

# Differential Modulation for Two-User Cooperative Diversity Systems

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**Abstract**— Cooperative transmission is an alternative approach to achieve diversity in wireless systems. So far the research has focused on the situation where perfect channel state information is available and coherent detection is employed at the destination receiver. This paper proposes a differential modulation scheme for a two-user cooperative diversity system which does not require channel state information at either users or destination. The relaying protocols such as fixed decode-and-forward, selection relaying, and incremental relaying protocols are compared. The proposed scheme achieves spatial diversity gains when the signal-to-noise ratios between cooperative users are sufficiently high.

## I. INTRODUCTION

Traditional space-time coding and multiple-input multiple-output (MIMO) techniques exploit spatial diversity through multiple transmit and/or receive antennas. *Cooperative diversity* has recently emerged as an alternative way to achieve spatial diversity when the users cannot afford multiple transmit antennas. The main idea is to share the resource (e.g., time, frequency) among cooperative users and each user serves as a relay for the transmissions of other users' information.

In [3], the concept of cooperative diversity (user cooperation diversity) was introduced and the capacity region, outage probabilities, coverage area as well as CDMA implementation were discussed. In [4], outage probabilities for several relaying protocols were derived. It was concluded that except for fixed decode-and-forward relaying protocol, all other protocols which are amplify-and-forward relaying, selection relaying and incremental relaying achieve second-order diversity. In [5], an improved multiple access scheme over [4] referred to as space-time coded protocol was proposed and it was shown to offer full spatial diversity.

All the previously mentioned works consider simple repetition codes and hard decisions at the relay mobiles or use a random coding argument to develop some performance bounds. The concept of *coded cooperation*, where the relay mobile is allowed to perform channel coding, was introduced in [6], and later extended in [7] where the idea of space-time coding and turbo coding were exploited. Another related work is [8] where Alamouti's space-time block code [1] was applied in a wireless relay system. In [9], a linear space-time block

code was applied in a cooperative relay system in purely line-of-sight channels.

In all the above research works, perfect channel state information (CSI) is assumed and coherent detection is employed at the receivers. In practice, this means channel estimation has to be done at the cooperative users and at the destination. This leads to a very complex system when many users are involved since channel estimation is needed among each pair of users, not to mention the possibility of asymmetric channels among the users.

In this paper, we propose a differential modulation scheme for two-user cooperative diversity systems which bypasses channel estimation. This modulation scheme can be considered as a nontrivial modification of differential phase-shift keying (DPSK) and differential space-time block codes (DSTBC) in [2]. The information of one user stays in the in-phase axis (I-axis) while that of the other user stays in the quadrature-phase axis (Q-axis). This is to ensure separability of each user information at the destination receiver. This paper considers the situation where both users have information to send and they share the same frequency band. Therefore, multiple access is done by time-division multiplexing (TDM). For practical purpose, we focus on a half-duplex transmission, i.e. the mobiles cannot transmit and receive signals at the same time. In addition, the mobiles are able to perform differential decoding and re-encoding of the partner's message as well as negative and conjugate operations before relaying the signals.

We focus on the decode-and-forward relaying protocol [4] in which the relaying mobiles perform differential decoding and re-encoding the partner's message. In the case of selection relaying and incremental relaying, cyclic redundancy check (CRC) bits are used to detect whether the information is received correctly. The simulation results show that our differential modulation scheme accompanied with the above relaying protocols achieves spatial diversity and performance gain over noncooperative transmission when SNRs between users are sufficiently high.

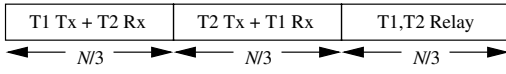


Fig. 1. Time-division multiple access scheme for two cooperative users

## II. SYSTEM MODEL

We consider a cellular system in which two cooperative users are transmitting their information to the same destination (e.g. the base station). Both users share the same frequency band and the mobile of each user cannot transmit and receive signal at the same time. The multiple access scheme is TDM as shown in the Fig. 1. The total time frames are divided into three parts. The first time frame belongs to the first user information. The second time frame belongs to the second user information. The third time frame is shared between both users and is used to relay each other's message. Notice that this partitioning of time frames is different from those in [4], [5]. Compared to the space-time coded medium access control, our partitioning is more efficient since the transmissions share both time and frequency resources in the relaying frames.

Each user's mobile can be an information source or a relay terminal at a specific time frame. The channels among users (interuser channels) and between the users and the destination are independent of each other. All channels experience frequency flat fading and are quasi-static, i.e. they are fixed during the multiple-access time frames (3 time frames) and change independently to the next.

