Collaboration in Contingent Capacities with Information Asymmetry

Amiya K. Chakravarty,[1] Jun Zhang[2]

[1] *College of Business Administration, Northeastern University, Boston, Massachusetts*

[2] *School of Management, The University of Texas at Dallas, Richardson, Texas*

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Abstract: We study the optimal contracting problem between two firms collaborating on capacity investment with information asymmetry. Without a contract, system efficiency is lost due to the profit-margin differentials among the firms, demand uncertainty, and information asymmetry. With information asymmetry, we demonstrate that the optimal capacity level is characterized by a newsvendor formula with an upward-adjusted capacity investment cost, and no first-best solution can be achieved. Our analysis shows that system efficiency can always be improved by the optimal contract and the improvement in system efficience is due to two factors. While the optimal contract may bring the system's capacity level closer to the first-best capacity level, it prevents the higher-margin firm from overinvesting and aligns the capacity-investment decisions of the two firms. Our analysis of a special case demonstrates that, under some circumstances, both firms can benefit from the principal having better information about the agent's costs. Naval Research Logistics 54: 000–000, 2007

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1. INTRODUCTION

To meet the challenge of ever-changing market environments, firms have responded by forming close relationships with their suppliers and distributors. In addition to vertical collaboration between firms at different stages in a supply chain, there is also a growing interest in horizontal collaboration among different firms at the same level of a supply chain. Such horizontal collaboration can create advantages for firms possessing complementary expertise; they can reach a wider base of customers than they would working individually. Certain products or services become feasible only when firms offering these services combine their technology or resources. For example, working in partnerships, construction contract manufacturers and other specialist manufacturers are often in a position to offer more cost-effective, innovative, and efficient solutions for clients' projects. According to Strategic Forum for Construction report, "50% of construction projects (are) to be undertaken by integrated teams and supply chains by 2007" [22]. Major collaborations for product development have also taken shape between large enterprises because, as the product life cycle shortens and competition intensifies, the time required to establish expertise or gain market share individually is likely to exceed the time required through collaboration [19]. IBM recently teamed up with Siebel to provide hosted CRM software service that customers can deploy at the click of a button. Boulton [3] describes this collaboration arrangement as "the latest fruit of a partnership in which a leading applications provider and leading infrastructure provider team up to reach more customers, as neither makes what the other specializes in."

Once firms decide to form a partnership, it is critical to understand how the partners may continue to maintain the benefits of the alliance. One key factor for the success of such a partnership is that all parties' decisions, such as capacity investment, are coordinated. In scenarios where decisions are decentralized, individual firms may invest in different amounts of capacities, independently of one another. It can be shown, using a simple newsvendor analysis, that under uncertain demand, the firm with a lower profit margin will invest in a smaller capacity than the firm with a higher profit margin. Thus, the capacity for offering the integrated service in the collaboration would be limited by the capacity level of the lower-margin firm. In order to encourage the lower-margin firm to increase its capacity (to sell more of the integrated service), the higher-margin firm may offer incentives in the form of contracts. Such contracts may take many forms such as purchasing capacity or providing a subsidy—all subject to the lower-margin firm committing to a certain level of capacity investment. Another equally important decision both

Correspondence to: J. Zhang (jun.zhang@utdallas.edu)

firms must make is how to charge for their individual service because pricing for each individual service will affect the demand for the integrated service and both firms' capacity investment levels. In this paper we do not explicitly address the price–demand relationship, where price becomes endogenous to the decision model. Instead, we focus on the impact of information asymmetry for a given (i.e., exogenous) total price of the two services. Therefore, our model is a first step toward a complete understanding on collaboration in complementary capacities with information asymmetry.

One of the authors has first-hand knowledge of such a collaboration between two cotton processing firms. One firm processes cotton seeds and the other processes cotton fibers. It is difficult for the firms to enter the other's market because their products belong to different markets. The two firms chose a colocation strategy and built their facilities next to each other. During the harvesting season, farmers call both firms to check for available capacities before sending their cotton to them. If both firms possess sufficient capacity, the farmer sends the cotton to the firm with the seed processing capacity, which then cleans the cotton fiber, takes out the seeds, and processes the seeds. The cotton fiber is then sent to the other firm to be further processed. If any one of the firms does not possess sufficient capacity, the farmer has his cotton processed using other alternatives. At the beginning of the harvesting season, both firms determine how many workers to hire and how many shifts to run. Because the cotton seed processing firm has a higher margin, it makes a payment up front so that the fiber processing firm is willing to run an additional shift.

In most research on supply-chain contracting, a common assumption is that all relevant information is transparent between the parties concerned. See, for example, a review by Cachon [4]. In the context of our research, however, it is clear that firms would keep information such as the cost of capacity investment very private, resulting in information asymmetry between the firms. A realistic incentive contract must incorporate this asymmetry in its design. Therefore, we focus on designing optimal contracts with private information on capacity cost and study the impact of this information asymmetry on the contracting firms' welfare and the coordination role of contracting.

A number of papers in the supply chain literature have considered optimal contracting with asymmetric information. Cachon and Lariviere [5] study the forecast sharing problem between a manufacturer and a supplier and examine the supply chain performance when the manufacturer has superior demand information. They demonstrate that it is always in the interest of the manufacturer with a high demand forecast to share its forecast with the supplier, even if sharing the forecast is costly. Porteus and Whang [20] study a model similar to that of Cachon and Lariviere [5] except that they have the supplier offering the contract rather than the manufacturer. As a result, the supplier offers a menu of contracts, one designed for each type of manufacturer, which is a form of screening rather than signaling, and screening is what we study in this paper.

Ha [12] considers the problem of designing a contract to maximize the supplier's profit in a one-supplier, one-retailer channel. He uses a mechanism design to determine the supplier's optimal contracts when the buyer's marginal cost is private and shows that the system-wide solution is impossible to obtain under asymmetric information. Corbett and de Groote [8] study a model with one buyer, one supplier, deterministic demand, and fixed ordering costs for each tier in the supply chain. As in Ha [12], supply chain coordination is not achieved if there is asymmetric information. Corbett [7] studies coordination with one supplier, one buyer, stochastic demand, fixed ordering costs, and asymmetric information with respect to either the fixed ordering cost or the backorder penalty cost. He finds that consignment stock influences incentives, sometimes in a beneficial way and sometimes in a destructive way. Corbett, Zhou, and Tang [9] study the value to a supplier of obtaining better information about a buyer's cost structure by analyzing three different contracts with full or incomplete information. They find that the value of information is higher under two-part contracts and show that information and contract generality are strategic complements. Lütze and Özer [16] study the optimal contract between a supplier and a buyer where the supplier offers a promised lead-time contract when the buyer's inventory-related cost information is private. The above studies focus on designing contracts to reduce system inefficiency caused by double marginalization. However, "there has not been much research on the loss of efficiency due to lack of horizontal collaboration" [11].

This paper is also related to the research on decentralized assembly systems. In such systems, the marginal differentials among different component suppliers cause system inefficiency. Wang and Gerchak [24] study a capacity investment game between an assembler and its component suppliers. They examine the behavior of the system under supplier pricing and under assembler pricing. They characterize the optimal prices and the resulting equilibrium production quantities for their model. Tomlin [23] studies the capacity investment game between a manufacturer and a supplier and demonstrates that a class of coordinating price-only contracts may allocate the supply chain profit arbitrarily. For systems with multiple components, supplied by different firms, he shows that a piecewise-linear contract can be structured. Bernstein and Decroix [2] study a more general multi-tier decentralized assembly system and characterize the optimal pricing and capacity investment decisions at equilibrium. In all these cases, information is assumed to be transparent to all parties and contracts are designed for vertical relationship between a buyer and supplier.

In contrast to the above research, our paper focuses on the incentive issues in a lateral relationship between two partners. Inefficiency in such a system is caused by the marginal differentials among the parties, demand uncertainties, and information asymmetry. Note that lateral collaborations have been used widely in many scenarios, especially in the infrastructure industry where firms exchange short-term capacities. A number of papers have specifically studied the incentive issues in lateral collaborations when information is assumed to be transparent (Anupindi, Bassok, and Zemel [1], Rudi, Kapur, and Pyke [21], and Chakravarty and Zhang [6]).

Our research contributes to the existing literature by providing an analysis of the issues related to incentives and asymmetric cost information in a lateral collaborative relationship. In particular, we study the collaboration between two firms with two contingent capacities, in the sense that both capacities are required to provide an integrated service [17]. Demand for the service is uncertain and the two firms' margins from offering this service might be different. Both firms agree to form a partnership to offer this service and each firm must determine its capacity investment level before demand materializes.

We begin with the analysis of two base-case scenarios. In the first base case, a central planner makes capacity investment decisions for both firms. This scenario yields the first-best capacity level. In the second base case, the two firms make capacity investment decisions individually and simultaneously. The second scenario consists of two cases: complete information and incomplete information. The incomplete information case is assumed to be the status quo. The marginal capacity investment cost of one firm (firm 1) is private and the other firm (firm 2) possesses a prior on firm 1's investment cost. Our analysis demonstrates that system efficiency is lost with decentralized decision makings because of margin differentials and demand uncertainty. Further, information asymmetry worsens the system efficiency because firm 2 may overinvest.

We then study how contracting on capacity investment can improve the system efficiency using a principal–agent framework, in which firm 2 acts as the principal and firm 1 acts as an agent. We show that, with information asymmetry, no first-best capacity level can be achieved and the optimal capacity investment level (the second-best capacity level) is characterized by a newsvendor formula that uses an adjusted capacity investment cost for the lower-margin firm. Essentially, information asymmetry "distorts" the lower-margin firm's capacity cost upward and the system performance deteriorates. Our analysis demonstrates that contracting improves the system efficiency in two different ways. On one hand, it may bring the system's capacity investment level closer to the first-best level. On the other hand, it rules out the possible overinvestment by the firm who is uncertain about its partner's investment cost. With optimal contracts, the firm with private information (firm 1) accrues information rent and this rent is not necessarily monotone with the firm's capacity investment cost. Our study of a special case indicates that higher uncertainty in the private cost information may hurt the welfare of both firms. Therefore, it may be in the interests of the two firms to reduce information asymmetry.

