that $G(0) < 1$. Let $G^{(k)}(\cdot)$ denote the k-fold convolution of $G(\cdot)$, *i.e.*, the distribution function of $\sum_{j=1}^{k} D_j$, and let $G^0(a) = 1$ (0) if $a \geq (<) 0$.

The following chronology of events occurs during each time period t: The inventory position is reviewed; the previously ordered items o_{t-L} arrive; the order size o_t is chosen, and finally demand occurs. Let y_t be the inventory in period t after deliveries of previously ordered goods but before demand occurs: $y_t = x_t + o_{t-L}$. For ease of exposition, define $a|b$ as a modulo b for integers a and b. Moreover, without loss of generality, let period t with $t|R = 0$ be a review period, that is, periods $0, R, 2R, \ldots$ are review periods. Clearly, if $t|R \neq 0$, $o_t = 0$. Because excess demand (if any) is backlogged, the inventory dynamics are $x_{t+1} = y_t - D_t$. The on-hand inventory that is available to satisfy demand in period t is $(y_t)^+ = \max\{y_t, 0\}$. The *fill rate*, β, is the long-run average fraction of demand that can be satisfied immediately from on-hand inventory. So,

$$\beta = \lim_{T \to \infty} E\left[\frac{\sum_{t=1}^{T} \min\{(y_t)^+, D_t\}}{\sum_{t=1}^{T} D_t}\right]. \tag{1}$$

The expectation and limit exist in (1) for the base-stock policies that are analyzed in subsequent sections.

Let τ be the base-stock level, then $o_t = (\tau - y_t)^+$ if $t|R = 0$ and $o_t = 0$ otherwise for $t = 1, 2, \cdots$. Without loss of generality, the initial inventory is never higher than τ, *i.e.*, $x_0 \leq \tau$. As a consequence, for every $t \geq L$,

$$y_t = \tau - \sum_{k=1}^{L+[(t-L)|R]} D_{t-k}, \quad \text{where } (t-L)|R = 0, 1, \cdots, R-1 \tag{2}$$

The following theorem gives an exact fill rate formula for our inventory system.

Theorem 1

$$\beta = \frac{1}{R\mu} \int_0^\tau [G^{(L)}(b) - G^{(L+R)}(b)] db. \tag{3}$$

Proof. By (1),

$$\begin{aligned}
\beta &= \lim_{T\to\infty} E\Big[\sum_{t=1}^{T} \min\{(y_t)^+, D_t\} / \sum_{t=1}^{T} D_t\Big] \\
&= \lim_{T\to\infty} E\Big\{[\sum_{t=1}^{T} \min\{(y_t)^+, D_t\}/T] / [\sum_{t=1}^{T} D_t/T]\Big\} \\
&= \frac{1}{\mu} \lim_{T\to\infty} E\Big[\sum_{t=1}^{T} \min\{(y_t)^+, D_t\}\Big]/T \\
&= \frac{1}{R\mu} \lim_{T\to\infty} E\Big[\sum_{t=1}^{T} \min\{(y_t)^+, D_t\}\Big]/(T/R) \\
&= \frac{1}{R\mu} \lim_{T\to\infty} E\Bigg\{\sum_{i=0}^{R-1}\Big[\sum_{t\in\{t:(t-L)|R=i\}} \min\{(y_t)^+, D_t\}\Big]\Bigg\}/(T/R) \\
&= \frac{1}{R\mu} \lim_{T\to\infty} E\Bigg\{\sum_{i=0}^{R-1}\Big[\sum_{t\in\{t:(t-L)|R=i\}} \min\{(y_t)^+, D_t\}/(T/R)\Big]\Bigg\},
\end{aligned}$$

where the third equation follows from the elementary renewal theorem (see, e.g., [12], p. 358) and the law of large numbers, the fifth equation is a result of grouping $\{y_t\}$ according to the values of $(t-L)|R$, and all other equations are based on straightforward algebra.