With the above assumptions, we now consider the received signals in the first time frame. The first user's mobile transmits its own message and the destination receiver and the second user's mobile receive the signal. At the destination receiver, we have a *direct transmission* and the received signal,  $r_{1d}[n]$ , written as

$$r_{1d}[n] = a_{1d}s_1[n] + w_d[n] \quad (1)$$

where  $s_1[n]$  is a symbol transmitted from the first user at the  $n$ th symbol interval,  $n = 0, 1, \dots, \frac{N}{3} - 1$ . At the second user's mobile, the received signal,  $r_{12}[n]$ , is written as

$$r_{12}[n] = a_{12}s_1[n] + w_2[n]. \quad (2)$$

In the second time frame, the roles of the first and the second users are switched. Therefore, at the destination receiver and at the first user's mobile, the respective received signals  $r_{2d}[n]$  and  $r_{21}[n]$  are

$$r_{2d}[n] = a_{2d}s_2[n] + w_d[n] \quad (3)$$

$$r_{21}[n] = a_{21}s_2[n] + w_1[n] \quad (4)$$

where  $s_2[n]$  is a symbol transmitted from the second user at the  $n$ th symbol interval,  $n = \frac{N}{3}, \frac{N}{3} + 1, \dots, \frac{2N}{3} - 1$ .

In the third time frame, both users' mobiles act as relays and transmit the relay signals at the same time. This is referred to as a *relay transmission*. The signal arriving at the destination is a linear combination of signals from each user which can be written as

$$r_d[n] = a_{1d}s_1[n] + a_{2d}s_2[n] + w_d[n] \quad (5)$$

where  $s_1[n], s_2[n]$  are relayed symbols transmitted from the first and the second user, respectively, at the  $n$ th symbol interval,  $n = \frac{2N}{3}, \frac{2N}{3} + 1, \dots, N - 1$ .

In (1)-(5), the channel gains,  $a_{ij}, i, j \in \{1, 2, d\}$ , are associated with each transmission path from  $i$  to  $j$ . The subscripts 1, 2 represent the first and the second users' mobiles and the subscript  $d$  represents the destination. The channel gains are modeled as independent zero-mean complex Gaussian random variables. Since the channels are fixed during the multiple-access time frame, the time-dependent variable is omitted. We also consider the channels between users to be symmetric for simplicity, which implies  $a_{12} = a_{21}$ . The additive white noise  $w_i[n], i \in \{1, 2, d\}$ , associated with the users' mobiles and the destination receiver, are modeled as independent circularly symmetric zero-mean complex Gaussian random variables, each with variance  $N_0$ . The SNR is defined as  $E[a_{ij}^2] \cdot \frac{E_b}{N_0}$  where  $E_b$  is the energy per bit. Note that the SNR defined here incorporates the effects of noise and channel path loss.

## III. DIFFERENTIAL ENCODING AND DECODING

For the proposed scheme to work, both users apply BPSK constellation. To ensure separability at the destination (to be clearly seen in the decoding part), the first user's data symbols lie on the I-axis while the second user's data symbols lie on the Q-axis. The first and the second user's data symbols are represented respectively by  $g_1[n]$  and  $j \cdot g_2[n]$ , where  $g_i[n] \in \{-1, 1\}, i = 1, 2$  and  $j = \sqrt{-1}$ . For the first user, the transmitted symbols in the first time frame are encoded by

$$s_1[n] = g_1[n]s_1[n-1], n = 1, 2, \dots, \frac{N}{3} - 1. \quad (6)$$

For the second user, the transmitted symbols in the second time frame are encoded by

$$s_2[n] = j \cdot g_2[n]s_2[n-1], n = \frac{N}{3} + 1, \frac{N}{3} + 2, \dots, \frac{2N}{3} - 1. \quad (7)$$

Note that while the transmission symbols whose message belong to the first user is still in the BPSK constellation, the transmission symbols whose message belong to the second user is expanded to the QPSK constellation. The necessary initial symbols are assumed to be one, i.e.  $s_1[0] = 1, s_2[N/3] = 1$ . These a priori known initial symbols are needed to perform differential re-encoding at the relay mobiles.