The remainder of the paper is organized as follows. In Section 2 we describe the model in greater detail, characterize the optimal capacity investment levels with complete information and incomplete information, and develop the optimal contract with asymmetric information. We characterize the impact of information asymmetry in Section 3. We conclude in Section 4.

2. CONTINGENT CAPACITY INVESTMENT DECISIONS

2.1. Model Preliminaries

We consider two risk-neutral firms (indexed by i) with contingent service capacities, in the sense that both capacities are required to complete a customer service. Without loss of generality, we assume one unit of each capacity is required to provide one unit of service. We assume the capacity investment cost to be linear with a marginal value of c_i and the contribution margin of the capacity to be p_i for $i = 1, 2$. Demand for the service is assumed to be a random variable ξ with distribution (CDF) $F(\cdot)$, and density (PDF) $f(\cdot)$. Demand materializes after capacity is built. If realized demand is higher than the capacity level, all unsatisfied demand is lost. We assume the salvage value for the unused capacity and the penalty cost for unmet demands to be zero; however, our analysis can be extended to the case where these values are not zero.

If the two firms are managed by a central planner, the central planner would choose an investment level q to maximize $-(c_1 + c_2)q + (p_1 + p_2)E[q \wedge \xi]$, where $a \wedge b = \min\{a, b\}$. And the optimal capacity investment level would satisfy

$$q^*(c_1) = \bar{F}^{-1}\left(\frac{c_1 + c_2}{p_1 + p_2}\right), \quad (1)$$

where $\bar{F}^{-1}(\cdot)$ is the inverse of $\bar{F}(\cdot)$ with $\bar{F}(\cdot) = 1 - F(\cdot)$. Note that we made no assumptions on the relationship between $c_1 + c_2$ and $p_1 + p_2$. In the unlikely case where $c_1 + c_2 > p_1 + p_2$, (1) still remains valid if we extend the definition of $\bar{F}^{-1}(\cdot)$ such that $\bar{F}^{-1}(z) = 0$ if $z > 1$. Throughout this paper, we keep the extended definition of $\bar{F}^{-1}(\cdot)$. Similarly, we extend the definition of $F^{-1}(\cdot)$ such that $F^{-1}(z) = 0$ if $z < 0$.

We now examine the capacity investment level under the assumption that each firm sets an investment level to maximize its own expected profit, taking into consideration the capacity decision of the other firm. When both firms know c_i, p_i, and $F(\cdot)$, the capacity investment game between the two firms is a special case of the assembler-as-leader game studied by Wang and Gerchak [24]. Define

$$q^o(c_1) = \min\left\{F^{-1}\left(\frac{p_1 - c_1}{p_1}\right), F^{-1}\left(\frac{p_2 - c_2}{p_2}\right)\right\}. \quad (2)$$

Proposition 1 characterizes the Nash equilibrium capacity investment levels and follows directly from Proposition 1 of Wang and Gerchak [24].

PROPOSITION 1: With complete information, for any given $q \le q^o(c_1)$, (q, q) constitutes a Nash equilibrium, and the equilibrium $(q^o(c_1), q^o(c_1))$ is Pareto optimal.

In this paper, whenever there are multiple equilibria, we will focus on the Pareto optimal equilibrium.

2.2. Bayesian Capacity Investment Game

We now examine the capacity investment levels when firm 1's capacity investment cost c_1 is private information. For this case, we assume that firm 2 possesses a prior distribution $\Phi(c_1)$ with a prior density function $\phi(c_1)$ over c_1 with support $[\underline{c}_1, \bar{c}_1]$. As in the standard practice in studying Bayesian games (see, e.g., Fudenberg and Tirole [10]), we assume that Firm 1 knows $\Phi(\cdot)$, i.e., the prior is a common knowledge. We first make two assumptions regarding the problem parameters:

- Assumption 1: $p_2 > c_2$.
- Assumption 2: $(p_1 - \bar{c}_1)/p_1 < (p_2 - c_2)/p_2$.

Assumption 1 ensures that firm 2 is willing to invest in the capacity. Note that we made no assumptions regarding the relationship between p_1 and c_1. Assumption 2 ensures that the margin of firm 1 is lower than that of firm 2 at least for the highest possible cost of firm 1.[1]

In the literature on optimal contracting with adverse selection, it is usually assumed that the continuous prior is log-concave (Corbett and de Groote [8]) for developing insights with respect to the impact of key problem parameters on system performance. In contrast, we use a condition on the prior distribution $\Phi(c)$ and the prior density $\phi(c)$ (Assumption 3), hitherto unexplored in the literature, to develop monotonicity results on the impact of key problem parameters. To state Assumption 3 we first define

$$H(c) = c + \frac{\Phi(c)}{\phi(c)} \text{ and} \quad (3)$$

$$h(c) = H'(c). \quad (4)$$

[1] It turns out that this assumption is not critical to our results. We make this assumption only to faciliate the exposition. After Proposition 2, we comment on how our results change if this assumption is voilated.

- Assumption 3: Firm 2's prior distribution $\Phi(\cdot)$ of c_1 satisfies that $H(c_1)$ increases[2] in c_1, that is, $c_1 + \Phi(c_1)/\phi(c_1)$ increases in c_1.

A sufficient condition for Assumption 3 to hold is that $\Phi(c)/\phi(c)$ is increasing in c, which, since it corresponds to the prior having a *decreasing* reversed hazard rate (DRH), is equivalent to requiring log-concavity of $\Phi(\cdot)$. To facilitate our exposition, we call a distribution $\Phi(\cdot)$ possesses the *decreasing generalized reversed hazard rate (DGRH)* when $H(\cdot)$ is increasing because it can be regarded as a generalization of the classic decreasing reversed hazard rate property (a distribution $\Phi(\cdot)$ has a *decreasing* reversed hazard rate if $\Phi(\cdot)/\phi(\cdot)$ is *increasing*). Note that we did not define a generalized reversed hazard rate of a random variable in order to define the DGRH property.

The DGRH assumption is not restrictive. Many common distributions satisfy this assumption; see, e.g., Corbett [7] for a discussion. One should note that there are some distributions that possess the DGRH property but not DGH; one such example is the student distribution with a degree of freedom of one. Our generalization of DRH to DGRH is similar to the generalization of Increasing Failure Rate (IFR) to IGFR by Lariviere and Porteus [15]. A continuous random variable η is IFR (IGFR) if and only if its failure rate $f_\eta(x)/\bar{F}_\eta(x)$ (generalized failure rate $xf_\eta(x)/\bar{F}_\eta(x)$, respectively) increases in x. There is a natural relationship between DRH and IFR: "c_1 is DRH" is equivalent to "$\bar{c}_1 - c_1$ is IFR"; see, e.g., Ha [12]. However, the relationship between DGRH and IGFR is not clear. Note that the student distribution with a degree of freedom of 1 is DGRH but not IGFR.

When firm 2 does not know firm 1's cost, the investment game becomes a Bayesian game in which firm 1's "type" is continuously distributed in $[\underline{c}_1, \bar{c}_1]$ and firm 2's type is degenerate at c_2.[3] Let $q_1(c_1)$ denote firm 1's capacity investment level when its cost is c_1 and q_2 denote firm 2's capacity

[2] Throughout this paper, we use the terms positive, negative, increasing, decreasing, and concave all in the nonstrict sense.

[3] In the Bayesian game, we assume that only firm 1's capacity investment cost is private information. Another possible scenario might be that both firms are uncertain about each other's cost. However, we use the Bayesian investment game only to provide a benchmark for the impact of contracting on the system efficiency. In the principle–agent framework, which we use to study the impact of contracting on system efficiency, the informational structure regarding firm 2's cost is irrelevant because firm 1 only gets to pick an item from the menu of contract to maximize its expected profit. Therefore, in our model, we assume that firm 2's cost is degenerate at c_2. In any case, although cumbersome, Proposition 2 can be extended to the case when both firms' cost information is incomplete.

investment level. The payoffs of both firms satisfy

$$G_1(q_1(c_1), q_2) = -c_1 q_1(c_1) + p_1 E_\xi[q_1(c_1) \wedge q_2 \wedge \xi], \quad (5)$$

$$G_2(q_1(c_1), q_2) = -c_2 q_2 + p_2 E_{c_1}\{E_\xi[q_1(c_1) \wedge q_2 \wedge \xi]\}. \quad (6)$$

Then, $\left(q_1^{AI}(c_1), q_2^{AI}\right)$ is a Bayesian Nash equilibrium if $q_1^{AI}(c_1) \in \arg\max_{q_1(c_1)}\{G_1(q_1(c_1), q_2^{AI})\}$ and $q_2^{AI} \in \arg\max_{q_2}\{G_2(q_1^{AI}(c_1), q_2)\}$. (The superscript "$AI$" represents Asymmetric Information.) Proposition 2 characterizes the pure-strategy Bayesian Nash equilibrium for the Bayesian capacity game. (All proofs are in the Appendix.)

