

CS6374
Predicate Logic Assignment

1. Let $P(X)$ and $Q(X)$ represent “ X is a rational number” and “ X is a real number” respectively. Symbolize the following sentences:
 - a. Every rational number is a real number.
 - b. Some real numbers are rational numbers.
 - c. Not every real number is a rational number.
2. Transform into Prenex Normal Form.
 - a. $(\forall X)(P(X) \implies (\exists Y)Q(X, Y))$
 - b. $(\exists X)(\sim ((\exists Y)P(X, Y)) \implies ((\exists Z)Q(Z) \implies R(X)))$
 - c. $\forall X \forall Y ((\exists Z)P(X, Y, Z) \wedge ((\exists U)Q(X, U) \implies (\exists V)Q(Y, V)))$
3. Assume that St. Francis is loved by everyone who loves someone. Also assume that noone loves nobody. Deduce that St. Francis is loved by everyone.
4. An Abelian group is a set A with a binary operator $+$ that has certain properties. Let $P(x, y, z)$ and $E(x, y)$ represent $x + y = z$ and $x = y$, respectively. Express the following axioms for Abelian groups symbolically.
 - For every x and y in A , there exists a z in A such that $x + y = z$ (closure)
 - If $x + y = z$ and $x + y = w$ then $z = w$ (uniqueness)
 - $(x + y) + z = x + (y + z)$
5. Consider the following interpretation:
Domain = $\{1, 2\}$
Assignment of constants: $a = 1$ and $b = 2$
Assignment of functions: $f(1) = 2$ and $f(2) = 1$
Assignment for predicate P :
 $P(1, 1) = T; P(1, 2) = T; P(2, 1) = F; P(2, 2) = F$
Evaluate the truth value of following formulas in the above interpretation:
 - $P(a, f(a)) \wedge P(b, f(b))$
 - $(\forall x)(\exists y)P(y, x)$
 - $(\forall x)(\forall y)(P(x, y) \rightarrow P(f(x), f(y)))$