Lateral Capacity Exchange and Its Impact on Capacity Investment Decisions

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Abstract

We study the problem of capacity exchange between two firms in anticipation of the mismatch between demand and capacity and its impact on firm's capacity investment decisions. For given capacity investment levels of the two firms, we demonstrate how capacity price may be determined and how much capacity should be exchanged when either manufacturer acts as a Stackelberg leader in the capacity exchange game. By benchmarking against the centralized system, we show that a side payment may be used to coordinate the capacity exchange decisions. We then study the firms' capacity investment decisions using a biform game framework in which capacity investment decisions are made individually and exchange decisions are made as in a centralized system. We demonstrate the existence and uniqueness of the Nash equilibrium capacity investment levels and study the impact of firms' share of the capacity exchange surplus on their capacity investment levels.

Keywords: Capacity exchange, capacity sharing, newsvendor model, Stackelberg game, biform game

1 Introduction

Competitive forces are driving companies towards collaborations. Such collaborations are possible between a supplier and a buyer in a vertical supply chain, or between two firms in a horizontal supply chain. Lateral capacity exchange is one such collaboration where two firms offering similar products or services exchange capacities between them. In practice, lateral capacity exchange has been used widely to manage emergent capacity shortage in industries requiring lengthy periods to build up capacity. For instance, to compensate for delays in infrastructure buildup in 2000, Brazilian corporate network service providers have collaborated with one another in manufacturing fiber for the network through capacity swaps and redundancy agreements ([21]). Following this trend, Intelig, a Brazilian long-distance operator, and Electronet, an association of electricity distributors, have formed a partnership to exchange capacity for the next ten years ([21]). Lateral capacity exchange is also used in the energy industry where capacity build-up may take a long time and demand can be seasonal depending on operating regions. In Australia, exchanging spare generation-capacity has been practiced between firms operating in different regions to take advantage of the seasonal demand ([16]).

Firms can conduct such lateral exchange before or after demand uncertainty is resolved. One stream of research has studied lateral exchange in the form of inventory transshipment or demand substitution in centralized environments. For example, the optimal ordering and transshipment (substitution) decisions have been studied in [2], [4], [8], [9], [10], [18] and [14] for systems without fixed costs, and in [7] and [17] for systems with fixed costs. Van Mieghem and Rudi [24] introduce a general framework which they call the "newsvendor networks" model to study the problems of stochastic capacity investment and inventory management. Their framework allows the study of assembly, commonality, distribution, flexibility, substitution and transshipment in a centralized setting. Another stream of research, closer to our work, has examined the inventory transshipment or demand substitution in decentralized environments. Rudi et. al. [19] study a transshipment game between two firms at different locations; one firm can use the leftover inventory from the other firm. They examine the impacts of such transshipment on the optimal inventory orders at each location and determine the optimal transshipment prices that induce the locations to choose inventory levels consistent with joint-profit maximization. Anupindi et. al. [1] study a more general decentralized distribution system involving more than two retailers in which transshipment mitigates the mismatch between supply and demand. They develop a general framework to study the problem of inventory ordering and allocation when ordering decisions are made competitively, and inventory transshipment decisions are made cooperatively. They reveal the existence of an allocation mechanism for inventory transshipment that achieves the first best solution for inventory ordering and allocation. Granot and Sošić [5] extend the work of Anupindi et. al. to a three-stage model in which residual inventory may not enter the transshipment stage.

Such research commonly assumes that capacity exchange/inventory transshipment between different firms occurs only after demand becomes known. The exceptions are the work by Karmarkar and Patel [9], Karmarkar [8], and Lee and Whang [12]. As seen in the real-world examples described earlier, lateral exchange can also be beneficial if instituted before the complete resolution of demand uncertainty, especially when the lead-time for converting capacity is long. Lee and Whang investigate the impacts of a secondary market where resellers can buy and sell excess inventories before actual demand happens. In this paper, we examine lateral capacity exchange, in anticipation of the mismatch between capacity and demand in a decentralized environment. Our work is fundamentally different from [12] for the following two reasons. First, Lee and Whang consider inventory rather than capacity, and demands are independent across periods in their model. Consequently, inventory exchange is driven by the supply side, that is, by the amount of inventory left over from the first period. In our paper, capacity exchange is driven by the demand side, that is, by the demand forecast update. Second, Lee and Whang consider a market with many players in which the competitive equilibrium results in an efficient inventory allocation. We, in contrast, consider a game between two players, which can result in an inefficient allocation.

We study the lateral capacity exchange between two firms in anticipation of the mismatch between demand and capacity, and its impact on firm's capacity investment decisions. For given capacity investment levels of the two firms, we demonstrate how capacity price may be determined and how much capacity should be exchanged when either firm acts as a Stackelberg leader in the capacity exchange game. By benchmarking against a centralized system, we show that a side payment may be used to coordinate the capacity exchange decisions. We then study the firms' capacity investment decisions assuming that capacity investment decisions are made individually, exchange decisions are made as in a centralized system, and firms share the capacity exchange surplus. We demonstrate the existence and uniqueness of the Nash equilibrium capacity investment levels and show that the higher the share of capacity exchange surplus a firm receives, the more it invests in capacity. Finally, using the normal demand distribution as an example, we develop insights on when capacity exchange may be more beneficial.

The paper is organized as follows. In §2 we present the basic modeling framework of capacity investment with exchange. §3 studies capacity exchange in a centralized system to provide a benchmark. Capacity exchange under two Stackelberg games are studied in §4. In §5, we characterize the first-best capacity exchange price. In §6 we study the capacity investment decision problem. We conclude in §7.

2 Structuring a Collaboration

We consider a system with two non-competing firm and a single period comprising two stages. Each firm is a monopoly in its own market and sells one product to its customers. At the beginning of the first stage, the firms estimate that their demand, denoted by $\overrightarrow{\xi} = (\xi_1, \xi_2)$, has a bivariate distribution $\Phi(\cdot)$ and density $\phi(\cdot)$, where ξ_i is the demand for firm i, $i = 1, 2$. Based on the initial estimate of the demand, each firm builds an initial capacity R_i, $i = 1, 2$, (i.e., the ability to produce R_i units of its product) in the first stage and incurs a linear investment cost $c_i R_i$. (The duration of stage 1 can be regarded as the lead-time for building capacity.) At the beginning of the second stage, firms utilize the demand signal (market information) $\overrightarrow{\eta}$ =(η_1, η_2) observed in the first stage to update the probability distribution of $\overrightarrow{\xi}$. The demand updating process is based on the distribution of the multivariate random vector $(\eta_1, \eta_2, \xi_1, \xi_2)$. Based on the updated demand distribution, the firms may decide to make their anticipated unused production capacities available to each other when needed.

Actual demands are observed after the capacity exchange between the firms has taken place. Then, two firms satisfy their demands with their respective available capacity. The demand of firm i is fulfilled by using the capacity of firm i or converted capacity from firm j ($j \neq i$). However, to fulfill its demand using firm j's capacity, firm i must convert firm j's capacity, at a cost t_{ji} per unit of conversion. No capacity exchanges are allowed after demands have been observed. Therefore, if the demand for firm i exceeds its capacity K_i (including the capacity it has acquired from or excluding the capacity sold to the other firm), excess demand is lost. If the demand falls short of capacity, the firm experiences idle capacity.

We denote the firm that reserves capacity with its partner as the capacity buyer. Similarly, the firm whose capacity is reserved becomes the capacity seller. We consider capacity exchange under two market-leading scenarios. In the first scenario, we assume that the seller (the firm with excess capacity) moves first, as the Stackelberg leader, setting the capacity price; the buyer (the company in need of capacity) selects the amount of capacity to buy. In the second scenario, the buyer moves first, as the Stackelberg leader, setting the capacity price; the seller follows by choosing the amount of capacity to sell. Both firms maximize their expected profits. All parameters and distributions are assumed to be common knowledge. First, we summarize our notation:

K_i = capacity available for use to firm i after capacity conversion;

R_i = capacity owned by firm i;

c_i = marginal investment cost of firm i's capacity;

m_i = margin of firm i's product;

p = capacity purchasing price per unit;

q = amount of capacity purchased and converted;

t_{ij} = cost per unit of converting capacity from firm i to j $(i, j = 1, 2,\ i \neq j)$;

$\overrightarrow{\eta} = (\eta_1, \eta_2)$ = demand signal vector;

$\overrightarrow{\xi} = (\xi_1, \xi_2)$ = demand vector;

$\Phi_i(\cdot|\overrightarrow{\eta})$ = the conditional marginal distribution of ξ_i;

$\overline{\Phi}_i(x|\overrightarrow{\eta}) = 1 - \Phi_i(x|\overrightarrow{\eta})$;

$\phi_i(\cdot|\overrightarrow{\eta})$ = the conditional marginal density of ξ_i.

We use superscripts s, b, and c to identify seller-leading, buyer-leading, and centralized systems, respectively. Finally, we assume that, for i, $j = 1, 2$, $i \neq j$, $t_{ji} < m_i$, and ξ_i's have finite means.

3 Capacity Exchange in a Centralized System

In this section, we study capacity exchange in a centralized system to establish a performance benchmark. For ease of exposition, we denote $\Phi_i(\cdot|\overrightarrow{\eta})$ as $\Phi_i(\cdot)$ and $\phi_i(\cdot|\overrightarrow{\eta})$ as $\phi_i(\cdot)$. Given capacity K_i, the expected gain of firm i from fulfilling demand (ignoring the capacity conversion cost) is

$$G_i(K_i) = m_i E[\min\{\xi_i, K_i\}] = m_i \left[\int_0^{K_i} x\phi_i(x)dx + \int_{K_i}^{+\infty} K_i \phi_i(x)dx \right].$$

Note that $K_1 + K_2 = R_1 + R_2$ and that the capacity price is not a concern for the centralized system. Define $\pi^c(K_1) \overset{def}{=} -t_{12}(R_1 - K_1)^+ - t_{21}(K_1 - R_1)^+ + G_1(K_1) + G_2(R_1 + R_2 - K_1)$ as the expected profit of the centralized firm, where K_1 is firm 1's capacity after capacity conversion. Here $(R_1 - K_1)^+ = \max(R_1 - K_1, 0)$ is the amount of capacity of firm 1 converted, if any, for use by firm 2. Similarly $(K_1 - R_1)^+$ is the amount of capacity converted at firm 2 for use by firm 1. $t_{12}(R_1 - K_1)^+ + t_{21}(K_1 - R_1)^+$ represents the total cost of capacity conversion, and $G_1(K_1) + G_2(R_1 + R_2 - K_1)$ represents the total gain of the two firms following capacity conversions.