PROPOSITION 2: (i) There exists a unique $c_1^o \in (\underline{c}_1, \bar{c}_1)$ such that $c_1^o \Phi(c_1^o)/p_1 = c_2/p_2$. (ii) For any q_2, define

$$q_1^r(q_2|c_1) = \begin{cases} \bar{F}^{-1}(c_1/p_1), & \text{if } c_1 \geq p_1 \bar{F}(q_2), \\ q_2, & \text{otherwise.} \end{cases}$$

Then, for all $q_2 \leq \bar{F}^{-1}(c_1^o/p_1)$, $(q_1^r(q_2|c_1), q_2)$ constitutes a pure-strategy Bayesian Nash equilibrium. Further, the Pareto optimal pure-strategy Bayesian Nash equilibrium satisfies

$$q_1^{AI}(c_1) = \begin{cases} \bar{F}^{-1}(c_1/p_1), & \text{if } c_1 \geq c_1^o, \\ \bar{F}^{-1}(c_1^o/p_1), & \text{otherwise.} \end{cases} \quad (7)$$

$$q_2^{AI} = \bar{F}^{-1}(c_1^o/p_1). \quad (8)$$

It is easy to verify that $c_1 \geq c_1^o$ is equivalent to $\bar{F}^{-1}(c_1/p_1) \leq q_2^{AI}$. From Proposition 2, we see that firm 1 chooses its capacity investment level by first solving a newsvendor problem. If the resulting capacity level $\bar{F}^{-1}(c_1/p_1)$ is higher than firm 2's equilibrium capacity investment level q_2^{AI}, firm 1 will invest in q_2^{AI}; otherwise, firm 1's capacity investment level is $\bar{F}^{-1}(c_1/p_1)$. Because $q_1^{AI}(c_1) \leq q_2^{AI}$, the effective capacity of the system is $q_1^{AI}(c_1)$. To facilitate our exposition, we define $q^{AI}(c_1) = q_1^{AI}(c_1)$ as the effective capacity investment level of the Bayesian capacity game. One should note that Assumption 2 is used to ensure the existence of $c_1^o \in (\underline{c}_1, \bar{c}_1)$. When Assumption 2 is violated, firm 1's margin is always higher than firm 2's margin. It is easy to verify that both firms, in that case, would invest in $\bar{F}^{-1}(c_2/p_2)$, as in the capacity game with complete information. This is so because it would be common knowledge that firm 2 was the lower-margin firm, and its margin would be common knowledge as well. $q^{AI}(c_1)$ will be replaced by $\bar{F}^{-1}(c_2/p_2)$.

Comparing (7) with (2), we see that $q^{AI}(c_1) \leq q^o(c_1)$ with the strict inequality holding for $c_1 < c_1^o$. Thus, system efficiency worsens because of information asymmetry. Further, firm 2 invests in unnecessary capacities when $c_1 > c_1^o$. Clearly, from (1) and (2) the joint expected profit of the two firms would be higher if they agree to invest $q^*(c_1)$, or, for that matter, any value between $q^{AI}(c_1)$ and $q^*(c_1)$, instead of $q^{AI}(c_1)$. Because firm 1 is worse off by choosing to invest $q^*(c_1)$ instead of $q^{AI}(c_1)$, a subsidy, from firm 2 to firm 1, would be required for firm 1 to commit to the investment level of $q^*(c_1)$ or any investment level not equal to $q^{AI}(c_1)$. Therefore, the capacity collaboration contract between the two firms would comprise two parts: a payment from firm 2 to firm 1 and a commitment by firm 1 to a certain level of investment in capacity.

In this paper, we study how a contract can improve the system's efficiency and the firms' expected profits using the principal–agent framework. Within this framework, firm 2 acts as a principal and offers a contract to firm 1; firm 1 determines whether it would accept the contract. If the contract is rejected, both firms pursue other investment opportunities, ensuring profits of $R_1(c_1)$ and R_2, respectively. $R_1(c_1)$ is the reservation profit of firm 1, which depends on firm 1's type. Firm 2 has a reservation profit level R_2. From the revelation principle (Myerson [18]), a contract can be summarized by a menu $\{q(c_1), r(c_1)\}$ such that firm 1's best response is to reveal its true cost c_1 and receive a profit above $R_1(c_1)$. In the contract, $q(c_1)$ specifies the capacity commitment firm 1 must make if it accepts the contract, and $r(c_1)$ is the payment firm 2 makes to firm 1 for its commitment.

2.3. The Optimal Contract with Information Asymmetry

Firm 2 chooses the optimal menu of contracts to maximize its expected profit. As in [9], we assume a cutoff level policy and therefore firm 2's problem may be formulated as follows:

$$\max \int_{\underline{c}_1}^{\beta} [-c_2 q(c_1) + (p_1 + p_2)E_\xi[q \wedge \xi] - r(c_1)]\phi(c_1)dc_1 + \int_{\beta}^{\bar{c}_1} R_2 \phi(c_1)dc_1 \quad (9)$$

$$s.t. \quad q = \arg\max_q\{-c_1 q + r(q)\}, \text{ for all } c_1 \in [\underline{c}_1, \beta] \text{ (IC)} \quad (10)$$

$$-c_1 q + r(q) \geq R_1(c_1), \text{ for all } c_1 \in [\underline{c}_1, \beta]. \text{ (IR)} \quad (11)$$

$$\underline{c}_1 \leq \beta \leq \bar{c}_1. \quad (12)$$

Constraint (10) is the *incentive-compatibility* (IC) constraint and accounts for firm 1's selection of the capacity investment level that maximizes its expected profit. Constraint (11) is the individual-rationality (IR) constraint and ensures firm 1's participation for all $c_1 \in [\underline{c}_1, \beta]$. The objective function (9) executes firm 2's cutoff policy: If firm 2 stands to earn less than R_2 by collaborating with firm 1, firm 2 will refuse to do so. Note that, in addition to the contract menu $\{q(c_1), r(c_1)\}$, β is also a decision variable in the contract design problem. Following Corollary 1, we discuss how optimal β can be established.

We first characterize the incentive-compatibility constraint. Presented with a contract $\{q(c), r(c)\}$, firm 1 reveals its investment cost by choosing c to maximize $\pi_1(c_1, c) \equiv -c_1 q(c) + r(c)$. Taking the first-order derivative of $\pi_1(c_1, c)$ with respect to c yields the first-order condition $\partial\pi_1(c_1, c)/\partial c = -c_1 q'(c) + r'(c) = 0$. Setting $c = c_1$ yields the incentive-compatibility constraint:

$$r'(c_1) = c_1 q'(c_1). \tag{13}$$

Note that (13) is only a necessary condition for firm 1 to reveal its true cost. In the Appendix (Lemma 6), we demonstrate that (13) is also a sufficient condition for firm 1 to reveal its true cost under the optimal contract. Given (13), firm 2's problem (9)–(11) can be reformulated as

$$\max \int_{\underline{c}_1}^{\beta} [-c_2 q(c_1) + (p_1 + p_2) E_\xi[q \wedge \xi] - r(c_1)]\phi(c_1) dc_1 + \int_{\beta}^{\bar{c}_1} R_2 \phi(c_1) dc_1$$

$$s.t. \quad r'(c_1) = c_1 q'(c_1), \text{for all } c_1 \in [\underline{c}_1, \beta] \text{ (IC)}, \tag{14}$$

$$- c_1 q(c_1) + r(c_1) \geq R_1(c_1), \text{for all } c_1 \in [\underline{c}_1, \beta]. \tag{15}$$

Proposition 3 characterizes the optimal contract firm 2 should offer for a fixed $\beta \in [\underline{c}_1, \bar{c}_1]$.

PROPOSITION 3: For a fixed $\beta \in [\underline{c}_1, \bar{c}_1]$, firm 2's optimal capacity purchase contract $(\hat{q}(c_1), \hat{r}(c_1))$ under asymmetric information satisfies

$$\bar{F}(q(c_1)) = \frac{c_1 + c_2 + \Phi(c_1)/\phi(c_1)}{p_1 + p_2}, \tag{16}$$

$$r(c_1) = \int_{c_1}^{\hat{c}_1} \frac{ch(c)}{(p_1 + p_2) f(q(c))} dc + R_1(\hat{c}_1) + \hat{c}_1 q(\hat{c}_1), \tag{17}$$

where $h(c)$ is defined in (4), and $\hat{c}_1$ is the minimizer of $-c_1\hat{q}(c_1) + \hat{r}(c_1) - R_1(c_1)$ over $[\underline{c}_1, \beta]$.

Define $\hat{\pi}_1^{AI}(c_1)$ as the profit of firm 1 under the optimal purchase contract with asymmetric information for a given capacity investment cost $c_1 \in [\underline{c}_1, \beta]$ of firm 1. Similarly, define $\hat{\pi}_2^{AI}$ as firm 2's expected profit under optimal purchase contract with asymmetric information. Corollary 1 follows directly from Proposition 3 and establishes some monotonicity results.

COROLLARY 1: Under the optimal purchase contract, (i) $\hat{q}'(c_1) = -h(c_1)/[(p_1 + p_2) f(\hat{q}(c_1))]$, (ii) $\hat{r}'(c_1) = -c_1 h(c_1)/[(p_1 + p_2) f(\hat{q}(c_1))]$, (iii) $\partial\hat{\pi}_1^{AI}(c_1)/\partial c_1 = -\hat{q}(c_1)$, and (iv) $\partial\hat{\pi}_2^{AI}(c_1)/\partial c_1 = \hat{q}'(c_1)\Phi(c_1)/\phi(c_1)$, where $\hat{\pi}_2^{AI}(c_1)$ is defined as firm 2's profit conditional on c_1.

From Corollary 1, we see that $\hat{q}'(c_1) \leq 0$. Therefore, as firm 1's cost increases, the capacity investment level becomes lower. Similarly, firm 2's payment to firm 1 also decreases in firm 1's marginal cost. Further, the profit of firm 2 decrease in the first firm's capacity investment cost, which implies that a cutoff policy is optimal for firm 2 (see, for example, [12]). The cutoff point, above which firm 2 refuses to offer a contract, satisfies $\hat{\pi}_2^{AI}(\beta^*) = R_2$. Clearly, if $\hat{\pi}_2^{AI}(\bar{c}_1) \geq R_2$, a full participation contract is optimal for firm 2. In the remainder of the paper, we assume that $\hat{\pi}_2^{AI}(\bar{c}_1) \geq R_2$ without loss of generality.