Now let us consider the transmission signals during the third time frame. In [4], a simple fixed amplify-and-forward protocol is shown to achieve spatial diversity where the relay signal has been multiplied with a factor whose value depends on the channel gains and noise. Since we are not going to use CSI at the users' mobiles, we choose to apply the decode-and-forward protocol in which differential decoding and re-encoding are performed at both users. At the first user's mobile, differential decoding is performed by determining

$$z_2[n] = r_{21}[n]r_{21}^*[n-1], n = \frac{N}{3} + 1, \frac{N}{3} + 2, \dots, \frac{2N}{3} - 1 \quad (8)$$

and the decoded data symbol,  $\hat{g}_2[n] = 1$ , if  $\text{Im}(z_2[n]) \geq 0$  and  $\hat{g}_2[n] = -1$ , if  $\text{Im}(z_2[n]) < 0$ , where  $\text{Im}(\cdot)$  is the

imaginary part. The second user performs differential decoding by determining

$$z_1[n] = r_{12}[n]r_{12}^*[n-1], n = 1, 2, \dots, \frac{N}{3} - 1 \quad (9)$$

and the decoded data symbol,  $\hat{g}_1[n] = 1$ , if  $\text{Re}(z_1[n]) \geq 0$  and  $\hat{g}_1[n] = -1$ , if  $\text{Re}(z_1[n]) < 0$ . To perform differential re-encoding, the first user employs (7) with  $\hat{g}_2[n]$  replacing  $g_2[n]$ . This results in a symbol  $\hat{s}_2[n]$  which contains the second user's message. Similarly, the second user employs (6) with  $\hat{g}_1[n]$  replacing  $g_1[n]$ . This results in a symbol  $\hat{s}_1[n]$  which contains the first user's message.

Now the first user's mobile transmits the relay signal as  $-\hat{s}_2^*$ , where  $*$  is a conjugate, and the second user's mobile transmits the relay signal as  $\hat{s}_1^*$ , simultaneously. This corresponds to the Alamouti's scheme where the relay signals are similar to the symbols in the second interval of the Alamouti's matrix [1]. The relay signal in (5) can be rewritten as

$$r_d[n] = -a_{1d}\hat{s}_2^*[n - N/3] + a_{2d}\hat{s}_1^*[n - 2N/3] + w_d[n], \quad (10)$$

where  $n = \frac{2N}{3}, \frac{2N}{3} + 1, \dots, N - 1$ . After the relay signals arrive at the destination, differential decoding is performed at the receiver. First, the receiver computes the combined signal from the direct transmissions from both users as

$$\begin{aligned} r_c[n] &= r_{1d}[n - 2N/3] + r_{2d}[n - N/3] \\ &= a_{1d}s_1[n - 2N/3] + a_{2d}s_2[n - N/3] \\ &\quad + w_d[n - 2N/3] + w_d[n - N/3], \end{aligned} \quad (11)$$

where  $n = \frac{2N}{3}, \frac{2N}{3} + 1, \dots, N - 1$ . The received signals in (10) and (11) are almost similar to those if the Alamouti's scheme is employed with two transmit antennas. The only difference is another additive noise in (11) and the delay from each user inherent in the cooperative system. Next, the receiver computes

$$z_d[n] = r_c[n]r_c^*[n-1] + r_d^*[n]r_d[n-1] \quad (12)$$

where  $n = \frac{2N}{3} + 1, \frac{2N}{3} + 2, \dots, N - 1$ . To make this clear, substituting (10) and (11) into (12) and after some manipulation, we can readily show that

$$\begin{aligned} z_d[n] &= |a_{1d}|^2 (s_1[n - 2N/3]s_1^*[n - 2N/3 - 1] \\ &\quad + \hat{s}_2[n - N/3]\hat{s}_2^*[n - N/3 - 1]) + \\ &\quad |a_{2d}|^2 (\hat{s}_1[n - 2N/3]\hat{s}_1^*[n - 2N/3 - 1] \\ &\quad + s_2[n - N/3]s_2^*[n - N/3 - 1]) + \text{noise} \\ &= |a_{1d}|^2 (g_1[n - 2N/3] + j \cdot \hat{g}_2[n - N/3]) \\ &\quad + |a_{2d}|^2 (\hat{g}_1[n - 2N/3] + j \cdot g_2[n - N/3]) \\ &\quad + \text{noise}. \end{aligned} \quad (13)$$

Since both  $g_1[n - 2N/3], g_2[n - N/3]$  are real, we can decode them separately by considering real and imaginary parts of (13). Therefore, the following decision rule is obtained:

$$\begin{aligned} \tilde{g}_1[n - 2N/3] &= \begin{cases} 1 & ; \text{if } \text{Re}(z_d[n]) > 0 \\ -1 & ; \text{else} \end{cases} \\ \tilde{g}_2[n - N/3] &= \begin{cases} 1 & ; \text{if } \text{Im}(z_d[n]) > 0 \\ -1 & ; \text{else} \end{cases} \end{aligned} \quad (14)$$

where  $n = \frac{2N}{3} + 1, \frac{2N}{3} + 2, \dots, N - 1$ .