The centralized system's problem may be formulated as: $\max_{0 \leq K_1 \leq R_1 + R_2}\{\pi^c(K_1)\}$. To simplify notation, we define

$$\alpha(K) \overset{def}{=} \frac{d}{dK}\left[G_1(K) + G_2(R_1 + R_2 - K)\right] = m_1\overline{\Phi}_1(K) - m_2\overline{\Phi}_2(R_1 + R_2 - K). \tag{1}$$

Proposition 1 characterizes the optimal policy for capacity exchange in the centralized system.

Proposition 1 *$\pi^c(K_1)$ is strictly concave in K_1. The optimal policy is to set K_1 to K_1^c where K_1^c satisfies*

$$K_1^c = \begin{cases} \{K_1 | t_{12} + \alpha(K_1) = 0\} & \text{if } t_{12} + \alpha(R_1) < 0 \text{ and } t_{12} + \alpha(0) \geq 0, \\ \{K_1 | -t_{21} + \alpha(K_1) = 0\} & \text{if } -t_{21} + \alpha(R_1) > 0 \text{ and } -t_{12} + \alpha(K_1 + K_2) \leq 0, \\ R_1 & \text{if } t_{12} + \alpha(R_1) \geq 0 \text{ and } -t_{21} + \alpha(R_1) \leq 0, \\ 0 & \text{if } t_{12} + \alpha(0) < 0, \\ K_1 + K_2 & \text{if } -t_{12} + \alpha(K_1 + K_2) > 0. \end{cases} \tag{2}$$

Proof. All proofs are in the appendix. ■

For simplicity of exposition, we will assume that all first-order conditions have interior solutions and ignore the last two scenarios of Proposition 1 throughout this paper. This assumption is valid if $t_{12} + \alpha(0) > 0$ and $-t_{12} + \alpha(R_1 + R_2) < 0$ respectively. In words, it is not optimal to convert the entire capacity of a firm for use by another firm, and this assumption should hold in most real-world scenarios. Our results can be extended to the cases in which the first-order conditions yield boundary values.

By Proposition 1, the relative value of $\alpha(R_1)$ and the conversion cost determine whether capacity conversion is needed and, if so, how much. Note that $\alpha(R_1)$ is the marginal benefit (ignoring conversion) of capacity exchange from firm 2 to firm 1. In the first case of Proposition 1, the marginal benefit of conversion from firm 1 to firm 2 is greater than the conversion cost, and so undertaking the capacity exchange is desirable. In the second case, the marginal benefit of conversion from firm 2 to firm 1 is greater than the marginal conversion cost t_{21}. Therefore, one would convert capacity at firm 2 for 1. Finally, if the marginal benefit of converting capacity in either direction is not significant compared to the conversion cost, no capacity exchange is required. Obviously, the capacity conversion decision is affected by the capacity flexibility (value of t_{ij}).

Corollary 1 *K_1^c is increasing in t_{12}, decreasing in t_{21}. Further, the total amount of capacity exchanged $|K_1^c - R_1|$ is decreasing in t_{12} and t_{21}.*

For the given capacity (R_1, R_2), denote $\pi^c(R_1, R_2 | \overrightarrow{\eta})$ as the optimal total expected payoff of the centralized system *conditional* on $\overrightarrow{\eta}$, and $E\pi^c(R_1, R_2)$ as the overall (*unconditional*) expected optimal centralized system payoff. We have:

Proposition 2 *$\pi^c(R_1, R_2 | \overrightarrow{\eta})$ and $E\pi^c(R_1, R_2)$ are increasing and concave in (R_1,R_2).*

Next, we study the capacity exchange decision in decentralized systems where capacity price requires consideration.

4 Capacity Exchange in Decentralized Systems

We first study the scenario where a single company trades capacity in a capacity market.

4.1 The single company problem

We drop subscripts from our notation in this section; we denote t as the marginal capacity conversion cost and m as the contribution margin of the product respectively. We assume that the firm has a built-up capacity of R and that it can buy or sell capacity in a market at a price p. Let K denote the capacity available to the firm after the trade takes place. Clearly, $(K-R)^+ = \max(K-R, 0)$ is the amount purchased from the market at a cost $p(K-R)^+$, and $(R-K)^+$ is the amount sold to the market for a revenue $p(R-K)^+$. The firm's expected profit $\pi(K)$ is

$$\pi(K) = -(p+t)(K-R)^+ + p(R-K)^+ + m\left[\int_0^K x\phi(x)dx + \int_K^{+\infty} K\phi(x)dx\right].$$

It is easy to verify that $\pi(K)$ is strictly concave in K and continuously differentiable except when $K = R$. Further, its first-order derivative is

$$\frac{d\pi(K)}{dK} = \begin{cases} -p + m\overline{\Phi}(K) & \text{if } K < R, \\ -(p+t) + m\overline{\Phi}(K) & \text{if } K > R. \end{cases}$$

Define K^* as the optimal capacity after exchange. Using an analysis similar to that in the proof of Proposition 1, we find the optimal K^* by (1) checking the signs of left-side and right-side derivatives of $\pi(K)$ at R to determine whether $K^* < R$, $K^* = R$ or $K^* > R$, and (2) solving the corresponding first-order equation.

Lemma 1 *The optimal capacity after exchange with the market is*

$$K^* = \begin{cases} \Phi^{-1}(1-\frac{p}{m}) & \text{if } m\overline{\Phi}(R) < p, \\ R & \text{if } p \le m\overline{\Phi}(R) \le p+t, \\ \Phi^{-1}(1-\frac{p+t}{m}) & \text{if } m\overline{\Phi}(R) > p+t. \end{cases}$$

Based on Lemma 1, when the price for the capacity is high (higher than $m\overline{\Phi}(R)$), the firm should sell $R - \Phi^{-1}(1-\frac{p}{m})$ of its capacity. It also follows that when the price is low (lower than $m\overline{\Phi}(R) - t$), the firm should buy $\Phi^{-1}(1-\frac{p+t}{m})-R$ amount of capacity. When the capacity price is somewhere in between, the firm should do nothing with respect to its capacity. We call

$$p^s = m\overline{\Phi}(R) \tag{3}$$

the *selling point* of the firm. This is the price above which the firm would sell capacity. Similarly, we call

$$p^b = m\overline{\Phi}(R) - t \tag{4}$$

the *buying point*, below which the firm would purchase capacity from the market.

Next, we study how two independent companies may exchange capacity between them. For any capacity exchange to occur, the selling point of one firm must be lower than the buying point of the other firm. Since we are only interested in the situation in which a capacity exchange takes place, we assume $p_1^s < p_2^b$ from now on. This may occur if firm 2 expects a high demand due to an increase in updated demand forecast. Clearly, for any $p \in [p_1^s, p_2^b]$, firm 1 is willing to sell some of its capacity (and therefore it is the *seller*), and firm 2 is willing to buy capacity (and therefore it is the *buyer*).

We consider capacity exchange under two scenarios. In the first scenario, we assume the capacity seller moves first as the Stackelberg leader setting the capacity price, and the capacity buyer follows by choosing how much capacity to buy from the seller. In the second scenario, the capacity buyer moves first as the Stackelberg leader setting the capacity price, and the capacity seller follows by choosing how much capacity to sell to the buyer.

4.2 Price set by the seller

The seller (firm 1) sets the price as a Stackelberg leader. In response to this price, the buyer (firm 2) chooses the amount of capacity to buy. As shown in Lemma 1, for a price p, between p_1^s and p_2^b, the quantity purchased by firm 2 would be

$$q = \Phi_2^{-1}\left(1 - \frac{p + t_{12}}{m_2}\right) - R_2. \tag{5}$$

In anticipation of the buyer's response, the seller chooses the value of p to maximize its profit, $pq + G_1(R_1 - q)$. To find the optimal seller's price, we reformulate the optimization problem in terms of q as in Lariviere and Porteus [11]. Now, the seller's problem is to find the value of q to maximize its profit. Note that by rearranging the terms in (5), we obtain $p = m_2\overline{\Phi}_2(R_2 + q) - t_{12}$.

Therefore, the seller's profit, $pq + G_1(R_1 - q)$, will be

$$\pi^s(q) = [m_2\overline{\Phi}_2(R_2 + q) - t_{12}]q + G_1(R_1 - q). \tag{6}$$

The seller's optimization problem may be formulated as (*the Seller's Problem*):

$$\begin{aligned} Max \;\; & \pi^s(q) = [m_2\overline{\Phi}_2(R_2 + q) - t_{12}]q + G_1(R_1 - q) \\ s.t. \;\; & p_1^s \le m_2\overline{\Phi}_2(R_2 + q) - t_{12} \le p_2^b, \\ & 0 \le q \le R_1. \end{aligned}$$

The first constraint ensures that both the buyer and the seller will participate in the transaction. The second constraint reflects that the seller may not sell more capacity than it possesses. Note that the right-hand side of the first constraint is automatically satisfied when $q \ge 0$, since $p_2^b = m_2\overline{\Phi}_2(R_2) - t_{12}$ from (4). For the left-hand side of the first constraint to hold, it is necessary that $q \le \Phi_2^{-1}\left(1 - \frac{p_1^s + t_{12}}{m_2}\right) - R_2$. Define $q^{UB} = \min\left\{R_1, \Phi_2^{-1}\left(1 - \frac{p_1^s + t_{12}}{m_2}\right) - R_2\right\}$. Then, the seller's problem is to choose $q \in [0, q^{UB}]$ to maximize $\pi^s(q)$. To solve the seller's problem, since $\pi^s(q)$ cannot be shown to be concave, we first introduce the notion of Increasing Generalized Failure Rate; see [11].