COROLLARY 2: Define $\Pi(c_1) = \hat{\pi}_1^{AI}(c_1) - R_1(c_1)$ as firm 1's information rent. Then, $\Pi'(c_1) = -\hat{q}(c_1) + R'(c_1)$. Further, if $R'(c_1) \leq \hat{q}(c_1)$ for $c_1 \in [\underline{c}_1, \bar{c}_1]$, then $\Pi(c_1)$ decreases in c_1, the individual rationality constraint is binding at $\bar{c}_1$, and $\hat{r}(c_1) = \int_{c_1}^{\bar{c}_1} ch(c)/[(p_1 + p_2) f(\hat{q}(c))]dc + R_1(\bar{c}_1) + \bar{c}_1\hat{q}(\bar{c}_1)$.

It is well known that when the agent's reservation profit $R_1(c_1)$ does not vary with its type, the individual rationality constraint is binding at its highest cost, that is, $\hat{c}_1 = \bar{c}_1$, and firm 1's information rent decreases in its cost [14]. Corollary 2 demonstrates that this result would hold for our problem if the optimal contract increases both firms' capacity investment levels. In Section 3.1, we demonstrate that the individual rationality constraint can be binding at interior points of the range for c_1 and that firm 1's information rent is not necessarily monotone in its cost.

The optimal contract $(\hat{q}(c_1), \hat{r}(c_1))$ can be implemented in different ways. A straightforward way for firm 2 to implement the optimal contract is to make a lump sum payment to firm 1, $\hat{r}(c_1)$, for firm 1's capacity investment commitment, $\hat{q}(c_1)$, before demand materializes and bear all risk. Instead of purchasing capacity from firm 1 by making a lump sum payment, firm 2 can also institute a revenue sharing contract with subsidy to achieve the second-best solution. To see this, note that $\hat{r}(c_1)$ can be written as $\hat{r}(c_1) = \theta(p_1+p_2)E[\hat{q}(c_1)\wedge\xi]+\hat{s}(c_1)$ for some $\theta \in [0, 1]$. It follows that firm 2 can implement the optimal contract by offering a contract $(\theta, \hat{q}(c_1), \hat{s}(c_1))$ that specifies the portion of the revenue (θ) firm 1 receives from offering the integrated service, firm 1's capacity commitment $\hat{q}(c_1)$, and a side payment $\hat{s}(c_1)$. Note that firm 1 may accrue revenues by receiving the payment from customers directly, and firm 2 would then only choose $\hat{s}(c_1)$. Further, $\hat{s}(c_1)$ can take negative values, so the side payment could be made both ways. In particular, when $\theta = p_1/(p_1 + p_2)$, firm 1 receives the same contribution margin p_1 for each service provided as in the Bayesian game and $\hat{s}(c_1)$ (if it is positive) can be regarded as a subsidy firm 2 pays to encourage firm 1 to invest more in its capacity. This contract is similar to the collaboration between the two cotton processing firms discussed in Section 1, where both firms receive payments from farmers directly and the seed processing firm subsidizes the fiber processing firm's capacity buildup.

3. IMPACT OF CONTRACTING WITH INFORMATION ASYMMETRY

We now study the impact of information asymmetry and the coordination role of the contract. Proposition 4 characterizes the impact of information asymmetry.

PROPOSITION 4: (i) Under asymmetric information, the capacity investment levels satisfy $\hat{q}(c_1) = q^*\big(c_1 + \frac{\Phi(c_1)}{\phi(c_1)}\big) < q^*$ for $c_1 \in [\underline{c}_1, \bar{c}_1]$. (ii) Firm 1's profit with information asymmetry is at least as high as with full information: $\hat{\pi}_1^{AI}(c_1) \geq \hat{\pi}_1^{FI}(c_1)$. (iii) Firm 2's expected net profit is lower with information asymmetry: $\hat{\pi}_2^{AI} \leq \hat{\pi}_2^{FI}$.

Claim (i) states that the capacity investment level under optimal contract with information asymmetry is always lower than the first-best capacity level. Therefore, information asymmetry has a negative impact on system efficiency. In addition, claim (i) establishes the relationship between the optimal contract and the first-best capacity level. The optimal contract for firm 1 with cost c_1 is the same as the first-best capacity level when its cost is augmented to $c_1 + \Phi(c_1)/\phi(c_1)$. Therefore, information asymmetry effectively distorts firm 1's capacity investment cost upward. When $\Phi(c_1)/\phi(c_1)$ increases in c_1, the distortion becomes higher when c_1 is larger. Claims (ii) and (iii) establish results on welfare implications of the optimal contract. Firm 1 earns rent due to information asymmetry. However, the joint profit as well as firm 2's profit is lower due to information asymmetry.

Note that it is possible for $\Phi(c_1)/\phi(c_1)$ to decrease in c_1 while $c_1 + \Phi(c_1)/\phi(c_1)$ increases in c_1 (the student distribution with a degree of freedom of 1 satisfies such a property). For this type of prior, firm 1's profit will decrease in c_1, but not as fast as in the full information scenario.

We next study the coordination role of contracting. With asymmetric information, one cannot achieve the first-best capacity level. However, as will be shown, at least for some values of firm 1's cost, contracting can bring the capacity level closer to the first-best solution.

PROPOSITION 5: There exists $c_1^c \in (\underline{c}_1, \bar{c}_1]$ such that $\hat{q}(c_1) \geq q_1^{AI}(c_1)$ for $c_1 \in [\underline{c}_1, c_1^c]$.

Therefore, when firm 1's cost is low enough, contracting ensures firm 1 to invest more in its capacity. So the system efficiency is improved. Note that it is possible that $c_1/(p_1 + p_2) < [c_1 + c_2 + \Phi(c_1)/\phi(c_1)]/(p_1 + p_2)$ for some c_1. Therefore, the capacity level with contracting might be less than the capacity level in the Bayesian game (see Propositions 2 and 3). For this case, both firms are still better off with contracting because the uncertainty regarding firm 1's capacity investment is removed once a commitment is made by firm 1. In summary, contracting improves the system efficiency in two different ways. One one hand, it may bring the system's capacity level closer to the first-best solution. On the other hand, it rules out the possible overinvestment by firm 2 because it removes the uncertainty firm 2 might have regarding firm 1's capacity investment level. Clearly, firm 2's belief on firm 1's cost also affects the capacity level and the welfare of both firms; we study this for a special case to gain additional insights.

3.1. A Special Case: Uniform Demand and Uniform Prior

Assume $\xi \sim U[0,1]$ and that firm 2's prior on c_1 is uniformly distributed with $\underline{c}_1 = \mu - \varepsilon$ and $\bar{c}_1 = \mu + \varepsilon$, where μ is the mean value of the prior c_1 and 2ε is its range. Then, the CDF, its complement, and the PDF of the demand are, respectively, $F(x) = x$, $\bar{F}(x) = 1 - x$, and $f(x) = 1$. And the CDF and the PDF of the prior are, respectively, $\Phi(x) = (x - \mu + \varepsilon)/(2\varepsilon)$ and $\phi(x) = 1/(2\varepsilon)$. Therefore, $H(x) = x + \Phi(x)/\phi(x) = 2x - \mu + \varepsilon$ and $h(x) = H'(x) = 2$. For this case, the first-best capacity level satisfies $q^*(c_1) = 1-(c_1+c_2)/(p_1+p_2)$ from (1). Applying Propositions 2 and 3, we obtain the (effective) capacity level of the Bayesian game and the capacity investment level with the optimal contract:

$$q^{AI}(c_1) = \begin{cases} 1 - c_1/p_1, & \text{if } c_1 \geq c_1^o, \\ 1 - c_1^o/p_1, & \text{otherwise,} \end{cases}$$

$$\hat{q}(c_1) = 1 - (2c_1 + c_2 - \mu + \varepsilon)/(p_1 + p_2), \qquad (18)$$

where $c_1^o \in [\mu-\varepsilon, \mu+\varepsilon]$ solves $c_1^o(c_1^o-\mu+\varepsilon)/p_1 = 2\varepsilon c_2/p_2$.

We now compare $q^{AI}(c_1)$ and $\hat{q}(c_1)$ graphically to examine the impact of the optimal contract on the system's capacity. As shown in Figure 1, $\hat{q}(c_1)$ (represented by the dotted line) is a straight line and $q^{AI}(c_1)$ (represented by the solid line) is two segments of straight lines with $\hat{q}(\mu - \varepsilon) > q^{AI}(\mu - \varepsilon)$ (Proposition 5). So, $\hat{q}(c_1)$ crosses $q^{AI}(c_1)$ at most twice in $[\mu - \varepsilon, \mu + \varepsilon]$ and the crossing of these two curves can be fully characterized by the values of $q^{AI}(\mu+\varepsilon)$ and $\hat{q}(\mu+\varepsilon)$, and $q^{AI}(c_1^o)$ and $\hat{q}(c_1^o)$. There are three scenarios.

Scenario 1. $q^{AI}(\mu + \varepsilon) \geq \hat{q}(\mu + \varepsilon)$, i.e., $(\mu + \varepsilon)/p_1 \leq (\mu + 3\varepsilon + c_2)/(p_1 + p_2)$. For this scenario, $\hat{q}(c_1)$ crosses $q^{AI}(c_1)$ only once from above. Denote c_a as the crossing point. Then, $q^{AI}(c_1) < \hat{q}(c_1)$ for $c_1 < c_a$ and $q^{AI}(c_1) >$

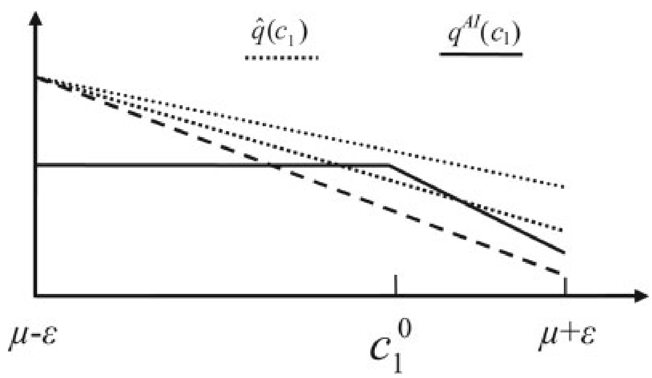

Figure 1. Comparison of capacity levels with or without contracting.