Note that differential decoding in (13) is somewhat similar to differential detection for differential space-time block codes in [2]. The difference is that more noise terms exist in (13) and that either  $g_1[n - 2N/3] + j \cdot \hat{g}_2[n - N/3]$  or  $\hat{g}_1[n - 2N/3] + j \cdot g_2[n - N/3]$  does not represent a single symbol. It represents an addition of an original data symbol and a re-encoded relay symbol, each owned by different users. If the relay symbols were re-encoded at the mobiles without error, i.e.  $\hat{g}_1[n - 2N/3] = g_1[n - 2N/3], \hat{g}_2[n - N/3] = g_2[n - N/3]$ , at the destination, it is equivalent to that each data symbol experiences a second-order diversity (from  $|a_{1d}|^2$  and  $|a_{2d}|^2$ ). Therefore, at high interuser SNRs, we expect the improvement from this spatial diversity which is from cooperative users' channels.

#### IV. RELAYING PROTOCOLS

In cooperative diversity systems, the relaying protocol is a crucial component to achieve diversity. Each protocol has its own trade-offs among performance, complexity and data transmission rate. The relaying protocols in this paper are taken from [4] with some difference.

##### A. Fixed Decode-and-Forward Relaying

With this protocol, the relay mobile fully decodes or symbol-by-symbol decodes the received codeword before transmitting the relay signal. In [4], 'decode' refers to coherent detection and 'forward' refers to transmission of repetition of decoded symbols from the relay mobiles. In this paper, 'decode' means differential decoding and 'forward' means relaying the re-encoded symbols according to the Alamouti's transmission matrix as described previously.

##### B. Selection Relaying

When the channels between cooperative users are not good, several decoded and re-encoded symbol errors occur at the relay mobiles. With the fixed decode-and-forward relaying, it turns out that several relay symbols in error will be transmitted. These relay symbols in error will mislead the original information when the destination receiver decodes the message. This results in a performance degradation even when the direct transmission channels between users and the destination are good. Therefore, it turns out that it is better not to transmit anything from the relay when the interuser channels are not good. In [4], selection relaying was proposed in which not all relay mobiles are allowed to transmit the relay signals. The permission to transmit the relay signals or not depends on the amplitudes of the fading gains between the source mobile and the relay mobile. If these values fall below a certain threshold, the relay mobile is not allowed to transmit the relay signal but instead transmit its own information signal. The destination knows whether the mobiles are transmitting the relay signals or not so that it can apply an optimal decoding scheme. In this paper, since we are not going to use the CSI, we assume that CRC bits have been added before differential encoding at the source mobile. Then, at the relay mobile, the receiver performs

CRC check after differential decoding to see if the whole frame has been received correctly. If the frame is received correctly, the relay mobile is allowed to perform differential re-encoding and forward the relay signal, otherwise it does nothing. The relay mobile is not allowed to transmit its own message at this time frame since doing that will destroy the Alamouti's transmission structure and differential decoding operation. The CRC check technique is also used in [7] with the possibly continuing its own message transmission of the relay mobile. In this paper, we assume that CRC check is perfect and the CRC bits are neglected when evaluating the performance. This is for simplicity and to show the potential of pure cooperative diversity gains. Differential detection in (13) and (14) can still be used when any of the mobiles stop their relay transmissions, since the disappearance of the relay transmission is equivalent to that the relay symbols are zero. Diversity can still be achieved for the data symbols that are received correctly and forwarded from the relay mobile. If the user's relay transmission is not performed, differential decoding in (8) or (9) is used for that user at the destination.

### C. Incremental Relaying

When the direct transmission channels are good, it might not be really necessary to have relay transmissions. This will not only avoid the misleading information transmitted from the relay mobile but also lead to an increase in the overall transmission rate. In [4], the incremental relaying was proposed which exploits a limited feedback from the destination. The feedback is in a form of a single bit to inform each relay whether it needs to transmit the relay signal. In [4], the feedback relies on the SNR between each user and the destination. If the SNR is sufficiently high, the feedback will notify the relay mobile not to transmit the relay signal. In this paper, instead of using SNR criterion, we employ error detection by CRC. Similar to the selection relaying, CRC bits have been added at the source mobiles. Then, CRC check is performed at the destination to see whether the frame is received correctly from the direct transmissions. If the frame is received correctly from the direct transmission, the feedback from the destination will notify the corresponding relay mobiles not to transmit the relay signal. In this protocol, a transmission rate increase can be achieved when the direct transmissions from all source mobiles are received correctly. Similar to selection relaying, we assume that CRC check is perfect and CRC bits as well as feedback bits are neglected when evaluating the performance of the protocol.