Definition 1 *A random variable ξ with CDF $\Phi(\cdot)$ has an increasing generalized failure rate if $\xi\frac{\phi(\xi)}{\overline{\Phi}(\xi)}$ (the generalized failure rate) is increasing for all ξ such that $\Phi(\xi) < 1$.*

Increasing generalized failure rate (IGFR) is a more general notion than the increasing failure rate (IFR), generally used in the literature. Most commonly used distributions, such as Normal, Gamma, and Weibull, are IGFR. (See [11] for details.) We now present two general results pertaining to random variables satisfying IGFR in Lemmas 2 and 3.

Lemma 2 *If a random variable ξ has an increasing generalized failure rate, then for any value R, the generalized failure rate of $\xi - R$ is $h_{\xi - R}(q) = q\frac{\phi(q+R)}{\overline{\Phi}(q+R)}$. Further, $h_{\xi - R}(q)$ is an increasing function; i.e., $\xi - R$ also has an increasing generalized failure rate.*

In other words, IGFR is invariant under subtractions of a constant.

Lemma 3 *Assume ξ with CDF $\Phi(q)$ has an increasing generalized failure rate and has a finite mean. Let $\bar{q}$ be the root of $q\frac{\phi(q+R)}{\overline{\Phi}(q+R)} = 1$ for $R \ge 0$. Then, $\bar{q}$ is finite.*

From now on, we will assume that ξ_i has an increasing generalized failure rate for $i = 1, 2$. Based on Lemma 3 and using the argument of Lariviere and Porteus, we can show that the objective function is unimodal, and the first-order condition (together with the constraint) is sufficient to identify the unique optimal value of q. We state this result as a proposition below.

Proposition 3 *$\pi^s(q)$ is unimodal. Let $\bar{q}$ be the root of $q\frac{\phi(q+R)}{\bar{\Phi}(q+R)} = 1$. For $q \le \bar{q}$, $\pi^s(q)$ is strictly concave, for $q \ge \bar{q}$, $\pi^s(q)$ is decreasing in q. The first-order condition satisfies:*

$$-t_{12} - \alpha(R_1 - q) - m_2 q \phi_2(R_2 + q) = 0. \tag{7}$$

A natural question is whether system efficiency in such a scenario suffers in terms of capacity exchange. The following result demonstrates that when the price is set by the seller in a decentralized system, the amount of capacity being exchanged is less than that in the centralized system.

Proposition 4 *(a) If exchanging capacity in the centralized system is desirable and the capacity is converted from firm i to j, then in the decentralized system, capacity conversion would lead to a Pareto optimal improvement; firm i would be the seller (firm j the buyer), and vice versa. (b) Capacity converted in the decentralized system with price set by the seller, denoted as q^s, is less than that in the centralized system: $q^s < R_1 - K_1^c$.*

The first part of Proposition 4 implies that if a capacity exchange is profitable in an centralized system, it will also be so in a decentralized system, and vice versa. The direction of conversion is identical in both systems. The second part of the proposition emphasizes that when the seller sets the capacity price, the amount of capacity exchanged in a decentralized system is less than that in the centralized system. Therefore, the benefits of capacity exchange may not be fully realized. This inefficiency occurs because the firms are motivated to maximize their individual profits, and not the joint profit of both firms. Note that this inefficiency is a form of quantity distortion driven by double marginalization. ([20])

4.3 Price set by the buyer

We next consider the case in which the buyer (firm 2) sets the capacity price, and the seller (firm 1) chooses the amount of capacity to sell based on this price. Based on Lemma 1, for any price $p \in [p_1^s, p_2^b]$ chosen by firm 2, the amount of capacity that firm 1 will sell is

$$q = R_1 - \Phi_1^{-1}\left(1 - \frac{p}{m_1}\right). \tag{8}$$

Therefore, the buyer's problem may be formulated as

$$\begin{aligned} Max \quad & \pi^b(p) = -(p + t_{12})q + G_2(R_2 + q) \\ s.t. \quad & p_1^s \le p \le p_2^b. \end{aligned}$$

Similar to our earlier analysis, we work with q and reformulate the buyer's problem as

$$\begin{aligned} Max \quad & \pi^b(q) = -t_{12}q - m_1 q\overline{\Phi}_1(R_1 - q) + G_2(R_2 + q) \\ s.t. \quad & p_1^s \le m_1\overline{\Phi}_1(R_1 - q) \le p_2^b. \end{aligned}$$

The following proposition characterizes the optimal solution to the buyer's problem.

Proposition 5 *$\pi^b(q)$ is strictly concave in q, and the first-order condition may be written as*

$$-t_{12} - \alpha(R_1 - q) - m_1 q\phi_1(R_1 - q) = 0. \tag{9}$$

As in Proposition 4, we can show that when the buyer sets the price, exchanged capacity will be lower than that in the centralized system.

Proposition 6 *(a) If it is optimal to exchange capacity in the centralized system and the capacity is converted from firm i to j, then in the decentralized system with the price set by the buyer, capacity conversion would also be optimal and firm i will be the seller (firm j the buyer), and vice versa. (b) Capacity converted in the decentralized system with price set by the buyer, denoted as q^b, is less than that in the centralized system: $q^b < R_1 - K_1^c$.*

Thus, according to Propositions 4 and 6, capacity exchanged in a decentralized system is lower than that in the centralized system. Since price drives the quantity of capacity conversion, the seller would set the price high and the buyer would set the price low. The natural question is whether the two parties can collaborate to set a capacity price that will guarantee increased benefits to both parties. We examine this question in the next section.

5 First-Best Capacity Exchange Price

In this section, we consider how to choose a price of capacity so that the impact of the capacity exchange decisions in the decentralized system is not worse than the impact of decisions in the centralized system. Clearly, with such a price denoted as p^c, firm 1 would willingly sell at least q^c

and firm 2 would willingly buy at least q^c. From (8), we know that in order for firm 1 to sell at least q^c, the price p^c must satisfy $R_1 - \Phi_1^{-1}\left(1 - \frac{p^c}{m_1}\right) \geq q^c$. Rearranging terms, we have:

$$p^c \geq m_1\overline{\Phi}_1(R_1 - q^c). \tag{10}$$

Similarly, from (5), in order for firm 2 to buy at least q^c, p^c must satisfy $\Phi_2^{-1}\left(1 - \frac{p^c + t_{12}}{m_2}\right) - R_2 \geq q^c$. Rearranging terms, we have

$$p^c \leq m_2\overline{\Phi}_2(R_2 + q^c) - t_{12}. \tag{11}$$

Note that $p_1^s < p_2^b$ implies $m_1\overline{\Phi}_1(R_1) < m_2\overline{\Phi}_2(R_2) - t_{12}$. Therefore, the first case of Proposition 1 applies. It follows that $K_1^c = R_1 - q^c$ is the root of $t_{12} + \alpha(K_1^c) = 0$. Substituting K_1^c with $R_1 - q^c$, we have

$$t_{12} + m_1\overline{\Phi}_1(R_1 - q^c) - m_2\overline{\Phi}_2(R_2 + q^c) = 0. \tag{12}$$

Conditions (11) and (12) yield

$$p^c \leq m_1\overline{\Phi}_1(R_1 - q^c). \tag{13}$$

Combining (10) and (13), we know that

$$p^c = m_1\overline{\Phi}_1(R_1 - q^c). \tag{14}$$

This is the only price at which the two firms can trade capacity exactly as they do in the centralized system. Proposition 7 summarizes this result.

Proposition 7 *Assume $p_1^s < p_2^b$, and q^c is the amount of capacity converted at firm 1 for firm 2 in the centralized system. If the capacity price is set as $p^c = m_1\overline{\Phi}_1(R_1 - q^c)$ in the decentralized system, firm 1 will willingly sell amount q^c, and firm 2 will willingly buy q^c.*

Define p^{buyer} (p^{seller}) as the price chosen by the buyer (seller), respectively, in the Stackelberg game. Corollary 2 follows immediately (without proof).

Corollary 2 $p^{buyer} \leq p^c \leq p^{seller}$.

Another way of deriving p^c is to examine the demand and supply curves of capacity under different values of price. Clearly as price increases, the supply of capacity increases, and the demand decreases. Any mismatch between the supply and demand implies the inefficiency of the capacity price p. Therefore, p^c is the price at which the two curves cross each other. See Figure 1 for an illustration.

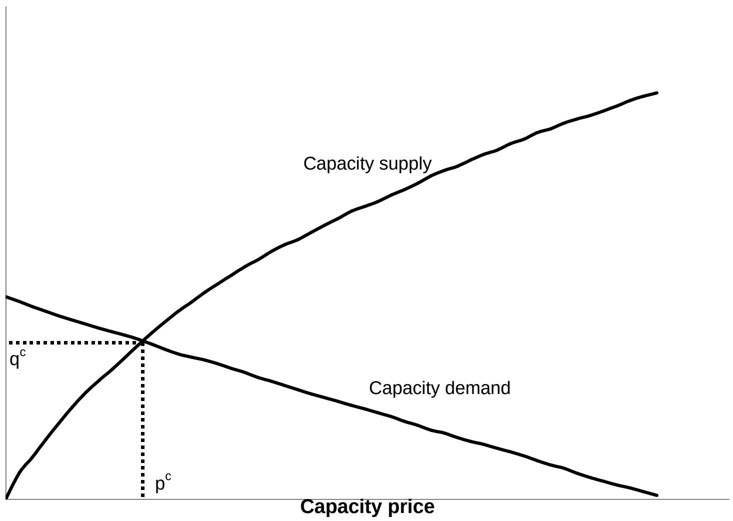

Figure 1: Capacity supply and demand curves

According to proposition 7, a price-only contract between the two firms is sufficient to guarantee the optimal capacity exchange if the price is chosen appropriately. Thus, when the two firms belong to a single organization, a central planner may set p^c as the internal transfer price for the capacity.

In a decentralized system, as we have shown, both firms can increase their total profit by co-operating to choose p^c as the capacity price. However, most of the gain would now accrue at one of the firms. Therefore, any such agreement must also ensure that the two parties stick to the agreed-price formula. For example, a side payment from the buyer to the seller or from the seller to the buyer may work to align the respective firm's incentive. We next study a capacity collaboration scheme in which the price is set by either the seller or the buyer.