Let $H_j = \sum_{t\in\{t:(t-L)|R=j\}} \min\{(y_t)^+, D_t\}$, $j = 0, 1, \cdots, R-1$. Using (2), for all $j \in \{0, 1, \cdots, R-1\}$, it can be shown that

$$\begin{aligned}
\lim_{T\to\infty} E\big[H_j/(T/R)\big] &= E\Big[\min\Big\{\big(\tau - \sum_{k=1}^{L+j} D_k\big)^+, D_{L+j+1}\Big\}\Big] \\
&= \mu - E\Big\{\big[D_{L+j+1} - (\tau - \sum_{k=1}^{L+j} D_k)^+\big]^+\Big\}.
\end{aligned}$$

Let $\beta(\tau)$ make explicit the dependence of β on τ. Then

$$\beta(\tau) = \frac{1}{R\mu}\sum_{j=0}^{R-1} H_j = 1 - \frac{1}{R\mu}\sum_{j=0}^{R-1}\Big\{E\Big\{\big[D_{L+j+1} - (\tau - \sum_{k=1}^{L+j} D_k)^+\big]^+\Big\}\Big\}.$$

Put $K(\tau) = \mu[1-\beta(\tau)]$, then

$$\begin{aligned}
K(\tau) &= \frac{1}{R}\sum_{j=0}^{R-1}\Big\{E\Big\{\big[D_{L+j+1} - (\tau - \sum_{k=1}^{L+j} D_k)^+\big]^+\Big\}\Big\} \\
&= \frac{1}{R}\sum_{j=0}^{R-1}\Big\{\int_0^\infty\int_{\tau-a}^{\tau}(a+b-\tau)dG^{(L+j)}(b)dG(a) + \int_0^\infty a\int_\tau^\infty dG^{(L+j)}(b)dG(a)\Big\} \\
&= \frac{1}{R}\sum_{j=0}^{R-1}\Big\{\mu\Big[1-G^{(L+j)}(\tau)\Big] + \int_0^\infty\int_{\tau-a}^{\tau}(a+b-\tau)dG^{(L+j)}(b)dG(a)\Big\}.
\end{aligned}$$

Leibnitz' Rule yields

$$\begin{aligned}
K'(\tau) &= \frac{1}{R}\sum_{j=0}^{R-1}\Big[-\int_0^\infty\int_{\tau-a}^{\tau} dG^{(L+j)}(b)dG(a)\Big] \\
&= \frac{1}{R}\sum_{j=0}^{R-1}\Big[-\int_0^\tau\int_{\tau-b}^{\infty} dG(a)dG^{(L+j)}(b)\Big] \\
&= \frac{1}{R}\sum_{j=0}^{R-1}\Big\{\int_0^\tau\Big[G(\tau-b)-1\Big]dG^{(L+j)}(b)\Big\} \\
&= \frac{1}{R}\sum_{j=0}^{R-1}\Big[G^{(L+j+1)}(\tau) - G^{(L+j)}(\tau)\Big] \\
&= \frac{1}{R}\Big[G^{(L+R)}(\tau) - G^{(L)}(\tau)\Big].
\end{aligned}$$

Since $\beta(0) = 0$, $K(0) = \mu[1-\beta(0)]$. Therefore,

$$\beta(\tau) = 1 - K(\tau)/\mu = 1 - [K(0) + \int_0^\tau K'(a)da]/\mu = \frac{1}{R\mu}\int_0^\tau [G^{(L)}(b) - G^{(L+R)}(b)]db. \qquad \blacksquare$$

Theorem 1 characterizes the dependence of the system fill rate on the base-stock level (τ), demand distribution (G), review period (R), and lead time (L).

When $R = 1$, Equation (3) becomes the fill rate formula provided in [15]. However, one should note that, in [15], there is an inconsistency between the fill rate definition and the proof of Theorem 1. In [15], y_t is used in the fill rate definition [(3) on page 43], but x_t is used to derive the fill rate formula in the proof of Theorem 1 (line 9, page 44). However, because of the chronology differences between his model and the present model, substituting x_t in the proof by y_t of our model results in the same fill rate formula. Formulas (4) and (5) in [15] remain valid here if $R = 1$.

We next establish the sensitivity results on the fill rate with respect to the lead time L and the review cycle R. First, we present a technical lemma.

Lemma 1 *Let $\bar{G}^{(L)}(b) = 1 - G^{(L)}(b)$. Then $\int_0^\tau \bar{G}^{(L)}(b)db$ is concave in L.*

Proof. Standard algebra shows that $\int_0^\tau \bar{G}^{(L)}(b)db = E[D^{(L)} \wedge \tau]$, where $a \wedge b = \min\{a, b\}$ and $D^{(L)} = D_1 + \cdots + D_L$. Because all D_is are nonnegative i.i.d. random variables, $\{D^{(L)} : L \in I^+\} \in SIL(sp)$, that is $D^{(L)}$ is stochastic linear in L in the sample path sense ([13], Example 4.3). Note that $a \wedge b$ is increasing and concave in a, so $\{D^{(L)} \wedge \tau : L \in I^+\} \in SICV(sp)$, that is, $D^{(L)} \wedge \tau$ is stochastic increasing and concave in L in the sample path sense ([13], Proposition 3.2). Consequently, $\int_0^\tau \bar{G}^{(L)}(b)db = E[D^{(L)} \wedge \tau]$ is increasing and concave in L from the definition of stochastic concavity. ■