## V. SIMULATION RESULTS AND DISCUSSION

We consider the case where the interuser channels are symmetric and the average SNRs between each user and the destination are equal. Since all the channels seen by each user is the same on average, all users experience the same average performance. Noncooperative transmission DPSK is chosen as a baseline performance. Included in all plots are the performance of DPSK and the performance of fixed decode-and-forward relaying when the users know each other's

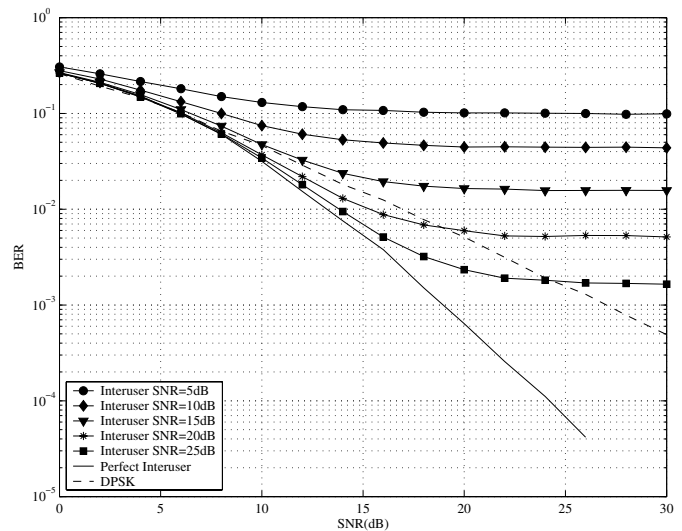


Fig. 2. BPSK differential modulation with fixed decode-and-forward relaying protocol at various interuser SNRs

message perfectly. The SNRs on the x-axis indicate the SNRs of the direct transmissions.

As shown in Fig. 2, the fixed decode-and-forward relaying protocol achieves a performance gain over DPSK when the interuser SNRs are greater than 15 dB. As the interuser SNRs increase, the protocol has some gains over DPSK as long as the interuser SNRs are greater than the direct transmission SNRs. When the direct transmission SNRs is greater than the interuser SNRs, the protocol experiences an error floor and cannot outperform DPSK. This error floor comes from the re-encoded symbols in error occurring at the relay mobile which causes a severe degradation. In the perfect interuser case, no error floor exists and it indeed achieves second-order diversity which can be seen from the slope of the curve.

The performance of the selection relaying protocol in Fig. 3 and the incremental relaying protocol in Fig. 4 show similar behavior while the latter slightly outperforms the former at high interuser SNRs. Even at 10-dB interuser SNR, both protocols achieve about 4-dB gain over DPSK. At 5-dB interuser SNR, the incremental relaying is slightly inferior to DPSK while the selection relaying is not. This is because it is still possible for the incremental relaying protocol to have re-encoded relay signals in error transmitted. At 25-dB interuser SNR, the performance of incremental relaying coincides with the perfect interuser case. This indicates that the limited feedback in the incremental relaying protocol from the destination yields a significant improvement to the performance. The performance improvement shown does not even take the increase in the transmission rate into account when no relay signals are needed and the users can transmit more of their data.

To make a fair comparison among the protocols, the transmission rate should be taken into account. We compute the normalized SNR which is defined as  $\text{SNR}_{\text{norm}} = \frac{\text{SNR}}{2^R - 1}$ , where  $R$  is the transmission rate in bits/s/Hz [4]. This normalization