5.1 Seller charges a fixed fee

In this arrangement, the seller sets $p^c(< p^s)$ as the price for capacity exchange and charges the buyer an additional fixed fee F^s. Through this arrangement, the joint profit of the two firms increases from $G_1(R_1-q^s)+G_2(R_2+q^s)-t_{12}q^s$, in the case of no side payment, to $G_1(R_1-q^c)+G_2(R_2+q^c)-t_{12}q^c$. Denote π_i^f as the profit of firm i when the price is p^c and the fixed payment is F^s for $i=1,2$. The profit of firm 1 changes from $\pi_1^s = G_1(R_1-q^s)+p^sq^s$ to $\pi_1^f = G_1(R_1-q^c)+p^cq^c+F^s$. The profit of firm 2 changes from $\pi_2^s = G_2(R_2+q^s)-(p^s+t_{12})q^s$ to $\pi_2^f = G_2(R_2+q^c)-(p^c+t_{12})q^c-F^s$. Clearly if $F^s = [G_2(R_2+q^c)-(p^c+t_{12})q^c]-[G_2(R_2+q^s)-(p^s+t_{12})q^s]$, firm 2 is no worse off with the side payment than it was without the side payment. However, because $G_1(R_1-q^c)+G_2(R_2+q^c)-t_{12}q^c >$

$G_1(R_1 - q^s) + G_2(R_2 + q^s) - t_{12}q^s$ (the left hand side is the joint profit of the centralized system), we have, by rearranging terms,

$$G_1(R_1 - q^c) + p^c q^c + F^s > G_1(R_1 - q^s) + p^s q^s.$$

This implies that the net profit of firm 1 is higher with the side payment than that without it. In summary, when the capacity price is chosen as p^c and the fixed charge is $F^s = [G_2(R_2 + q^c) - (p^c + t_{12})q^c] - [G_2(R_2 + q^s) - (p^s + t_{12})q^s]$, firm 2's profit remains the same as when the price is set as p^s without a side payment, and firm 1's profit is higher than that without a side payment. Obviously, firm 2 may ask for a lower fixed side payment to increase its net profit. As long as the fixed payment is no higher than F^s, firm 2's situation is not worse than that under a linear price arrangement. Proposition 8 summarizes this result (without proof).

Proposition 8 *Assume* $p_1^s < p_2^b$, *and the seller sets the capacity price. Consider a payment scheme where* $p = m_1\overline{\Phi}_1(R_1 - q^c)$, *and* F^s *is the side payment from the buyer to the seller. If* F^s *lies between* $[G_1(R_1 - q^s) + p^s q^s] - [G_1(R_1 - q^c) + p^c q^c]$ *and* $[G_2(R_2 + q^c) - (t_{12} + p^c)q^c] - [G_2(R_2 + q^s) - (t_{12} + p^s)q^s]$, *both firms accrue higher profits than under a linear price arrangement with the seller sets the capacity price.*

5.2 Buyer charges a fixed fee

From Proposition 6, we know that without a side payment and with the price set by the buyer, the exchanged capacity between the two firms is reduced because the buyer offers a lower price for the exchanged capacity ($p^b \leq p^c$). At price p^c, firm 1 is induced to sell more capacity to firm 2 than at price p^b. Of course, when the capacity price is set as p^c, the buyer's profit is less than that when the price is p^b. However, the buyer may charge a fixed fee F^b to the seller to recover the lost profit.

Proposition 9 *Assume* $p_1^s < p_2^b$, *and the buyer sets the capacity price. Consider a payment scheme where* $p = m_1\overline{\Phi}_1(R_1 - q^c)$, *and* F^b *is the side payment from the seller to the buyer. If* F^b *lies between* $[G_2(R_2 + q^b) - (p^b + t_{12})q^b] - [G_2(R_2 + q^c) - (p^c + t_{12})q^c]$ *and* $[G_1(R_1 - q^c) + p^c q^c] - [G_1(R_1 - q^b) + p^b q^b]$, *both firms accrue higher profits than under a linear price arrangement with the buyer sets the capacity price.*

Next, we examine how the capacity investment may be affected when the capacity investment decisions are made unilaterally and the capacity exchange is conducted as in the centralized system.

6 Capacity Investment Decisions

We first consider the capacity investment decision in a centralized system. Recall that $E\pi^c(R_1, R_2)$, defined in §3, is the overall expected profit of the centralized system for given capacity investment levels (R_1, R_2). The investment decision is then to choose (R_1, R_2) to maximize the total expected profit $-c_1R_1 - c_2R_2 + E\pi^c(R_1, R_2)$. Proposition 10 characterizes the optimal capacity investment decision in the centralized system.

Proposition 10 *$-c_1R_1 - c_2R_2 + E\pi^c(R_1, R_2)$ is concave, the optimal capacity investment level is unique and is determined by the first-order condition:*

$$-c_i + \frac{\partial}{\partial R_i} E\pi^c(R_1, R_2) = 0, \ \ i = 1, 2. \tag{15}$$

We now study the capacity investment decision in the decentralized system when the capacity exchange is conducted as in a centralized system. For ease of exposition, we define $E\pi^c(R_1, R_2) - \sum_{i=1}^{2} G_i(R_i)$, the expected additional value generated from capacity exchange, as *capacity exchange surplus*. We assume firm i will receive $\theta_i \in (0, 1)$ portion of the capacity exchange surplus for $i = 1, 2$ with $\theta_1 + \theta_2 = 1$. One possible way for both firms to agree on exchanging capacity (as in a centralized system) and splitting the capacity exchange surplus is through negotiation before capacity investment decisions are made. As in Van Mieghem [23], θ_i can be regarded as the "bargaining power" of firm i; the higher the value of θ_i, the greater power firm i has in negotiation. Note that since capacity exchange is conducted before demand uncertainty is resolved, the split of the surplus will be in terms of "expectation". Under this scenario, firms do not make pricing decisions for the capacity, and the only decision that needs to be made is the amount of capacity to be exchanged. Capacity investment decisions are typically made individually. Hence, we consider the scenario in which each firm sets its capacity investment level that maximizes its expected payoff, taking into account the investment decision of the other firm. Therefore, the two firms play a non-cooperative game in which the payoffs depend on an embedded cooperative capacity exchange game. This formulation is an example of a biform game and has been used recently in the operations literature; see, e.g., [15]. To determine the amount of capacity investment, firm i has to consider the capacity investment cost c_iR_i in the first stage and the expected profit through capacity exchange in the second stage. Therefore, firm i's expected total profit $\pi_i(R_1, R_2)$, when firms choose capacity levels (R_1, R_2), can be written as

$$\pi_i(R_1, R_2) = -c_iR_i + G_i(R_i) + \theta_i \left[E\pi^c(R_1, R_2) - \sum_{i=1}^{2} G_i(R_i) \right]. \tag{16}$$

Given that firm j chooses R_j, firm i's reaction $r_i(R_j)$ satisfies:

$$r_i(R_j) = \arg\max_{R_i}\{\pi_i(R_1, R_2)\}. \tag{17}$$

Lemma 4 characterizes a firm's payoff function $\pi_i(R_1, R_2)$ and its response curve $r_i(R_j)$.

Lemma 4 *For $i, j = 1, 2$ and $i \neq j$, (i) $\pi_i(R_1, R_2)$ is strictly concave in R_i for a given R_j; (ii) $r_i(R_j)$ is a function and $-1 < r_i'(R_j) \leq 0$.*

The following result establishes the existence and uniqueness of a pure strategy Nash equilibrium for capacity investment.

Proposition 11 *A pure strategy Nash equilibrium exists and the Nash equilibrium is unique in the decentralized system when capacity exchange is conducted as in a centralized system. The capacity investment levels at Nash equilibrium satisfy the following first-order conditions:*

$$-c_i + G_i'(R_i) + \theta_i \left[\frac{\partial}{\partial R_i} E\pi^c(R_1, R_2) - G_i'(R_i)\right] = 0,\ i = 1, 2. \tag{18}$$

From Proposition 11, we can see the capacity investment decisions in the decentralized system are also affected by the value of θ_i. We now study the impact of θ_i on firms' capacity investment decisions. For ease of exposition, we define $\theta = \theta_1$ as the portion of the capacity exchange surplus that goes to firm 1 ($(1-\theta)$ of the surplus goes to firm 2). Further, we write $r_i(R_j)$ as $r_i(R_j, \theta)$ to emphasize the fact that firm i's response curve depends on the value of θ. Lemma 5 is used to characterize the impact of θ on their capacity investment levels.

Lemma 5 *$r_1(R_2, \theta)$ increases in θ and $r_2(R_1, \theta)$ decreases in θ.*

Proposition 12 demonstrates that the higher share of the capacity exchange surplus a firm gets, the more capacity it will invest. Figure 2 illustrates this result.

Proposition 12 *Firm 1's capacity investment level at Nash equilibrium, R_1^*, increases in θ. Firm 2's capacity investment level at Nash equilibrium, R_2^*, decreases in θ.*

Intuitively, as a firm receives a higher share of the capacity exchange surplus, the value created by an additional unit of capacity increases. Hence, it will invest more in capacity.