Theorem 2 *For a given τ, (i) $\beta(R, L) \geq \beta(R, L + 1)$, (ii) $\beta(R, L) \geq \beta(R + 1, L)$, (iii) $\beta(R, L) \leq \beta(R + 1, L - 1)$.*

Proof. By Theorem 1,

$$\beta(R, L) - \beta(R, L + 1) = \frac{1}{R\mu} \int_0^\tau [\bar{G}^{(L+R)}(b) - \bar{G}^{(L)}(b)]db - \frac{1}{R\mu} \int_0^\tau [\bar{G}^{(L+R+1)}(b) - \bar{G}^{(L+1)}(b)]db.$$

Then, $\beta(R, L) > \beta(R, L + 1)$ follows directly from the fact that $\int_0^\tau [\bar{G}^{(L)}(b)db$ is concave in L.

In order to show (ii), note that

$$\beta(R, L) - \beta(R+1, L) = \frac{1}{R(R + 1)\mu} \{ \int_0^\tau [\bar{G}^{(L+R)}(b) - \bar{G}^{(L)}(b)]db - R \int_0^\tau [\bar{G}^{(L+R+1)}(b) - \bar{G}^{(L+R)}(b)]db\}. \tag{4}$$

From the concavity of $\int_0^\tau [\bar{G}^{(L)}(b)db$ in L, we obtain

$$\int_0^\tau [\bar{G}^{(L+R)}(b) - \bar{G}^{(L+R-1)}(b)]db \geq \int_0^\tau [\bar{G}^{(L+R+1)}(b) - \bar{G}^{(L+R)}(b)]db,$$
$$\int_0^\tau [\bar{G}^{(L+R-1)}(b) - \bar{G}^{(L+R-2)}(b)]db \geq \int_0^\tau [\bar{G}^{(L+R+1)}(b) - \bar{G}^{(L+R)}(b)]db,$$
$$\cdots\cdots\cdots$$
$$\int_0^\tau [\bar{G}^{(L+1)}(b) - \bar{G}^{(L)}(b)]db \geq \int_0^\tau [\bar{G}^{(L+R+1)}(b) - \bar{G}^{(L+R)}(b)]db.$$

Combining the above R inequalities with (4) yields $\beta(R, L) \geq \beta(R + 1, L)$, which proves (ii).

(iii) can be proved similarly by noticing that $\beta(R+1, L-1) - \beta(R, L) = \frac{1}{R(R+1)\mu}\{R \int_0^\tau [\bar{G}^{(L)}(b) - \bar{G}^{(L-1)}(b)]db - \int_0^\tau [\bar{G}^{(L+R)}(b) - \bar{G}^{(L)}(b)]db\}$. ■

While claims i and ii are intuitively clear, claim iii is not as obvious. Indeed, in most operations management textbooks, a periodic review policy is fully characterized by the vulnerable period $L+R$; see, for example, [6]. However, according to claim iii of Theorem 2, how the vulnerable period is allocated between L and R can greatly impact the system's fill rate for a given base-stock level. In particular, as L decreases (while keeping $L+R$ constant), the system's fill rate increases. This is because for a shorter L, an inventory shortfall can be quickly corrected as soon as the shortfall is identified. However, for a longer L, even though the inventory shortfall can be quickly identified, it cannot be corrected as quickly which yields a larger backlogged demand. Note that Moses and Seshadri [10] have observed the fact that OM textbooks make a mistake in computing the safety stock for non-zero lead time inventory problems with lost sales. From Theorem 2, we see that, to improve the system's fill rate for a given base-stock level, a manager can reduce the lead time or the review cycle length (claims i and ii); however, when facing the choice of shortening the lead time or the review cycle, the manager should reduce the lead time before reducing the review cycle, other factors, including $L+R$, being equal (claim iii).

3 Normal Demand

In practice, demands are usually assumed and characterized as normally distributed. Many researchers also investigate the fill rate of an inventory system with normally distributed demands. This section specializes (3) for the normal demand distribution with mean μ and standard deviation σ and gives two easy-to-use approximations.