$\hat{q}(c_1)$ for $c_1 > c_a$. Recall that firm 1's information rent $\Pi(c_1)$ satisfies $\Pi'(c_1) = -\hat{q}(c_1) + q^{AI}(c_1)$ from Proposition 6. So for this scenario, firm 1's information rent decreases in c_1 for $c_1 < c_a$ and increases in c_1 for $c_1 > c_a$.

Scenario 2. $q^{AI}(\mu+\varepsilon) < \hat{q}(\mu+\varepsilon)$ and $q^{AI}(c_1^o) < \hat{q}(c_1^o)$. For this scenario, $\hat{q}(c_1) \geq q^{AI}(c_1)$ for all c_1 and firm 1's information rent decreases in c_1.

Scenario 3. $q^{AI}(\mu+\varepsilon) < \hat{q}(\mu+\varepsilon)$ and $q^{AI}(c_1^o) \geq \hat{q}(c_1^o)$. For this scenario, $\hat{q}(c_1)$ crosses $q^{AI}(c_1)$ twice. For this case, denote c_b and c_c as the crossing points of the two curves. Then $\hat{q}(c_1) \geq q^{AI}(c_1)$ when $c_1 \leq c_b$ or $c_1 \geq c_c$; $\hat{q}(c_1) \leq q^{AI}(c_1)$ when $c_1 \in [c_b, c_c]$. It follows that firm 1's information rent first decreases in its cost (up to c_b), then increases in its cost (up to c_c), and finally decreases again.

We next study the impact of the prior on both firms' welfare by examining Scenario 2. In order to develop insights, we further assume that firm 1's reservation profit $R_1(c_1)$ is its expected profit under the Bayesian game studied in Section 2. For this scenario, $\hat{q}(c_1) \geq q_1^{AI}(c_1)$ for $c_1 \in [\underline{c}_1, \bar{c}_1]$ and $-c_1\hat{q}(c_1) + \hat{r}(c_1) - R_1(c_1)$ would obtain its minimum at $c_1 = \bar{c}_1 = \mu + \varepsilon$ (Corollary 2), i.e., $\hat{c}_1 = \mu + \varepsilon$. From Proposition 3, the optimal contract would satisfy

$$\hat{q}(c_1) = 1 - (2c_1 + c_2 - \mu + \varepsilon)/(p_1 + p_2), \tag{19}$$

$$\hat{r}(c_1) = \int_{c_1}^{\mu+\varepsilon} \frac{2c}{p_1 + p_2} dc + R_1(\mu+\varepsilon) + (\mu+\varepsilon)\hat{q}(\mu+\varepsilon). \tag{20}$$

With optimal contract, the profits of the two firms can be written as

$$\hat{\pi}_1^{AI}(c_1) = -c_1\hat{q}(c_1) + \int_{c_1}^{\mu+\varepsilon} \frac{2c}{p_1+p_2} dc + R_1(\mu+\varepsilon) + (\mu+\varepsilon)\hat{q}(\mu+\varepsilon), \tag{21}$$

$$\hat{\pi}_2^{AI} = \int_{\mu-\varepsilon}^{\mu+\varepsilon} [-c_2\hat{q}(c_1) + (p_1+p_2)E_\xi[\hat{q}(c_1) \wedge \xi] - \hat{r}(c_1)] \frac{1}{2\varepsilon} dc_1. \tag{22}$$

Substituting (19) and (20) into (22) and simplifying (see Appendix for the detail), we obtain

$$\frac{\partial \hat{\pi}_2^{AI}}{\partial \varepsilon} = \frac{(c_2 + \mu + \varepsilon/3)}{p_1 + p_2} - \frac{\mu+\varepsilon}{p_1}, \tag{23}$$

$$\frac{\partial \hat{\pi}_2^{AI}}{\partial \mu} = \frac{(c_2 + \mu + \varepsilon)}{p_1 + p_2} - \frac{\mu+\varepsilon}{p_1}. \tag{24}$$

Because $(c_2 + \mu + \varepsilon)/(p_1 + p_2) \leq (\mu+\varepsilon)/p_1$ based on Assumption 2, we know that

$$\frac{\partial \hat{\pi}_2^{AI}}{\partial \varepsilon} \leq 0 \quad \text{and} \quad \frac{\partial \hat{\pi}_2^{AI}}{\partial \mu} \leq 0 \tag{25}$$

from (23) and (24). Therefore, when the prior is uniform, the higher the uncertainty in the private cost information of firm 1 (i.e., higher ε), the lower the expected profit of firm 2. Moreover, as the prior increases stochastically (i.e., μ becomes greater), the expected profit of the second firm decreases.

We now examine the impact of the changes in the prior on firm 1's profit. Straightforward algebra shows that

$$\frac{\partial \hat{\pi}_1^{AI}(c_1)}{\partial \varepsilon} = \frac{\mu+\varepsilon}{p_1} - \frac{c_1 + c_2 + 2(\mu + 2\varepsilon)}{p_1 + p_2} \text{ and} \tag{26}$$

$$\frac{\partial \hat{\pi}_1^{AI}(c_1)}{\partial \mu} = \frac{\mu+\varepsilon}{p_1} - \frac{c_1 + c_2 + 2\varepsilon}{p_1 + p_2}. \tag{27}$$

From our assumption, $(\mu+\varepsilon)/p_1 \geq (c_1+c_2+2\varepsilon)/(p_1+p_2)$. So, $\partial\hat{\pi}_1^{AI}(c_1)/\partial\mu \geq 0$. Therefore, an overestimate of c_1 always benefits firm 1. From (26), we see that $\partial\hat{\pi}_1^{AI}(c_1)/\partial\varepsilon < 0$ if and only if $(\mu+\varepsilon)/p_1 - 2(\mu+2\varepsilon)/(p_1+p_2) < (c_1+c_2)/(p_1+p_2)$. So, for certain sufficiently high range of μ and ε, it is possible that both $\hat{\pi}_1^{AI}(c_1)$ and $\hat{\pi}_2^{AI}$ decrease in ε. If this is the case, firm 1 would have the incentive to signal its cost information to firm 2 so that ε can be reduced to benefit both firms. We summarize these results in the following proposition.

PROPOSITION 6: Assume $\xi \sim U[0,1]$, $c_1 \sim U[\mu-\varepsilon, \mu+\varepsilon]$, $q^{AI}(\mu+\varepsilon) < \hat{q}(\mu+\varepsilon)$, and $q^{AI}(c_1^o) < \hat{q}(c_1^o)$. Then, (i) $\hat{\pi}_2^{AI}$ decreases in μ and ε, and $\hat{\pi}_1^{AI}(c_1)$ increases in μ. (ii) $\hat{\pi}_1^{AI}(c_1)$ decreases in ε if $(\mu+\varepsilon)/p_1 - 2(\mu+\varepsilon)/(p_1+p_2) < (c_1+c_2)/(p_1+p_2)$; it increases in ε if $(\mu+\varepsilon)/p_1 - 2(\mu+\varepsilon)/(p_1+p_2) \geq (c_1+c_2)/(p_1+p_2)$.

Based on Proposition 6, we see that firm 2's welfare is hurt by uncertainty and overestimate of firm 1's cost. By contrast, the impact of the information asymmetry on firm 1's welfare depends on the relationship between the service margin and the cost prior. If the margin of the service is relatively high, i.e., $(c_1+c_2)/(p_1+p_2) \le (\mu+\varepsilon)/p_1 - 2(\mu+\varepsilon)/(p_1+p_2)$, firm 1 will welcome the uncertainty and would like firm 2 to overestimate firm 1's cost up to a certain point. When the margin is relatively low, i.e., $(c_1+c_2)/(p_1+p_2) \ge (\mu+\varepsilon)/p_1 - 2(\mu+\varepsilon)/(p_1+p_2)$, firm 1's welfare will be hurt by information uncertainty. (Interestingly, when $p_1 = p_2$, this inequality always holds; for this case, firm's payoff is always hurt by the increase in uncertainty regarding firm 1's investment cost.) The counter-intuitive result that higher information uncertainty hurts both the principal (firm 2) and the agent (firm 1) can be explained as follows. From (18), we see that as information uncertainty ε becomes higher, the system's capacity investment level $\hat{q}(c_1)$ becomes lower. Consequently, even though the agent may be better off with higher information uncertainty because it may accrue higher information rent, the system's efficiency is hurt by higher information uncertainty and the system's total expected profit becomes smaller as information uncertainty becomes higher. As a result, higher information uncertainty may hurt the welfare of both firms. Therefore, obtaining better cost information by firm 2 would not only benefit firm 2, but also benefit firm 1, for a wide range of the service margin.

4. CONCLUSIONS AND DISCUSSION

We have studied the issues of incentive and information in designing a capacity collaboration contract between two firms with contingent capacities. Without a contract, system efficiency is lost because of the firms' marginal differentials, demand uncertainty, and information asymmetry. Further, the firm with the higher margin might overinvest in its capacity as a result of information asymmetry. When the capacity investment cost of one firm is private, we derive the optimal contracts and characterize their properties under a general assumption on the prior of the private cost information. In general, the first-best capacity level cannot be achieved; however, the optimal contract brings the capacity investment level closer to the first-best level at least when the agent's cost is lower. It is possible that both firms invest less in capacity with the optimal contract than without a contract. However, when this happens, the optimal contract improves the system efficiency because the overinvesting effect is removed by contracting between the two firms.

As in standard contract theory research, the firm with private information accrues information rent. Therefore, firm 1 will be expected to hold its cost information private. In addition, we also establish in a special case that the principal's profit will deteriorate as the uncertainty regarding the private cost information increases. Therefore, the principal may be prepared to pay for a better estimate of the prior. Clearly, the economics of contracts with third parties (consultants) for obtaining a better estimate of the prior would be of interest to the principal. The fact that the agent's profit may increase with uncertainty in its private investment cost, when the profit margin of the integrated service is high, could be a factor in the agent's decision on whether to collaborate with the principal in revealing some private cost information and how much to charge the principal for revealing its private information.