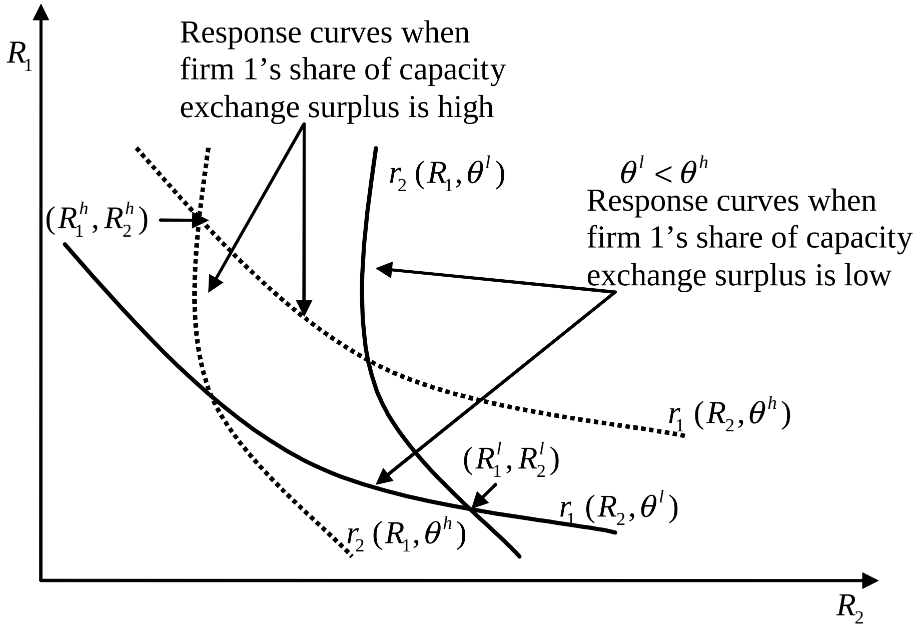

Figure 2: Impact of firms' bargaining powers on their capacity investment levels

6.1 Analysis of a Special Case

In this section, we study a special case where $(\overrightarrow{\eta}, \overrightarrow{\xi})$ is a multivariate normal random vector. The normal distribution is widely used to represent demand variability because its correlation structure is amenable to analysis; see, for example, Gurnani and Tang [6]. Assume $E[\eta_i] = \mu_{\eta_i}$, $E[\xi_i] = \mu_{\xi_i}$, and the covariance matrix of $(\overrightarrow{\eta}, \overrightarrow{\xi})$ to be Σ, which is assumed to be non-singlular with inverse $\Omega = (w_{ij})_{4\times 4}$. Then, the conditional density of ξ_i given $\overrightarrow{\eta}$ is also normal. Further,

$$E[\xi_i|\overrightarrow{\eta}] = \mu_{\xi_i} + a_{1i}(\eta_1 - \mu_{\eta_1}) + a_{2i}(\eta_2 - \mu_{\eta_2}), \tag{19}$$

$$Var[\xi_i|\overrightarrow{\eta}] = w_{ii}^{-1}, \tag{20}$$

where $a_{ji} = -w_{j(i+2)}/w_{(i+2)(i+2)}$ for $i, j = 1, 2$ (see Theorem 5 in Chapter III of [3]).

Proposition 13 *Suppose $(\eta_1, \eta_2, \xi_1, \xi_2)$ is multivariate Normal, and $a_{1i} \times a_{2i} \geq 0$ for $i = 1, 2$. Then the total profit of the centralized system is decreasing in the correlation coefficient between η_1 and η_2 (denoted as $\rho(\eta_1, \eta_2)$) if capacity investment is either held fixed or adjusted optimally as correlation changes.*

Thus, capacity exchange is more beneficial in the centralized system when demand signals η_1 and η_2 are less correlated with $a_{1i} \times a_{2i} \geq 0$. This observation is slightly different from the well-known result of risk pooling in the inventory literature in that we need an additional technical condition ($a_{1i} \times a_{2i} \geq 0$) to establish this result. This difference can be explained by the fact that in our model demand uncertainty is resolved sequentially whereas in the traditional inventory model with risk pooling, the demand uncertainty is resolved simultaneously.

The impact of correlation between $\overrightarrow{\eta}$ and $\overrightarrow{\xi}$ is complex. In order to keep tractability, we assume that $\rho(\eta_1, \eta_2) = 0$, $\rho(\xi_1, \xi_2) = 0$, and $\rho(\eta_i, \xi_j) = 0$ for $i \neq j$, and study the impact of $\rho(\eta_i, \xi_i)$ on the total expected profit in the centralized system and on the capacity investment level corresponding to the Nash equilibrium in the decentralized system. Denote $\rho^2(\eta_i, \xi_i)$ as the square of $\rho(\eta_i, \xi_i)$.

Proposition 14 *Suppose* $(\eta_1, \eta_2, \xi_1, \xi_2)$ *is multivariate Normal, and* $\rho(\eta_1, \eta_2) = 0, \rho(\xi_1, \xi_2) = 0$, *and* $\rho(\eta_i, \xi_j) = 0$ *for* $i \neq j$. *Then the total profit of the centralized system is increasing in* $\rho^2(\eta_i, \xi_i)$ *if capacity investment is either held fixed or adjusted optimally as correlation changes.*

Next result illustrates the effect of demand correlation on firms' capacity investment levels.

Proposition 15 *Suppose the two firms are symmetric and* $\rho(\eta_1, \eta_2) = 0, \rho(\xi_1, \xi_2) = 0$, *and* $\rho(\eta_i, \xi_j) = 0$ *for* $i \neq j$. *Then the optimal capacity investment level at Nash equilibrium,* R_i^*, *decreases in* $\rho^2(\eta_i, \xi_i)$ *if* $R_i^* > \mu_\xi$, *increases in* $\rho^2(\eta_i, \xi_i)$ *if* $R_i^* < \mu_\xi$ *and remains unchanged with respect to* $\rho^2(\eta_i, \xi_i)$ *if* $R_i^* = \mu_\xi$.

Thus, the impact of correlation between the demand signal and the realized demand on a firm's capacity investment level depends on the relationship between its capacity investment level and the mean demand. Clearly, for a high margin capacity, firms tend to invest more in capacity (more than the mean demand). Therefore, higher correlation allows firms to invest less until a firm's capacity investment level equals its mean demand. On the other hand, when the margin is low, firms tend to invest less in capacity. For this case, increasing correlation causes the capacity investment level to converge to its mean demand.

7 Conclusions

We have shown how capacity price may be determined in a capacity exchange game between two firms; either firm acting as the Stackelberg leader. By benchmarking against the centralized system, we demonstrate how a side payment may be used to coordinate the capacity exchange decisions. We then study the firms' capacity investment decisions using a biform game framework in which capacity investment decisions are made individually and exchange decisions are made as in a centralized system. We demonstrate the existence and uniqueness of the Nash equilibrium capacity investment levels and study the impact of firms' share of the capacity exchange surplus on their capacity investment levels. Finally, using normal demand distribution as an example, we develop insights on when capacity exchange may be more beneficial.

In our analysis, we have assumed that information is complete in that each firm observes the other firm's demand signal to facilitate the analysis. The extension to the incomplete information case involves a mechanism for both firms to learn the roles they are going to play (a buyer's role or a seller's role) in the exchange. Clearly, this is well beyond the scope of this paper and we leave it for future research. A second study of interest would be the scenario in which *ex ante* capacity collaboration is combined with *ex post* capacity collaboration. A third research avenue would be to examine capacity collaboration between firms in different geographical locations, and the role of transportation cost in such collaborations.

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A Proofs of Results

PROOF of Proposition 1. Note that $dG_i(K_i)/dK_i = m_i\overline{\Phi}_i(K_i)$ for $i = 1, 2$. The first-order derivative of $\pi^c(\cdot)$ is

$$\frac{d\pi^c(K_1)}{dK_1} = \begin{cases} t_{12} + \alpha(K_1) & \text{if } K_1 < R_1, \\ -t_{21} + \alpha(K_1) & \text{if } K_1 > R_1. \end{cases} \tag{21}$$

It is straightforward to show that $\pi^c(K_1)$ is strictly concave in K_1 and continuously differentiable except when $K_1 = R_1$. It follows that, to determine the optimal solution K_1^c, one needs to find whether K_1^c is less than R_1 (or greater than R_1) by checking the signs of the left-side derivative and the right-side derivative of $\pi^c(\cdot)$ at R_1. From (21), the left-side derivative of $\pi^c(\cdot)$ at R_1 would be $t_{12} + \alpha(R_1)$, and the right-side derivative would be $-t_{21} + \alpha(R_1)$.

First, assume $t_{12} + \alpha(R_1) < 0$, which implies $-t_{21} + \alpha(R_1) < 0$. Therefore, $\pi^c(K_1)$ is decreasing for $K_1 \geq R_1$. Hence, K_1^c must be less than R_1 and can be found by solving $t_{12} + \alpha(K_1) = 0$. In the case where $t_{12} + \alpha(K_1) = 0$ has no solution between 0 and R_1, $K_1^c = 0$.

Next, assume $t_{12} + \alpha(R_1) > 0$. Then, $-t_{21} + \alpha(R_1)$ can be either positive or negative. If $-t_{21} + \alpha(R_1) \leq 0$, we know $\pi^c(K_1)$ is decreasing when $K_1 \geq R_1$ and is increasing when $K_1 \leq R_1$. Therefore, $K_1^c = R_1$. If $-t_{21} + \alpha(R_1) > 0$, since $\pi^c(K_1)$ must now increase when $K_1 \leq R_1$, the maximum of $\pi^c(K_1)$ is obtained at a value between R_1 and $R_1 + R_2$. Thus, K_1^c may be obtained by solving the first-order condition, $-t_{21} + \alpha(K_1) = 0$. In the case where $-t_{21} + \alpha(K_1) = 0$ has no solution between R_1 and $R_1 + R_2$, $K_1^c = R_1 + R_2$. This completes the proof. ■

PROOF of Corollary 1. Note that $\frac{d\pi^c}{dK_1}$ is increasing in t_{12} when $K_1^c < R_1$ and is constant in t_{12} when $K_1^c \geq R_1$. Therefore, if the increase in t_{12} does not change the range of K_1^c, then K_1^c as a function of t_{12} is increasing in t_{12}. If the increase in t_{12} does change the range of K_1^c, then, according to Proposition 1, the increase in t_{12} can only cause K_1^c to change from $K_1^c < R_1$ to $K_1^c = R_1$. So K_1^c is increasing in t_{12}. Similarly, we can prove that K_1^c is decreasing in t_{21}.