3.1 An Exact Expression

Let $\Phi(\cdot)$ and $\phi(\cdot)$ denote the distribution and density function, respectively, of a standard normal random variable (with mean 0 and variance 1), and let $b(a,j)=(a-j\mu)/(\sigma\sqrt{j})$. The normality and independence of demand imply that sums of demands are normally distributed; that is, $G^{(S)}(a)=\Phi[b(a,S)]$ for $S\in I^+$. So, formula (3) yields

$$\beta = \frac{1}{R\mu}\int_0^\tau \Big\{\Phi[b(a,L)]-\Phi[b(a,L+R)]\Big\}da \qquad (5)$$

$$= \frac{1}{R\mu}\Bigg[\sigma\sqrt{L}\int_{b(0,L)}^{b(\tau,L)}\Phi(x)dx-\sigma\sqrt{L+R}\int_{b(0,L+R)}^{b(\tau,L+R)}\Phi(x)dx\Bigg]. \qquad (6)$$

It is well known that $\int_t^\infty \big[1-\Phi(x)\big]dx = \phi(t) + t\Phi(t) - t$ (see, e.g. [7], [15], and [17]). This equation implies

$$\int_t^s \Phi(x)dx = \phi(s) - \phi(t) + s\Phi(s) - t\Phi(t). \tag{7}$$

It follows from simple algebra after substituting (7) into (6) that

$$\begin{aligned}\beta = \frac{1}{R\mu}\Big\{&\sigma\sqrt{L}\Big\{\phi[b(\tau,L)] - \phi[b(0,L)]\Big\} - \sigma\sqrt{L+R}\Big\{\phi[b(\tau,L+R)] - \phi[b(0,L+R)]\Big\}\\ &+ (\tau - L\mu)\Big\{\Phi[b(\tau,L)] - \Phi[b(\tau,L+R)]\Big\}\\ &+ R\mu\Phi[b(\tau,L+R)] + L\mu\Phi[b(0,L)] - (L+R)\mu\Phi[b(0,L+R)]\Big\}.\end{aligned} \tag{8}$$

Equation (8) involves the density function and the distribution function of the standard normal distribution. It can be easily implemented in commercial spreadsheet software (such as Microsoft Excel) using built-in functions, and will be used subsequently to develop an approximation.

3.2 An Approximation

We now develop a simple approximation for the fill rate using basic properties of the density function and the distribution function of standard normal distribution. It is easy to see that $\phi[b(0,L)] \simeq 0$, $\phi[b(0,L+R)] \simeq 0$, $\Phi[b(0,L+R)] \simeq 0$, and $\Phi[b(0,L)] \simeq 0$. Moreover, the base-stock level τ is usually set to protect the demand for $L+R$ periods. Consequently, we expect that $\tau \gg L\mu$, which implies $\phi[b(\tau,L)] \simeq 0$, $\Phi[b(\tau,L)] \simeq 1$. With these assumptions, we obtain the following simple approximation for the fill rate.

$$\beta_A = \frac{1}{R\mu}\Big\{-\sigma\sqrt{L+R}\phi[b(\tau,L+R)] + (\tau - L\mu)\Big\{1-\Phi[b(\tau,L+R)]\Big\} + R\mu\Phi[b(\tau,L+R)]\Big\}. \tag{9}$$

Put $\tau = (L+R)\mu + z\sqrt{L+R}\sigma$. Substituting it into (9) and simplifying, we obtain

$$\beta_A = 1 - \frac{1}{R\mu}\sqrt{L+R}\sigma[\phi(z) - z\bar{\Phi}(z)].$$

It is easy to verify that $\sqrt{L+R}\sigma[\phi(z) - z\bar{\Phi}(z)]$ is the system's expected backlogged orders per order cycle. Consequently, (9) coincides with the *modified fill rate* (also known as γ fill rate) used by researchers (see, e.g., [8]) to simplify the fill rate computation.

3.3 An Approximation Based on Logistic Distributions

Equation (9) still involves computing the normal density function and the normal distribution function. In this subsection, we develop a closed-form approximation that only involves *elementary* *functions* by approximating normal distributions with logistic distributions.

A logistic distribution with mean μ and variance σ^2 has a CDF: $F(x) = 1/\left[1 + e^{-(x-m)/r}\right]$, where $m = \mu$ and $r = \sigma\sqrt{3}/\pi$. Also,

$$\int_{-\infty}^{t} F(x)dx = r\text{Log}(1 + e^{(t-m)/r}) \tag{10}$$

Due to the closed-form expression of $\int_{-\infty}^{t} F(x)dx$ in (10), we use logistic distributions to approximate normal demand distributions to develop a closed-form expression for the fill rate. If demand in our fill rate model is normally distributed, then $G^{(L)}$ and $G^{(L+R)}$ are normal distribution functions with parameters $(L\mu,\, L\sigma^2)$ and $((L+R)\mu,\, (L+R)\sigma^2)$. If we approximate $G^{(L)}$ and $G^{(L+R)}$ with logistic distributions with the same means and variances, then, from (6) and (10), we obtain the following fill rate approximation that only involves elementary functions.