In our model, we assume the contribution margin for each individual capacity is known exogenously and the total contribution margin is the sum of the two individual contribution margins. This is true for the situations where both firms interact with customers directly and customers pay each firm separately, such as in the cotton processing example or in the collaboration between small and midsize construction contractors. In some situations, firms with complementary expertise pursue to team up in order to offer a new service so that they can charge a premium price for the integrated service. How such price premium is established will be an interesting research area. Another limitation of our research is that we study the impact of information asymmetry using a static game. In practice, the interaction between two firms collaborating on contingent capacities is an on-going process. Therefore, firm 2 can learn firm 1's cost information through its interaction with firm 1, for example, if and when firm 1 rejects the contract. Examining the impact of this learning on the firm's capacity investment decisions and the supply chain efficiency will also be an interesting future research area.

APPENDIX

Proofs of Results

PROOF OF PROPOSITION 2: (i) follows by noting that (a) $c_1\Phi(c_1)/p_1$ is continuous and increases strictly in c_1 for $c_1 \in [\underline{c}_1, \bar{c}_1]$, (b) $\underline{c}_1\Phi(\underline{c}_1)/p_1 = 0 < c_2/p_2$, and (c) $\bar{c}_1\Phi(\bar{c}_1)/p_1 = \bar{c}_1/p_1 > c_2/p_2$ (Assumption 2). We next prove (ii) with a sequence of lemmata (Lemmata 1–5). We first characterize the response function of firm 1 in Lemma 1.

LEMMA 1: Firm 1's response function $q_1^r(q_2|c_1)$, for given c_1 and $q_2 \ge 0$, satisfies

$$q_1^r(q_2|c_1) = \begin{cases} \bar{F}^{-1}(c_1/p_1), & \text{if } \bar{F}^{-1}(c_1/p_1) < q_2, \\ q_2, & \text{otherwise.} \end{cases} \quad (28)$$

PROOF: For given c_1 and $q_2 \ge 0$, firm 1 chooses q_1 to maximize its expected profit $-c_1q_1 + p_1E[q_1 \wedge q_2 \wedge \xi]$. It is easy to see that firm 1's optimal capacity level must fall in $[0, q_2]$. So, firm 1's problem can be formulated as $\max_{q_1\in[0,q_2]}\{-c_1q_1 + p_1E[q_1 \wedge \xi]\}$. Because $-c_1q_1 + p_1E[q_1 \wedge \xi]$ is maximized at $\bar{F}^{-1}(c_1/p_1)$, firm 1's optimal response must satisfy (28). □

Lemma 2 characterizes the response function of firm 2.

LEMMA 2: For given $q_1(c_1)$, firm 2's response satisfies $q_2^r = \min\{q| - c_2 + p_2\Pr[q_1(c_1) > q_2]\Pr[\xi > q_2] \le 0\}$.

PROOF: From (28), we see that at equilibrium, firm 1's capacity investment level is continuous and decreasing with respect to c_1. We therefore examine firm 2's response only for a continuous and decreasing $q_1(c_1)$. Because $G_2(\cdot)$ defined in (6) is concave in q_2 for any given $q_1(c_1)$, firm 2's response is characterized by the first-order condition. We evaluate $\partial G_2/\partial q_2$ using a sample-path argument. Note that a marginal increase in q_2 increases firm 2's revenue (by p_2) only when $q_1(c_1) > q_2$ and $\xi > q_2$. Therefore, the sample-path partial derivative of $G_2(\cdot)$ with respect to q_2 is $-c_2 + p_2 I[q_1(c_1) > q_2 \text{ and } \xi > q_2]$, where $I[\cdot]$ is the indicator function. It follows that

$$\partial G_2/\partial q_2 = -c_2 + p_2 \Pr[q_1(c_1) > q_2] \Pr[\xi > q_2], \tag{29}$$

where we use the fact that c_1 and ξ are independent. So, firm 2's optimal capacity investment level is $q_2^r = \min\{q| - c_2 + p_2 \Pr[q_1(c_1) > q_2]\Pr[\xi > q_2] \le 0\}$. Note that the minimum can always be achieved because $-c_2 + p_2 \Pr[q_1(c_1) > q_2] \Pr[\xi > q_2]$ is upper semi-continuous in q_2. □

LEMMA 3: At the Bayesian Nash equilibrium, $q_2 \le q_2^o = \bar{F}^{-1}(c_1^o/p_1)$.

PROOF: With contradiction. Suppose there exists a Bayesian Nash equilibrium $\left(q_1^{\triangle}(c_1), q_2^{\triangle}\right)$ such that $q_2^{\triangle} > q_2^o = \bar{F}^{-1}\left(c_1^o/p_1\right)$. From Lemma 1, we see that firm 1's capacity investment level at all possible equilibria must satisfy $q_1^r(q_2|c_1) \le \bar{F}^{-1}\ (\underline{c}_1/p_1)$ for all c_1. So, $q_2^{\triangle}$ must be no greater than $\bar{F}^{-1}\ (\underline{c}_1/p_1)$. It follows that $q_2^{\triangle} \in \left(\bar{F}^{-1}\ \left(c_1^o/p_1\right), \bar{F}^{-1}\ (\underline{c}_1/p_1)\right]$. Therefore, there exists $c_1^{\triangle} \in [\underline{c}_1, c_1^o)$ such that $q_2^{\triangle} = \bar{F}^{-1}\left(c_1^{\triangle}/p_1\right)$. Applying Lemma 1, we know that

$$q_1^{\triangle}(c_1) = q_1^r\left(q_2^{\triangle}|c_1\right) = \begin{cases} \bar{F}^{-1}\ (c_1/p_1), & \text{if } c_1 > c_1^{\triangle}, \\ \bar{F}^{-1}\ \left(c_1^{\triangle}/p_1\right), & \text{otherwise.} \end{cases}$$

From (29), we have $\partial G_2\left(q_1^{\triangle}(c_1), q_2^o\right)/\partial q_2 = -c_2 + p_2 \Pr\left[q_1^{\triangle}(c_1) > q_2^o\right]\Pr\left[\xi > q_2^o\right]$. Note that $q_1^{\triangle}(c_1) > q_2^o$ if and only if $c_1 < c_1^o$. (This is true because for $c_1 \le c_1^{\triangle}$, $q_1^{\triangle}(c_1) = \bar{F}^{-1}\ \left(c_1^{\triangle}/p_1\right) = q_2^{\triangle} > q_2^o$; for $c_1^{\triangle} < c_1 < c_1^o$, $q_1^{\triangle}(c_1) = \bar{F}^{-1}\ (c_1/p_1) > \bar{F}^{-1}\ \left(c_1^o/p_1\right) \ge q_2^o$; for $c_1 \ge c_1^o$, $q_1^{\triangle}(c_1) = \bar{F}^{-1}\ (c_1/p_1) \le \bar{F}^{-1}\ \left(c_1^o/p_1\right) = q_2^o$.) It follows that $\partial G_2\left(q_1^{\triangle}(c_1), q_2^o\right)/\partial q_2 = -c_2 + p_2 \Pr\left[c_1 < c_1^o\right]\Pr\left[\xi > q_2^o\right] = -c_2 + p_2\Phi\left(c_1^o\right)\bar{F}\left(q_2^o\right) = -c_2 + p_2\Phi\left(c_1^o\right)c_1^o/p_1 = 0$, where the second and the last equality are based on the definitions of q_2^o and c_1^o, respectively. Therefore, we have found a q_2 value q_2^o such that $q_2^o < q_2^{\triangle}$ and $\partial G_2\left(q_1^{\triangle}(c_1), q_2^o\right)/\partial q_2 \le 0$. This contradicts the fact that $q_2^{\triangle}$ is the smallest q_2 value such $\partial G_2\left(q_1^{\triangle}(c_1), q_2\right)/\partial q_2 \le 0$ from Lemma 2. □

LEMMA 4: $\left(q_1^r(q_2|c_1), q_2\right)$ constitutes a Bayesian Nash equilibrium of the Bayesian game for all $q_2 \le q_2^o = \bar{F}^{-1}\ \left(c_1^o/p_1\right)$.

PROOF: If $q_2 \le \bar{F}^{-1}\ (\bar{c}_1/p_1)$, then $q_1^r(q_2|c_1) = q_2$. Clearly, (q_2, q_2) is a Bayesian Nash equilibrium for all $q_2 \le \bar{F}^{-1}\ (\bar{c}_1/p_1)$. If $q_2 \in \left[\bar{F}^{-1}\ (\bar{c}_1/p_1), \bar{F}^{-1}\ \left(c_1^o/p_2\right)\right]$, then the capacity level $q_1^r(q_2|c_1)$ that maximizes firm 1's payoff is given by (28):

$$q_1^r(q_2|c_1) = \begin{cases} \bar{F}^{-1}\ (c_1/p_1), & \text{if } \bar{F}^{-1}\ (c_1/p_1) < q_2, \\ q_2, & \text{otherwise.} \end{cases} \tag{30}$$

Further, $-c_2 + p_2 \Pr[q_1^r(q_2|c_1) > q_2]\Pr[\xi > q_2] = -c_2 \le 0$ because $q_1^r(q_2|c_1) \le q_2$. For any $\varepsilon > 0$,

$$\begin{aligned} &\partial G_2\left(q_1^r(q_2|c_1), q_2 - \varepsilon\right)/\partial q_2 \\ &= -c_2 + p_2 \Pr\left[q_1^r(q_2|c_1) > q_2 - \varepsilon\right]\Pr[\xi > q_2 - \varepsilon] \\ &> -c_2 + p_2 \Pr\left[q_1^r(q_2|c_1) \ge q_2\right]\Pr[\xi \ge q_2] \\ &= -c_2 + p_2 \Pr\left[\bar{F}^{-1}\ (c_1/p_1) \ge q_2\right]\Pr[\xi \ge q_2] \\ &= -c_2 + p_2 \Pr\left[c_1 \le p_1\bar{F}(q_2)\right]\Pr[\xi \ge q_2] \\ &\ge -c_2 + p_2 \Pr\left[c_1 \le p_1\bar{F}\left(q_2^o\right)\right]\Pr[\xi \ge q_2^o] = 0, \end{aligned}$$

where the first equality is based on (29), the second inequality is based on $q_2 \le q_2^o$, and the last equality is based on the definitions of c_1^o and q_2^o. So $\partial G_2(q_1^r(q_2|c_1), q_2)/\partial q_2 < 0$, and, for any $\varepsilon > 0$, $\partial G_2(q_1^r(q_2|c_1), q_2 - \varepsilon)/\partial q_2 > 0$. Therefore, $q_2 = \min\{q|\partial G_2(q_1^r(q_2|c_1), q)/\partial q_2 \le 0\}$. So q_2 is the best response of firm 2 for a given $q_1^r(q_2|c_1)$ from Lemma 2. And $(q_1^r(q_2|c_1), q_2)$ is a Bayesian Nash equilibrium. □

LEMMA 5: The equilibrium $\left(q_1^r(q_2^o|c_1), q_2^o\right)$ is Pareto optimal.