To prove the second part, note that $|K_1^c - R_1| = R_1 - K_1^c$ if $K_1^c < R_1$. Hence, if the increase in t_{12} does not change the the fact that $K_1^c < R_1$, then $|K_1^c - R_1|$ is decreasing in t_{12}. If the increase in t_{12} cause the violation of $K_1^c < R_1$, then K_1^c can only change from $K_1^c < R_1$ to $K_1^c = R_1$, which implies $|K_1^c - R_1|$ decreases. Hence, $|K_1^c - R_1|$ is decreasing in t_{12}. Similarly, we can prove that $|K_1^c - R_1|$ is decreasing in t_{21}. ■

PROOF of Proposition 2. We only have to show that $\pi^c(R_1, R_2|\overrightarrow{\eta})$ is increasing and concave in (R_1, R_2). The monotonicity of $\pi^c(R_1, R_2|\overrightarrow{\eta})$ is trivial. Note that $\pi^c(\cdot)$ is jointly concave in (K_1, R_1, R_2) and the feasible region of K_1 is convex. Since maximizing a concave function retains concavity, the optimal objective function value $\pi^c(\cdot)$ as a function of (R_1, R_2) is concave. ■

PROOF of Lemma 2. Let $h(q) = q\frac{\phi(q)}{\overline{\Phi}(q)}$ be the generalized failure rate. Since the CDF of ξ is $\Phi(q)$, the CDF of $\xi - R$ is $\Phi(q + R)$, and the PDF is $\phi(q + R)$. So, the generalized failure rate of $\xi - R$ may be written as $h_{\xi - R}(q) = q\frac{\phi_{\xi - R}(q)}{\overline{\Phi}_{\xi - R}(q)} = q\frac{\phi(q+R)}{\overline{\Phi}(q+R)}$. To show that $\xi - R$ also has an increasing generalized failure rate, we only need to show that $h_{\xi - R}(q)$ is increasing in q. For $q_1 \leq q_2$,

$$q_1\frac{\phi(q_1 + R)}{\overline{\Phi}(q_1 + R)} = q_1 \times \frac{q_1 + R}{q_1 + R} \times \frac{\phi(q_1 + R)}{\overline{\Phi}(q_1 + R)} \leq \frac{q_1}{q_1 + R}(q_2 + R)\frac{\phi(q_2 + R)}{\overline{\Phi}(q_2 + R)} \leq q_2\frac{\phi(q_2 + R)}{\overline{\Phi}(q_2 + R)},$$

where the first inequality is based on $\Phi(q)$ is IGFR and the second inequality is based on the fact that $\frac{q_1}{q_1+R}(q_2 + R) \leq q_2$ for $q_1 \leq q_2$. ■

PROOF of Lemma 3. Since ξ is IGFR and has a finite mean, $\xi - R$ is also IGFR (based on Lemma 2) and has a finite mean. From Lemma 2 of [11], we know that the root of $q\frac{\phi(q+R)}{\overline{\Phi}(q+R)} = 1$ is finite, where the left-hand side of the equation is the generalized failure rate of $\xi - R$. ■

PROOF of Proposition 3. Take the first-order derivative of $\pi^s(q)$ given in (6) with respect to q, $d\pi^s/dq = -t_{12} - m_1\overline{\Phi}_1(R_1 - q) + m_2\overline{\Phi}_2(R_2 + q)\left[1 - q\frac{\phi_2(R_2+q)}{\overline{\Phi}_2(R_2+q)}\right]$.

For $q \leq \bar{q}$, $q\frac{\phi_2(R_2+q)}{\overline{\Phi}_2(R_2+q)}$ is increasing in q from Lemma 2 since $\Phi(\cdot)$ is IGFR. Further, $1 - q\frac{\phi_2(R_2+q)}{\overline{\Phi}_2(R_2+q)} \geq 0$ for $q \leq \bar{q}$. Note that $\overline{\Phi}(R_2 + q)$ is decreasing in q. So, $m_2\overline{\Phi}_2(R_2 + q)\left(1 - q\frac{\phi_2(R_2+q)}{\overline{\Phi}_2(R_2+q)}\right)$ is decreasing in q for $q \leq \bar{q}$. The first two terms of $\frac{d\pi^s}{dq}$, $-t_{12} - m_1\overline{\Phi}_1(R_1 - q)$, is also decreasing in q. So, $\pi^s(q)$ is strictly concave for $q \leq \bar{q}$.

For $q \geq \bar{q}$, $1 - q\frac{\phi_2(R_2+q)}{\overline{\Phi}_2(R_2+q)} \leq 0$ since $q\frac{\phi_2(R_2+q)}{\overline{\Phi}_2(R_2+q)}$ is increasing in q. Therefore $m_2\overline{\Phi}_2(R_2 + q)[1 - q\frac{\phi_2(R_2+q)}{\overline{\Phi}_2(R_2+q)}] \leq 0$. Note that $-t_{12} - m_1\overline{\Phi}_1(R_1 - q) < 0$ for all q. Hence $\pi^s(q)$ is strictly decreasing for $q \geq \bar{q}$. Therefore, $\pi^s(q)$ is unimodal. There is a unique optimal solution to the seller's problem. (7) follows by setting $d\pi^s/dq$ to zero. ■

PROOF of Proposition 4. Since there is capacity exchange in the centralized system and firm 1 is a capacity seller, we know that $t_{12} + \alpha(R_1) < 0$ from Proposition 1. That is, $t_{12} + m_1\overline{\Phi}_1(R_1) - m_2\overline{\Phi}_2(R_2) < 0$, or $m_1\overline{\Phi}_1(R_1) < m_2\overline{\Phi}_2(R_2) - t_{12}$. So $p_1^s < p_2^b$. Hence, in the decentralized system, firm 1 is still a capacity seller, and firm 2 is still a capacity buyer. Similarly, we can argue that if there is capacity exchange in the decentralized system, there is also capacity exchange in the centralized system, and both firms will play the same roles in both systems.

To prove the second part of the result, we need to compare the first-order conditions under the centralized system and under the decentralized system. Under the centralized system, K_1^c solves: $-t_{12} - \alpha(K_1^c) = 0$ (Proposition 1). Let $q^c = R_1 - K_1^c$, q^c solves:

$$-t_{12} - \alpha(R_1 - q) = 0. \tag{22}$$

Under the decentralized system, q^s satisfies (Proposition 3):

$$-t_{12} - \alpha(R_1 - q) - m_2 q\phi_2(R_2 + q) = 0. \tag{23}$$

Clearly, the left-hand side of (22) is greater than the left-hand side of (23), and both of them are decreasing in q. So $q^c > q^s$. ■

PROOF of Proposition 5.

$$\frac{d\pi^b(q)}{dq} = -t_{12} - m_1\overline{\Phi}_1(R_1 - q)\left[1 + q\frac{\phi_1(R - q)}{\overline{\Phi}_1(R - q)}\right] + m_2\overline{\Phi}_2(R_2 + q).$$

Since ξ_1 has an IGFR, $q\frac{\phi_1(R+q)}{\overline{\Phi}_1(R+q)}$ is increasing in q (Lemma 3) which implies $1+q\frac{\phi_1(R-q)}{\overline{\Phi}_1(R-q)}$ is increasing in q. Note $m_1\overline{\Phi}_1(R_1-q)$ is increasing in q, and $m_2\overline{\Phi}_2(R_2+q)$ is decreasing in q. Therefore $d\pi^b(q)/dq$ is decreasing in q. $\pi^b(q)$ is a strictly concave function. The optimal solution of the buyer's problem may be found by solving (9). ■

PROOF of Proposition 6. Similar to the proof of Proposition 4. ■

PROOF of Proposition 7. We first prove that $p^c \in [p_1^s, p_2^b]$. First, $p^c = m_1\overline{\Phi}_1(R_1 - q^c) \geq m_1\overline{\Phi}_1(R_1) = p_1^s$. Since $p_1^s \leq p_2^b$, we have $-t_{12} - m_1\overline{\Phi}_1(K_1^c) + m_2\overline{\Phi}_2(R_1 + R_2 - K_1^c) \geq 0$ from

Proposition 1; that is, $-t_{12} - m_1\overline{\Phi}_1(R_1 - q_1^c) + m_2\overline{\Phi}_2(R_2 + q_1^c) \geq 0$, which yields $m_1\overline{\Phi}_1(R_1 - q^c) \leq -t_{12} + m_2\overline{\Phi}_2(R_2 + q_1^c)$. Therefore, $p^c \leq -t_{12} + m_2\overline{\Phi}_2(R_2 + q_1^c) \leq -t_{12} + m_2\overline{\Phi}_2(R_2) = p_2^b$. Hence, firm 1 will be a capacity seller, and firm 2 will be a buyer under capacity price p^c.

Based on Lemma 1, the seller's optimal selling quantity is: $R_1 - \Phi_1^{-1}(1 - \frac{p^c}{m_1}) = R_1 - \Phi_1^{-1}(1 - \overline{\Phi}_1(R_1 - q^c)) = q^c$. Similarly, the optimal purchasing quantity for the buyer is $\Phi_2^{-1}(1 - \frac{p^c + t_{12}}{m_2}) - R_2$. To show $\Phi_2^{-1}\left(1 - \frac{p^c+t_{12}}{m_2}\right) - R_2$ is equal to q^c, we only need to show $\Phi_2(R_2 + q^c) = 1 - \frac{p^c+t_{12}}{m_2}$, or $-t_{12} - p^c + m_2\overline{\Phi}_2(R_2 + q^c) = 0$, and this equation follows from Proposition 1. ■

PROOF of Proposition 10. The concavity follows from Proposition 2. So, the optimal capacity investment value is unique and satisfies the first-order condition. ■

PROOF of Lemma 4. For each realization of $\overrightarrow{\eta}$, firm i's expected profit can be written as (see, also, equation 16)

$$\pi_i(R_1, R_2|\overrightarrow{\eta}) = (1 - \theta_i)G_i(R_i|\overrightarrow{\eta}) + \theta_i\pi^c(R_1, R_2|\overrightarrow{\eta}) - \theta_i G_j(R_j|\overrightarrow{\eta}). \tag{24}$$

The first two terms of the right-hand side of (24) are concave in R_i, and the third term does not change with respect to R_i. Therefore, $\pi_i(R_1, R_2|\overrightarrow{\eta})$ is concave in R_i for each realization of $\overrightarrow{\eta}$ which implies $\pi_i(R_1, R_2)$ is concave in R_i for a given R_j.

That $r_i(R_j)$ is a function follows directly from the concavity of $\pi_i(R_1, R_2)$. In order to demonstrate $-1 < r_i'(R_j) \leq 0$, we first show that for any realization of $\overrightarrow{\eta}$, $\frac{\partial^2\pi^c(R_1,R_2|\overrightarrow{\eta})}{\partial R_1^2} \leq \frac{\partial^2\pi^c(R_1,R_2|\overrightarrow{\eta})}{\partial R_1 \partial R_2} \leq 0$ by considering three scenarios of capacity exchange based on the realization of $\overrightarrow{\eta}$.