$$\beta_{Log} = \frac{1}{R\mu\pi}\left[\sqrt{3L}\sigma\text{Log}(1 + e^{\frac{(\tau - L\mu)\pi}{\sqrt{3L}\sigma}}) - \sqrt{3(L+R)}\sigma\text{Log}(1 + e^{\frac{(\tau-(L+R)\mu)\pi}{\sqrt{3(L+R)}\sigma}})\right]. \tag{11}$$

3.4 Performance of the Approximations

To examine the performance of the two approximations, we conducted extensive computational studies. Our computational studies (see [16] for details) indicate that, when $\tau \geq (L+R)\mu$, both approximations perform well. Further, both approximations perform well when the demand coefficient of variation is low or when the fill rate is high. In general, both approximations perform better for higher values of R and lower values of L for a given $L + R$. Because managers usually set very high fill rate targets for their customer service levels, both (9) and (11) are good approximations for the fill rate of finished goods . However, it has been widely recognized in the literature ([2], [3], [4], and [14]) that the internal service level of a supply chain (the service level provided by upstream stages to downstream stages) need not be very high in order to optimize the inventories of a supply chain. Our computational studies show that our approximations are effective in estimating internal service levels as long as the demand coefficient of variation is low.

4 Conclusion

The results of this research will serve as a starting point for studying the multi-echelon inventory positioning problems with general review intervals and fill-rate constraints. It would be also interesting to investigate the impact of the proposed approximations on the stock-level or the inventory cost estimate in the future.

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References

[1] V. Agrawal and S. Seshadri. Distribution free bounds for service constrained (Q, r) inventory systems. *Naval Research Logistics*, 47(8):635–656, 2000.

[2] S. Axsäter. Supply chain operations: Serial and distribution inventory systems. In S. Graves and T. de Kok, editors, *Handbooks in Operations Research and Management Science: Supply Chain Management*. Elsevier Science Publisher, North–Holland, 2003.

[3] R. Bollapragada, U. S. Rao, and J. Zhang. Managing a two-stage serial inventory system under demand and supply uncertainty and customer service level requirements. *IIE Transactions*, 36(1):73–85, 2004.

[4] R. Bollapragada, U. S. Rao, and J. Zhang. Managing inventory and supply performance in assembly systems with random supply capacity and demand. *Management Science*, 50(12):1729–1743, 2004.

[5] T. Boyaci and G. Gallego. Serial production/distribution systems under service constraints. *Manufacturing & Service Operations Management*, 3:43–50, 2001.

[6] R. B. Chase, F. R. Jacobs, and N. J. Aquilano. *Operations Management for Competitive Advantage*. McGraw-Hill Irwin, New York, 11th edition, 2005.

[7] G. J. Hadley and T. M. Whitin. *Analysis of Inventory Systems.* Prentice Hall, Englewood Cliffs, NJ, 1963.

[8] M. E. Johnson, H. Lee, T. Davis, and R. Hall. Expressions for item fill rates in periodic inventory systems. *Naval Research Logistics*, 42(1):57–80, 1995.

[9] L. J. Krajewski and L. P. Ritzman. *Operations Management: Strategy and Analysis.* Prentice Hall, Upper Saddle River, NJ, 6th edition, 2002.

[10] M. Moses and S. Seshadri. Policy mechanisms for supply chain coordination. *IIE Transactions*, 32(3):245–262, 2000.

[11] U. S. Rao. Properties of the periodic review (R, T) inventory control policy for stationary, stochastic demand. *Manufacturing* & *Service Operations Management*, 5(1):37–53, 2003.

[12] S. Ross. *Introduction to Probability Models*. Academic Press, San Diego, CA, 6th edition, 1997.

[13] M. Shaked and J. G. Shanthikumar. Stochastic convexity and its applications. *Adv. Appl. Prob.*, 20:427–446, 1988.

[14] K. Shang and J-S Song. Analysis of serial supply chains with a service constraint. Working Paper, Duke University, Durham, NC, 2005.

[15] M. J. Sobel. Fill rates of single-stage and multi-stage supply systems. *Manufacturing & Service Operations Management*, 6:41–52, 2004.

[16] J. Zhang and J. Zhang. Fill rate of single-stage general periodic review inventory systems: Extended version. Working Paper, School of Management, The University of Texas at Dallas, Richardson, TX 75083, 2006.

[17] P. Zipkin. *Foundations of Inventory Management*. McGraw-Hill, New York, NY, 2000.