PROOF: Substituting $\left(q_1^r(q_2|c_1), q_2\right)$ into (5) yields

$$G_1\left(q_1^r(c_1), q_2\right) = \begin{cases} -c_1\bar{F}^{-1}\ (c_1/p_1) + p_1 E_\xi\left[\bar{F}^{-1}\ (c_1/p_1) \wedge \xi\right], & \text{if } \bar{F}^{-1}\ (c_1/p_1) < q_2, , \\ -c_1 q_2 + p_1 E_\xi[q_2 \wedge \xi], & \text{otherwise.} \end{cases}$$

It follows that

$$\frac{\partial G_1}{\partial q_2} = \begin{cases} 0, & \text{if } \bar{F}^{-1}\ (c_1/p_1) < q_2, \\ -c_1 + p_1\bar{F}(q_2), & \text{otherwise.} \end{cases}$$

Note that $-c_1 + p_1\bar{F}(q_2) \ge 0$ if $\bar{F}^{-1}\ (c_1/p_1) \ge q_2$. So, $\partial G_1/\partial q_2 \ge 0$ for all q_2.

Similarly, substituting $\left(q_1^r(q_2|c_1), q_2\right)$ into (6) yields $G_2(q_1(c_1), q_2) = -c_2q_2 + p_2\int_{p_1\bar{F}(q_2)} E_\xi\left[\bar{F}^{-1}\ (c_1/p_1) \wedge \xi\right]\phi(c_1)dc_1 + p_2\int^{p_1\bar{F}(q_2)} E_\xi[q_2 \wedge \xi]\phi(c_1)dc_1$. Taking the first-order derivative of $G_2(\cdot)$ with respect to q_2 and collecting similar terms, we obtain $\partial G_2/\partial q_2 = -c_2 + p_2\int^{p_1\bar{F}(q_2)} \bar{F}(q_2)\phi(c_1)dc_1 = -c_2 + p_2\bar{F}(q_2)\Phi[p_1\bar{F}(q_2)]$. It follows that, for $q_2 \le q_2^o$, $\partial G_2/\partial q_2 \ge -c_2 + p_2\bar{F}\left(q_2^o\right)\Phi\left[p_1\bar{F}\left(q_2^o\right)\right] = 0$, where the equality is based on the definition of q_2^o.

So, for all Bayesian Nash equilibria $\left(q_1^r(q_2|c_1), q_2\right)$, both firms' payoffs increase in q_2. It follows that the equilibrium where q_2 takes its maximum value q_2^o is Pareto optimal. □

We complete the proof of Proposition 2. □

PROOF OF PROPOSITION 3: To solve firm 2's problem, we fix β and introduce $u(c_1) = q'(c_1)$. Ignoring constraint (15) (we will verify it afterward), firm 2's problem can be written as

$$\begin{aligned} &\max \int_{\underline{c}_1}^{\beta} \{-c_2 q + (p_1 + p_2)E_\xi[q \wedge \xi] - r(c_1)\}\phi(c_1)dc_1 \\ &s.t.\ r'(c_1) = c_1 u(c_1), \\ &\quad q'(c_1) = u(c_1). \end{aligned} \tag{31}$$

This is an optimal control problem with two state variables: $r(c_1)$ and $q(c_1)$ and one control variable: $u(c_1)$. Define the Hamiltonian of the associated maximization problem:

$$H = \{-c_2 q + (p_1 + p_2)E_\xi[q \wedge \xi] - r(c_1)\}\phi(c_1) + \lambda_q u(c_1) + \lambda_r c_1 u(c_1).$$

It is well known that $\hat{q}$, $\hat{r}$ must satisfy (see, e.g., Kamien and Schwartz [13, page 133])

$$D_u H = 0\colon \lambda_q + \lambda_r c_1 = 0, \tag{32}$$

$$D_q H + \lambda_q' = 0\colon [-c_2 + (p_1 + p_2)\bar{F}(q(c_1))]\phi(c_1) + \lambda_q' = 0, \tag{33}$$

$$D_r H + \lambda_r' = 0\colon -\phi(c_1) + \lambda_r' = 0. \tag{34}$$

The boundary at $c_1 = \underline{c}_1$ is unconstrained. Hence, the transversality condition at $c_1 = \underline{c}_1$ is $\lambda_r = 0$. Therefore, (34) yields $\lambda_r = \Phi(c_1)$. Substitution for λ_r into (32) yields $\lambda_q = -c_1\Phi(c_1)$, which in turn yields

$\lambda_q' = -c_1\phi(c_1) - \Phi(c_1)$. Substitution for λ_q' into (33) and collecting similar terms yields $\phi(c_1)[-c_1 - c_2 + (p_1+p_2)\bar{F}(q(c_1))] - \Phi(c_1) = 0$. We obtain $-c_1 - c_2 + (p_1+p_2)\bar{F}(q(c_1)) = \Phi(c_1)/\phi(c_1)$.

In order to prove that the optimal contract satisfies (16), we only need to verify that (15) is satisfied. Constraint (15) will definitely be satisfied if $-\hat{c}_1\hat{q}(\hat{c}_1) + \hat{r}(\hat{c}_1) - R_1(\hat{c}_1) = 0$, where $\hat{c}_1$ minimizes $-c_1\hat{q}(c_1) + \hat{r}(c_1) - R_1(c_1)$ in $[\underline{c}_1, \beta]$. Note that once $\hat{q}(c_1)$ is determined, $\hat{r}'(c_1)$ is determined from (31), which implies that $\hat{r}(c_1)$ is determined up to a constant. Therefore, $\hat{c}_1$ that minimizes $-c_1\hat{q}(c_1) + \hat{r}(c_1) - R_1(c_1)$ in $[\underline{c}_1, \beta]$ can be determined. So, if we choose $\hat{r}(\hat{c}_1)$ such that

$$\hat{r}(\hat{c}_1) = \hat{c}_1\hat{q}(\hat{c}_1) + R_1(\hat{c}_1), \tag{35}$$

the individual-rationality constraint will be satisfied for all $c_1 \in [\underline{c}_1, \beta]$, and we prove (16).

Next, taking the derivative of both sides of (16) with respect to c_1 yields

$$-1 + (p_1+p_2)f(q(c_1))q'(c_1) = \left(\frac{\Phi(c_1)}{\phi(c_1)}\right)'. \tag{36}$$

Based on (4), we have $h(c_1) = 1 + \left(\frac{\Phi(c_1)}{\phi(c_1)}\right)'$, substituting for $\left(\frac{\Phi(c_1)}{\phi(c_1)}\right)'$ in (36) yields

$$-f(\hat{q}(c_1)\hat{q}'(c_1) = \frac{h(c_1)}{p_1+p_2}. \tag{37}$$

It follows that $\hat{q}'(c_1) = -h(c_1)/[(p_1+p_2)f(\hat{q}(c_1))]$. Substituting $\hat{q}'(c_1)$ into (14) yields $\hat{r}'(c_1) = -\frac{c_1 h(c_1)}{(p_1+p_2)f(\hat{q}(c_1))}$. Note that $\hat{r}(\hat{c}_1) = \hat{c}_1\hat{q}(\hat{c}_1) + R_1(\hat{c}_1)$ from (35). Therefore, $\hat{r}(c_1) = \int_{c_1}^{\hat{c}_1} ch(c)/[(p_1+p_2)f(\hat{q}(c))]dc + R_1(\hat{c}_1) + \hat{c}_1 q(\hat{c}_1)$, and we prove (17).