Scenario 1: $t_{12} + \alpha(R_1|\overrightarrow{\eta}) < 0$. Based on Proposition 1, firm 1's capacity is converted to be used by firm 2 for this scenario and

$$\pi^c(R_1, R_2|\overrightarrow{\eta}) = -t_{12}(R_1 - K_1^c) + G_1(K_1^c) + G_2(R_1 + R_2 - K_1^c), \text{ with} \tag{25}$$

$$t_{12} + m_1\overline{\Phi}_1(K_1^c) - m_2\overline{\Phi}_2(R_1 + R_2 - K_1^c) = 0. \tag{26}$$

From (25), we obtain

$$\begin{aligned}
\frac{\partial \pi^c(R_1, R_2|\overrightarrow{\eta})}{\partial R_1} &= -t_{12}\left(1 - \frac{\partial K_1^c}{\partial R_1}\right) + m_1\overline{\Phi}_1(K_1^c)\frac{\partial K_1^c}{\partial R_1} + m_2\overline{\Phi}_2(R_1 + R_2 - K_1^c)\left(1 - \frac{\partial K_1^c}{\partial R_1}\right) \\
&= -t_{12} + m_2\overline{\Phi}_2(R_1 + R_2 - K_1^c) + [t_{12} + m_1\overline{\Phi}_1(K_1^c) - m_2\overline{\Phi}_2(R_1 + R_2 - K_1^c)]\frac{\partial K_1^c}{\partial R_1} \\
&= -t_{12} + m_2\overline{\Phi}_2(R_1 + R_2 - K_1^c) \\
&= m_1\overline{\Phi}_1(K_1^c),
\end{aligned} \tag{27}$$

where the third equality and the last equality are based on (26). Therefore,

$$\begin{aligned} \frac{\partial^2 \pi^c(R_1, R_2|\overrightarrow{\eta})}{\partial R_1^2} &= -m_1\phi_1(K_1^c)\frac{\partial K_1^c}{\partial R_1}, \\ \frac{\partial^2 \pi^c(R_1, R_2|\overrightarrow{\eta})}{\partial R_1 R_2} &= -m_1\phi_1(K_1^c)\frac{\partial K_1^c}{\partial R_2}. \end{aligned}$$

From (26), it is easy to verify that see that $\frac{\partial K_1^c}{\partial R_1} = \frac{\partial K_1^c}{\partial R_2} = \frac{m_2\phi_2(R_1+R_2-K_1^c)}{m_1\phi_1(K_1^c)+m_2\phi_2(R_1+R_2-K_1^c)} > 0$ using the implicit function theorem. Hence, $\frac{\partial^2 \pi^c(R_1,R_2|\overrightarrow{\eta})}{\partial R_1^2} = \frac{\partial^2 \pi^c(R_1,R_2|\overrightarrow{\eta})}{\partial R_1 R_2} < 0$.

Scenario 2: $-t_{21} + \alpha(R_1|\overrightarrow{\eta}) > 0$. For this scenario, firm 2's capacity is converted to be used by firm 2 from Proposition 1. Similar to scenario 1, we can demonstrate that $\frac{\partial^2 \pi^c(R_1,R_2|\overrightarrow{\eta})}{\partial R_1^2} = \frac{\partial^2 \pi^c(R_1,R_2|\overrightarrow{\eta})}{\partial R_1 R_2} < 0$.

Scenario 3: $t_{12} + \alpha(R_1|\overrightarrow{\eta}) \geq 0$ and $-t_{21} + \alpha(R_1|\overrightarrow{\eta}) \leq 0$. For this scenario, no capacity is exchanged between the two firms from Proposition 1 and $\pi^c(R_1, R_2|\overrightarrow{\eta}) = G_1(R_1) + G_2(R_1)$. It is straightforward to verify that $\frac{\partial^2 \pi^c(R_1,R_2|\overrightarrow{\eta})}{\partial R_1^2} = -m_1\phi_1(R_1) < 0$ and $\frac{\partial^2 \pi^c(R_1,R_2|\overrightarrow{\eta})}{\partial R_1 R_2} = 0$. Hence $\frac{\partial^2 \pi^c(R_1,R_2|\overrightarrow{\eta})}{\partial R_1^2} < \frac{\partial^2 \pi^c(R_1,R_2|\overrightarrow{\eta})}{\partial R_1 R_2} \leq 0$.

Therefore, $\partial^2 E\pi^c(R_1, R_2)/\partial R_1^2 \leq \partial^2 E\pi^c(R_1, R_2)/\partial R_1 \partial R_2 \leq 0$. Note that we exchanged the order of taking expectation and taking derivative; this is valid because the sample-path derivatives are bounded.

From the definition of $\pi_1(R_1, R_2)$ (see equation 16), we obtain

$$\frac{\partial^2 \pi_1(R_1,R_2)}{\partial R_1^2} = \frac{\partial^2 E\pi^c(R_1, R_2)}{\partial R_1^2} \text{ and } \frac{\partial^2 \pi_1(R_1,R_2)}{\partial R_1^2} = \frac{\partial^2 E\pi^c(R_1, R_2)}{\partial R_1 \partial R_2}.$$

Therefore,

$$\frac{\partial^2 \pi_1(R_1,R_2)}{\partial R_1^2} < \frac{\partial^2 E\pi^c(R_1, R_2)}{\partial R_1^2} \leq \frac{\partial^2 \pi_1(R_1,R_2)}{\partial R_1 \partial R_2} \leq 0, \tag{28}$$

where the second inequality is based on our analysis of the three scenarios for different realization of $\overrightarrow{\eta}$. From the implicit function theorem, we have

$$r_1'(R_2) = -\frac{\frac{\partial^2 \pi_1(R_1,R_2)}{\partial R_1 \partial R_2}}{\frac{\partial^2 \pi_1(R_1,R_2)}{\partial R_1^2}}. \tag{29}$$

Combining (28) and (29), we obtain $-1 < r_1'(R_2) \leq 0$. Switching the subscripts 1 and 2, we can show that $-1 < r_2'(R_1) \leq 0$. ■

Proof of Proposition 11. The existence of the Nash equilibrium follows from the concavity of the payoff function of firm i and the uniqueness of the Nash equilibrium follows directly from Lemma 4. ■

Proof of Lemma 5. We first show that $\pi^c(R_1, R_2|\overrightarrow{\eta}) - \sum_{i=1}^2 G_i(R_i|\overrightarrow{\eta})$ increases in R_i for all realizations of $\overrightarrow{\eta}$.

Scenario 1: $t_{12} + \alpha(R_1|\overrightarrow{\eta}) < 0$. Based on Proposition 1, firm 1's capacity is converted to be used by firm 2 for this scenario. From (27), we obtain $\partial\pi^c(R_1, R_2|\overrightarrow{\eta})/\partial R_1 = m_1\overline{\Phi}_1(K_1^c) \geq m_1\overline{\Phi}_1(R_1)$ where the inequality is based on $K_1^c \leq R_1$. Note that $\partial\sum_{i=1}^2 G_i(R_i|\overrightarrow{\eta})/\partial R_1 = m_1\overline{\Phi}_1(R_1)$. So, $\partial\pi^c(R_1, R_2|\overrightarrow{\eta})/\partial R_1 - \partial\sum_{i=1}^2 G_i(R_i|\overrightarrow{\eta})/\partial R_1 \geq 0$.

Scenario 2: $-t_{21} + \alpha(R_1|\overrightarrow{\eta}) > 0$. For this scenario, firm 2's capacity is converted to be used by firm 2 from Proposition 1. Similar to scenario 1, we can demonstrate that $\partial\pi^c(R_1, R_2|\overrightarrow{\eta})/\partial R_1 - \partial\sum_{i=1}^2 G_i(R_i|\overrightarrow{\eta})/\partial R_1 \geq 0$.

Scenario 3: $t_{12} + \alpha(R_1|\overrightarrow{\eta}) \geq 0$ and $-t_{21} + \alpha(R_1|\overrightarrow{\eta}) \leq 0$. For this scenario, no capacity is exchanged between the two firms from Proposition 1 and $\pi^c(R_1, R_2|\overrightarrow{\eta}) = \sum_{i=1}^2 G_i(R_i|\overrightarrow{\eta})$. Hence, $\partial\pi^c(R_1, R_2|\overrightarrow{\eta})/\partial R_1 - \partial\sum_{i=1}^2 G_i(R_i|\overrightarrow{\eta})/\partial R_1 = 0$.

Therefore, the left-hand side of (18) increases as θ_i increases. Because the left-hand side of (18) decreases in R_i, we see that the root of (18) increases in θ_i. This completes the proof. ■

PROOF of Proposition 12. Consider $0 < \theta^l < \theta^h < 1$. Denote (R_1^i, R_2^i) as the unique Nash equilibrium capacity investment level when firm 1 receives θ^i portion of the surplus for $i \in \{l, h\}$. We will demonstrate that $R_1^l \leq R_1^h$ and $R_2^l \geq R_2^h$.

With contradiction. Suppose $R_1^l > R_1^h$ instead. Then, $r_1(R_2^l, \theta^h) \geq r_1(R_2^l, \theta^l) = R_1^l > R_1^h = r_1(R_2^h, \theta^h)$ where the first inequality is based on the monotonicity of $r_1(\cdot)$ with respect to θ. Because $r_1(\cdot)$ is decreasing in R_2, we obtain $R_2^l < R_2^h$.