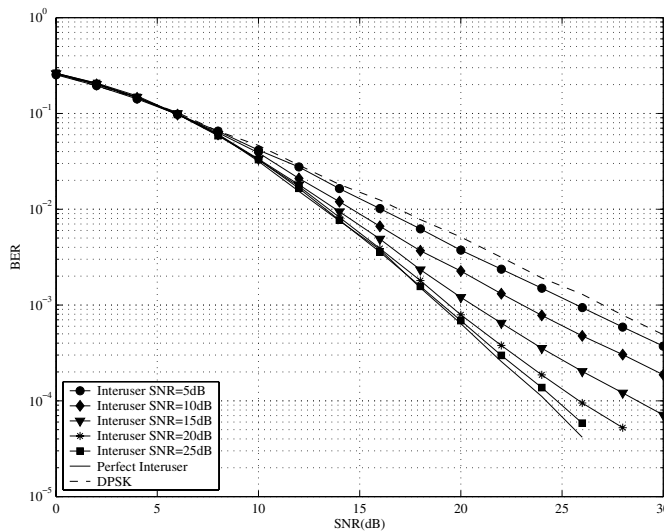


Fig. 3. BPSK differential modulation with selection relaying protocol at various interuser SNRs

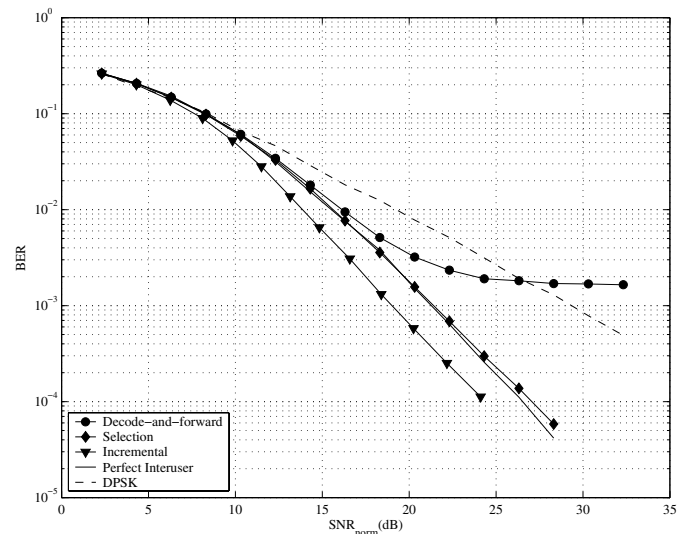


Fig. 5. Performance comparison among decode-and-forward, selection, incremental relaying protocols at 25-dB interuser SNR

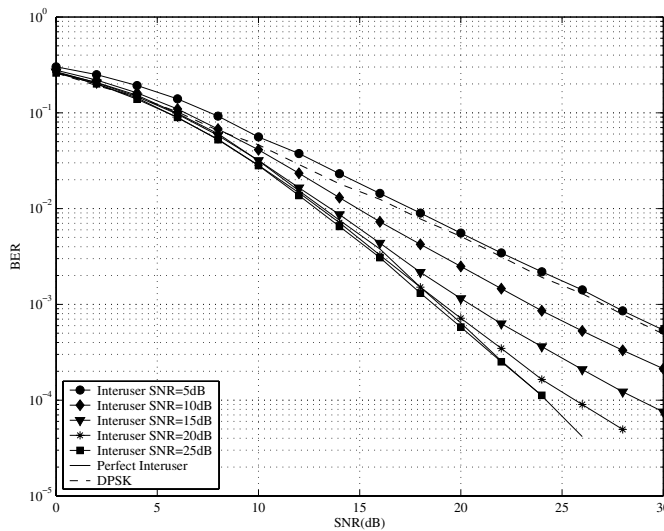


Fig. 4. BPSK differential modulation with incremental relaying protocol at various interuser SNRs

is relative to the minimum required SNR to transmit at rate  $R$  bits/s/Hz according to the Shannon limit. Fig. 5 shows the performance of the relaying protocols vs  $SNR_{norm}$  at the 25-dB interuser SNR. We can see that both the selection relaying and incremental relaying protocols achieve full spatial diversity while the decode-and-forward relaying protocol still significantly outperforms DPSK in the medium SNR range. The incremental relaying even outperforms the perfect interuser with decode-and-forward relaying since it exploits the limited feedback which results in an increase in the overall transmission rate.

## VI. CONCLUSIONS

A differential modulation scheme is proposed in a two-user cooperative diversity system. This scheme can achieve

second-order diversity with appropriate relaying protocols at sufficiently high interuser SNRs and it does not require CSI either at the user mobiles or at the destination. We have shown that the decode-and-forward relaying still significantly outperforms DPSK as long as the interuser SNRs are greater than the direct transmission SNRs. We have found that the incremental relaying protocol performs best since it exploits a limited feedback to allow only necessary relaying which results in an increase in the transmission rate. Further extension to accommodate more cooperative users is an interesting problem.

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