The sufficiency of (16) and (17) comes from the concavity of $-c_2 q + (p_1+p_2)E_\xi[q \wedge \xi] - r(c_1)$ with respect to (q, r), the concavity of $c_1 u(c_1)$ with respect to u, and the concavity of $u(c_1)$ with respect to u (see, e.g., Kamien and Schwartz [13, pages 122–123]). This completes the proof. □

PROOF OF COROLLARY 1: (i) follows directly from (37). Because $\hat{r}'(c_1) = c_1\hat{q}'(c_1)$ from (14), we know (ii) holds. We next prove (iii). Under the optimal contract, $\hat{\pi}_1^{AI}(c_1) = -c_1\hat{q}(c_1) + \hat{r}(c_1)$. Taking the first-order derivative of $\hat{\pi}_1^{AI}(c_1)$ with respect to c_1 yields $\partial\hat{\pi}_1^{AI}(c_1)/\partial c_1 = -\hat{q}(c_1) - c_1\hat{q}'(c_1) + \hat{r}'(c_1) = -\hat{q}(c_1)$, where the second equality is based on (14). We next prove (iv). Under the optimal contract, $\hat{\pi}_2^{AI}(c_1) = -c_2\hat{q}(c_1) + (p_1+p_2)E[\hat{q}(c_1) \wedge \xi] - \hat{r}(c_1)$. Taking the first-order derivative of $\hat{\pi}_2^{AI}(c_1)$ with respect to c_1 yields

$$\begin{aligned}
\frac{\partial\hat{\pi}_2^{AI}(c_1)}{\partial c_1} &= -c_2\hat{q}'(c_1) + (p_1+p_2)\bar{F}(\hat{q}(c_1))\hat{q}'(c_1) - \hat{r}'(c_1) \\
&= -c_2\hat{q}'(c_1) + (p_1+p_2)\bar{F}(\hat{q}(c_1))\hat{q}'(c_1) - c_1\hat{q}'(c_1) \text{ (based on Eq. 17)} \\
&= [-c_2 - c_1 + (p_2+p_1)\bar{F}(\hat{q}(c_1))]\hat{q}'(c_1) = \hat{q}'(c_1)\frac{\Phi(c_1)}{\phi(c_1)} \text{(based on Eq. 16)}
\end{aligned}$$

□

PROOF OF COROLLARY 2: Note that $R_1(c_1) = -c_1 q^{AI}(c_1) + p_1 E[q^{AI}(c_1)) \wedge \xi_1]$ by definition. So, $R_1'(c_1) = -q^{AI}(c_1) + \big[-c_1 + p_1\bar{F}(q^{AI}(c_1))\big]\frac{\partial q^{AI}(c_1)}{\partial c_1}$. From Proposition 2, we see that for $c_1 \geq c_1^o$, $-c_1 + p_1\bar{F}(q^{AI}(c_1)) = 0$ and for $c_1 < c_1^o$, $\frac{\partial q^{AI}(c_1)}{\partial c_1} = 0$, so the second term of $R_1'(c_1)$ is always zero. Therefore, $R_1'(c_1) = -q^{AI}(c_1)$. Further, $\Pi'(c_1) = -\hat{q}(c_1) - c_1\hat{q}'(c_1) + \hat{r}'(c_1) - R_1'(c_1) = -\hat{q}(c_1) + q^{AI}(c_1)$, where the second equality is based on the fact that $-c_1\hat{q}'(c_1) + \hat{r}'(c_1) = 0$ (Eq. 14). Because $q^{AI}(c_1) \leq -\hat{q}(c_1)$ from the assumption, $\Pi'(c_1) \leq 0$. Further, the individual rationality constraint is binding at $\bar{c}_1$. Applying (17), we obtain the expression for $\hat{r}(c_1)$. □

LEMMA 6: (13) is sufficient for firm 1 to reveal its true cost under the optimal purchase contract.

PROOF: Recall that $\pi_1(c_1, c) = -c_1 q(c) + r(c)$ is firm 1's profit when its marginal cost c_1 and it chooses to reveal c. In order to show that (13) is sufficient, we use the argument similar to that of Laffont and Tirole [14, page 121]. The proof follows by contradiction. Assume there is a $\hat{c} \neq c_1$ such that $\pi_1(c_1, \hat{c}) > \pi_1(c_1, c_1)$. This is equivalent to

$$\int_{c_1}^{\hat{c}} D_2\pi_1(c_1, x)dx > 0, \tag{38}$$

where $D_2\pi_1(c_1, x)$ is defined as the first-order partial derivative of $\pi_1(c_1, x)$ with respect to its second argument. From (13) we know that $D_2\pi_1(x, x) = 0$ for all x, so that (38) is equivalent to $\int_{c_1}^{\hat{c}_1}[D_2\pi_1(c_1, x) - D_2\pi_1(x, x)]dx > 0$, which is equivalent to

$$\int_{c_1}^{\hat{c}}\int_{x}^{c_1} D_{21}\pi_1(u, x)dudx > 0, \tag{39}$$

where $D_{21}\pi_1(u, x)$ is defined as the mixed second-order partial derivative. It is straightforward to verify that

$$D_{21}\pi_1(u, x) = -q'(x) \geq 0, \tag{40}$$

where the inequality is based on Corollary 1.

If $c_1 < \hat{c}$, then $x > c_1$ for all $x \in [c_1, \hat{c}]$. Using (40) in conjunction with (39), we obtain that the left-hand side of (39) is non-positive, which is a contradiction to (39). A similar contradiction follows for the case $\hat{c} < c_1$, which completes the proof. □

PROOF OF PROPOSITION 4: Part (i) follows directly from (16). $\hat{\pi}_1^{AI}(c_1) \geq \hat{\pi}_1^{FI}(c_1)$ holds because with complete information, $\hat{\pi}_1^{FI}(c_1) = R_1(c_1)$ and with asymmetric information $\hat{\pi}_1^{AI}(c_1) \geq R_1(c_1)$ based on the individual rationality constraint. We complete the proof of (ii). Part (iii) follows from (i) and (ii) directly. Because the total profit of the two firms is lower with asymmetric information and firm 1's profit is higher with asymmetric information. □

PROOF OF PROPOSITION 5: (i) We first demonstrate $\hat{q}(\underline{c}_1) > q_1^{AI}(\underline{c}_1)$. From Proposition 3, we obtain $\bar{F}(\hat{q}(\underline{c}_1)) = [\underline{c}_1 + c_2 + \Phi(\underline{c}_1)/\phi(\underline{c}_1)]/(p_1+p_2) = (\underline{c}_1+c_2)/(p_1+p_2)$. From Proposition 2, we know that $q_1^{AI}(\underline{c}_1) = q_2^{AI}$ and $\bar{F}\big(q_1^{AI}(\underline{c}_1)\big) = c_1^o/p_1$ because $\underline{c}_1 < c_1^o$. From the definition of c_1^o, $c_1^o\Phi\big(c_1^o\big)/p_1 = c_2/p_2$, we know that $c_1^o/p_1 > c_2/p_2$ because $c_1^o < \bar{c}_1$. Because $\underline{c}_1 < c_1^o$, $p_1\underline{c}_1 < p_1 c_1^o$. It follows that $(\underline{c}_1 + c_2)/(p_1+p_2) < c_1^o/p_1$, which implies $\hat{q}(\underline{c}_1) > q_1^{AI}(\underline{c}_1)$. From the continuity of $\hat{q}(c_1)$ and $q_1^{AI}(c_1)$, there must exists $c_1^c \in [\underline{c}_1, \bar{c}_1]$ such that $\hat{q}(c_1) \geq q_1^{AI}(c_1)$ for $c_1 \in \big[\underline{c}_1, c_1^c\big]$. □

PROOF OF (26): For ease of exposition, let $p = p_1 + p_2$. In order to show that (26) holds, we evaluate π_2^{AI}. First we compute $E_\xi[\hat{q}(c_1) \wedge \xi]$.

$$E_\xi[\hat{q}(c_1) \wedge \xi] = \int_0^{\hat{q}(c_1)} xdx + \int_{\hat{q}(c_1)}^1 \hat{q}(c_1)dx = \hat{q}(c_1) - \frac{1}{2}\hat{q}(c_1)^2. \tag{41}$$

Next, we compute $\hat{r}(c_1)$. Substituting (19) into (20) and simplifying, we obtain

$$\hat{r}(c_1) = \frac{(\mu+\varepsilon)^2 - c_1^2}{p} + R_1(\mu+\varepsilon) + (\mu+\varepsilon)\left(1 - \frac{c_2+\mu+3\varepsilon}{p}\right). \tag{42}$$

Now, we evaluate the integrand of (22). From (41) and (42), we have

$$\begin{aligned} -c_2\hat{q}(c_1) + pE_\xi[\hat{q}(c_1)\wedge\xi] - r(c_1) \\ = \frac{p-c_2}{p}(p-2c_1-c_2+\mu-\varepsilon) - \frac{1}{2p}(p-2c_1-c_2+\mu-\varepsilon)^2 \\ + \frac{c_1^2}{p} - \frac{(\mu+\varepsilon)^2}{p} - R_1(\mu+\varepsilon) - \frac{\mu+\varepsilon}{p}(p-c_2-\mu-3\varepsilon). \quad (43) \end{aligned}$$

Next,

$$\int_{\mu-\varepsilon}^{\mu+\varepsilon} \frac{p-c_2}{p}(p-2c_1-c_2+\mu-\varepsilon)dc_1 = \frac{p-c_2}{p}(p-c_2-\mu-\varepsilon)2\varepsilon. \quad (44)$$

$$\begin{aligned} \int_{\mu-\varepsilon}^{\mu+\varepsilon} -\frac{1}{2p}(p-2c_1-c_2+\mu-\varepsilon)^2 dc_1 \\ = \frac{(p-c_2-\mu-3\varepsilon)^3 - (p-c_2-\mu+\varepsilon)^3}{12p}. \quad (45) \end{aligned}$$

$$\int_{\mu-\varepsilon}^{\mu+\varepsilon} \frac{c_1^2}{p}dc_1 = \frac{1}{3p}[(\mu+\varepsilon)^3 - (\mu-\varepsilon)^3]. \quad (46)$$

Using (43) through (46), we are ready to evaluate $\hat{\pi}_2^{AI}$:

$$\begin{aligned} \hat{\pi}_2^{AI} = \frac{p-c_2}{p}(p-c_2-\mu-\varepsilon) - \frac{(p-c_2-\mu-3\varepsilon)^2}{6p} \\ - \frac{(p-c_2-\mu+\varepsilon)^2}{6p} + \frac{3\mu^2+\varepsilon^2}{3p} \\ - \frac{(p-c_2-\mu-3\varepsilon)(p-c_2-\mu+\varepsilon)}{6p} \\ - \frac{(\mu+\varepsilon)^2 + (\mu+\varepsilon)(p-c_2-\mu-3\varepsilon)}{p} - R_1(\mu+\varepsilon). \end{aligned}$$

Taking the first-order derivative of $\hat{\pi}_2^{AI}$ with respect to ε (or μ) yields (26) (or 27). □

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