Now consider four points on the (R_1, R_2) plane: $(R_1^l, r_2(R_1^l, \theta^h))$, (R_1^l, R_2^l), (R_1^h, R_2^h), and $(r_1(R_2^h, \theta^l), R_2^h)$; see Figure 3 for an illustration. Because $r_2(\cdot)$ decreases in θ for given R_1, we know that $r_2(R_1^l, \theta^h) \leq r_2(R_1^l, \theta^l) = R_2^l$. Because $r_1(\cdot)$ increases in θ for given R_2, we know that $r_1(R_2^h, \theta^l) \leq r_1(R_2^h, \theta^h) = R_1^h$. Note that both $(R_1^l, r_2(R_1^l, \theta^h))$ and (R_1^h, R_2^h) are on the response curve of firm 2 for θ^h: $r_2(\cdot)$. Because $-1 < r_2'(\cdot) \leq 0$, we obtain $0 \leq [r_2(R_1^h, \theta^h) - r_2(R_1^l, \theta^h)]/(R_1^l - R_1^h) < 1$, that is,

$$0 \leq \frac{R_2^h - r_2(R_1^l, \theta^h)}{R_1^l - R_1^h} < 1. \tag{30}$$

Similarly, both (R_1^l, R_2^l) and $(r_1(R_2^h, \theta^l), R_2^h)$ are on the response curve of firm 1 for θ^l : $r_1(\cdot)$ and $-1 < r_1'(\cdot) < 0$. Therefore,

$$0 \leq \frac{R_1^l - r_1(R_2^h, \theta^l)}{R_2^h - R_2^l} < 1. \tag{31}$$

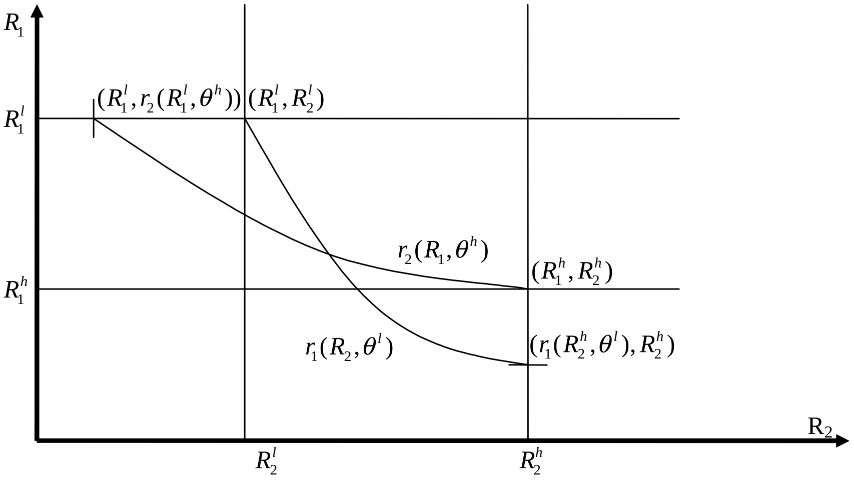

Figure 3: Illustration of the Proof for Proposition 12

Note that $R_2^h - r_2(R_1^l, \theta^h) \geq R_2^h - R_2^l$ and $R_1^l - R_1^h < R_1^l - r_1(R_2^h, \theta^l)$, we have

$$\frac{R_2^h - r_2(R_1^l, \theta^h)}{R_1^l - R_1^h} \geq \frac{R_2^h - R_2^l}{R_1^l - r_1(R_2^h, \theta^l)} > 1, \tag{32}$$

where the second inequality is based on (31). Note that (32) contradicts (30). We complete the proof. ■

PROOF of Proposition 13. We first show that for $i = 1, 2$, $G_{\xi_i}(K)$ is submodular in each realization of $\overrightarrow{\eta}$ for an arbitrary given capacity K. From (20), we see the realization of $\overrightarrow{\eta}$ does not affect the variance of ξ_i, hence, $\overrightarrow{\eta}$ affects $G_{\xi_i}(K)$ only through the expected value of ξ_i. Note that $G_{\xi_i}(K) = \int_K^{+\infty} K\phi_{\xi_i}(x)dx + \int_{-\infty}^{K} x\phi_{\xi_i}(x)dx = \int_{(K-E[\xi_i|\overrightarrow{\eta}])*w_{ii}}^{+\infty} K\phi(z)dz + \int_{-\infty}^{(K-E[\xi_i|\overrightarrow{\eta}])*w_{ii}} (E[\xi_i|\overrightarrow{\eta}] + z/w_{ii})\phi(z)dz$, where $\phi(z)$ is the density of standard Normal and the second equality is achieved by normalizing random variable ξ_i. Straightforward algebra shows that $G_{\xi_i}(K)$ is concave and increasing in the expected value of ξ_i for each realization of $\overrightarrow{\eta}$. Since $E[\xi_i|\overrightarrow{\eta}]$ is linear in (η_1, η_2) from (19), it is submodular in (η_1, η_2). Because $a_{1i} \times a_{2i} \geq 0$, $E[\xi_i|\overrightarrow{\eta}]$ is either increasing or decreasing in (η_1, η_2). So $G_{\xi_i}(K)$ is submodular in each realization of (η_1, η_2) (See Table 1 of [22]).

Therefore, the objective function $\pi^c(K)$, the summation of two submodular functions, is submodular in each realization of (η_1, η_2). When $\rho(\eta_1, \eta_2)$ increases, (η_1, η_2) increases in supermodular order (Theorem 11 of [13]). Consequently, $E\pi^c(R_1, R_2)$ decreases in $\rho(\eta_1, \eta_2)$, which implies $-c_1R_1 - c_2R_2 + E\pi^c(R_1, R_2)$ decreases in $\rho(\eta_1, \eta_2)$ when (R_1, R_2) is fixed. When (R_1, R_2) is adjusted optimally, $\partial[-c_1R_1 - c_2R_2 + E\pi^c(R_1, R_2)]/\partial R_i = 0$ from Proposition 10, so $\frac{d[-c_1R_1 - c_2R_2 + E\pi^c(R_1, R_2)]}{d\rho(\eta_1, \eta_2)} =$

$\frac{\partial[-c_1R_1-c_2R_2+E\pi^c(R_1,R_2)]}{\partial\rho(\eta_1,\eta_2)}+\sum_{i=1}^2\frac{\partial[-c_1R_1-c_2R_2+E\pi^c(R_1,R_2)]}{\partial R_i}\frac{\partial R_i}{\partial\rho(\eta_1,\eta_2)}=\frac{\partial[-c_1R_1-c_2R_2+E\pi^c(R_1,R_2)]}{\partial\rho(\eta_1,\eta_2)}$. Hence $-c_1R_1-c_2R_2+E\pi^c(R_1,R_2)$ decreases in $\rho(\eta_1,\eta_2)$ when (R_1,R_2) is adjusted optimally. ■

PROOF of Proposition 14. Straightforward algebra will show that,

$$E[\xi_i|\overrightarrow{\eta}]=\mu_{\xi_i}+\rho(\eta_i,\xi_i)\sigma_{\xi_i}\frac{\eta_i-\mu_{\eta_i}}{\sigma_{\eta_i}};\ Var[\xi_i|\overrightarrow{\eta}]=\sigma_{\xi_i}^2[1-\rho^2(\eta_i,\xi_i)]. \tag{33}$$

Equation (33) suggests that as $\rho^2(\eta_i,\xi_i)$ increases, $Var[\xi_i|\overrightarrow{\eta}]$ decreases, but $E\{E[\xi_i|\overrightarrow{\eta}]\}$ does not change. Therefore when $\rho^2(\eta_i,\xi_i)$ increases, ξ_i conditional on $\overrightarrow{\eta}$ decreases in convex order (See, for example, Theorem 4 of [13]). Since the objective function $\pi^c(K_1)$ is concave in ξ_i, $\pi^c(K_1)$ is decreasing when ξ_i is increasing convexly. So, as $\rho^2(\eta_i,\xi_i)$ increases, $\pi^c(K_1)$ increases. Therefore, $\pi^c(R_1,R_2)$ increases in $\rho^2(\eta_i,\xi_i)$ for fixed (R_1,R_2). From the argument used to prove Proposition 13, we can see this holds when (R_1,R_2) is adjusted optimally. ■

PROOF of Proposition 15. Because the two firms are symmetric, their capacity investment level at Nash equilibrium is identical: $R_1=R_2=R$. It is well known that the conditional distribution of ξ_i is normal with

$$E[\xi_i|\eta_i=y_i]=\mu_\xi+\rho\sigma_\xi\frac{y_i-\mu_\eta}{\sigma_\eta}\text{ and }Var[\xi_i|\eta_i=y_i]=\sigma_\xi^2[1-\rho^2]. \tag{34}$$

Because $t_{12}=t_{21}=0$, the three scenarios of Proposition 1 reduces into two scenarios and capacity exchange always occurs. And the capacity level of firm 1 after exchange satisfies

$$\overline{\Phi}_1(K_1^c|\eta_i=y_i)=\overline{\Phi}_2(2R-K_1^c|\eta_i=y_i). \tag{35}$$

Solving (35) yields

$$K_1^c=R-\rho\sigma_\xi\frac{y_2-y_1}{2\sigma_\eta}. \tag{36}$$

We now examine the expected profit of firm 1 in the second stage conditional on $\overrightarrow{\eta}$. Because $\theta=1/2$, we have $\pi_1(R,R|\overrightarrow{\eta})=G_1(K_1^c|\overrightarrow{\eta})+G_2(R_1+R_2-K_1^c|\overrightarrow{\eta})-[G_1(R)+G_2(R)]$. Therefore,

$$\frac{\partial\pi_1(R,R|\overrightarrow{\eta})}{\partial R}=m\overline{\Phi}_{\xi_1}(K_1^c|\overrightarrow{\eta})\frac{\partial K_1^c}{\partial R}+m\overline{\Phi}_{\xi_1}(2R-K_1^c|\overrightarrow{\eta})\left(2-\frac{\partial K_1^c}{\partial R}\right)=2m\Phi_{\xi_1}(K_1^c|\overrightarrow{\eta}).$$

With some algebra, one can show that

$$\frac{\partial\pi_1(R,R)}{\partial R}=-c+m_i\overline{\Phi}\left(\frac{R-\mu_\xi}{\sqrt{1-\rho^2/2}\sigma_\xi}\right). \tag{37}$$

From (37), we see that as ρ^2 increases $\partial\pi_1(R,R)/\partial R$ increases if and only if $R-\mu_\xi\leq 0$. Therefore, a firm's capacity investment level at Nash equilibrium increases in ρ^2 when $R-\mu_\xi\leq 0$ decreases in ρ^2 when $R-\mu_\xi\geq 0$. ■