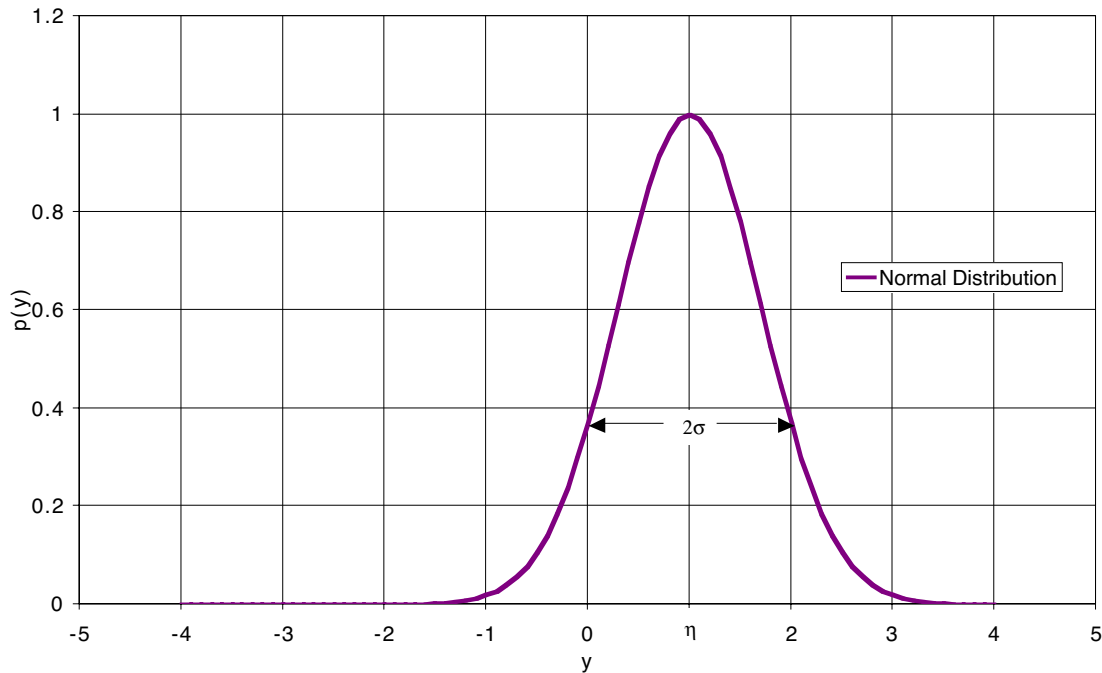


Lecture 1:
Basic gas Properties – Fluid Dynamics

As gas atoms/molecules (or particles) move through out a volume they collide and randomly distribute their energy. A basic tenant of statistics is that random processes result in Normally distributed results. This is know as the central limit theorem, see for example Box Hunter and Hunter, “Statistics for experimenters” (Yu Lei has the book right now so I don’t have the title quite right.) The normal distribution is

$$p(y) = const \frac{1}{\sigma} \exp\left[\frac{-(y - \eta)^2}{\sigma^2}\right].$$

Here σ^2 is the population variance, σ is the population standard deviation, and η is the central value. The distribution looks like



Concept of Temperature

Maxwell and Boltzman proposed that this same distribution can be used to model the velocity distribution of particles in thermal equilibrium. (This is known as the Maxwell Distribution, Boltzman Distribution and the Maxwell-Boltzman Distribution.) Using this assumption we have that the velocity distribution is

$$f(\bar{v}) = const \exp\left[\frac{-m(\bar{v})^2}{2kT}\right],$$

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while the energy distribution is

$$f(\epsilon) = \text{const} \exp\left[\frac{-\epsilon}{kT}\right].$$

Here k is Boltzman's constant and T is the temperature. Notice that the temperature is simply a measure of the random energy of the system. NOTE that often 'kT' is simply written as 'T' and hence 'T' is given units of energy.

Problem 1

Show that if in one dimension

$$\int f(v)dv = n$$

then

$$\text{const} = n \left(\frac{m}{2\pi kT} \right)^{1/2}.$$

Also show that if in three dimensions

$$\int f(\vec{v})dv^3 = n$$

then

$$\text{const} = n \left(\frac{m}{2\pi kT} \right)^{3/2}.$$

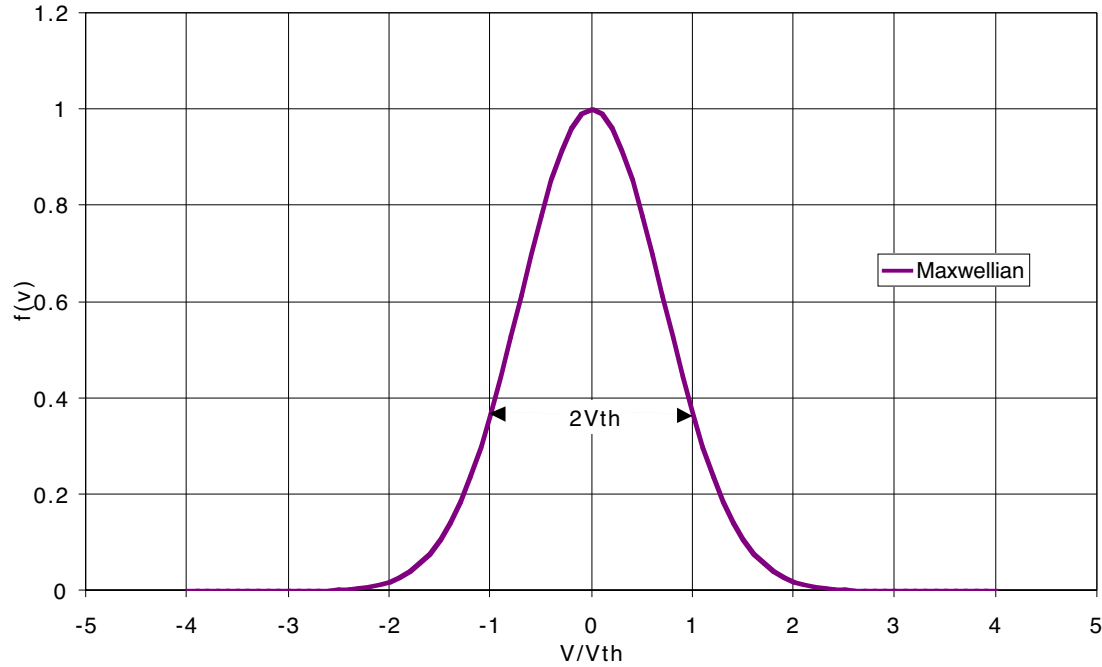
Here n is the particle density.

Typically, the velocity distributions are well behaved and we have distributions as such:

The Maxwellian Distribution

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Where

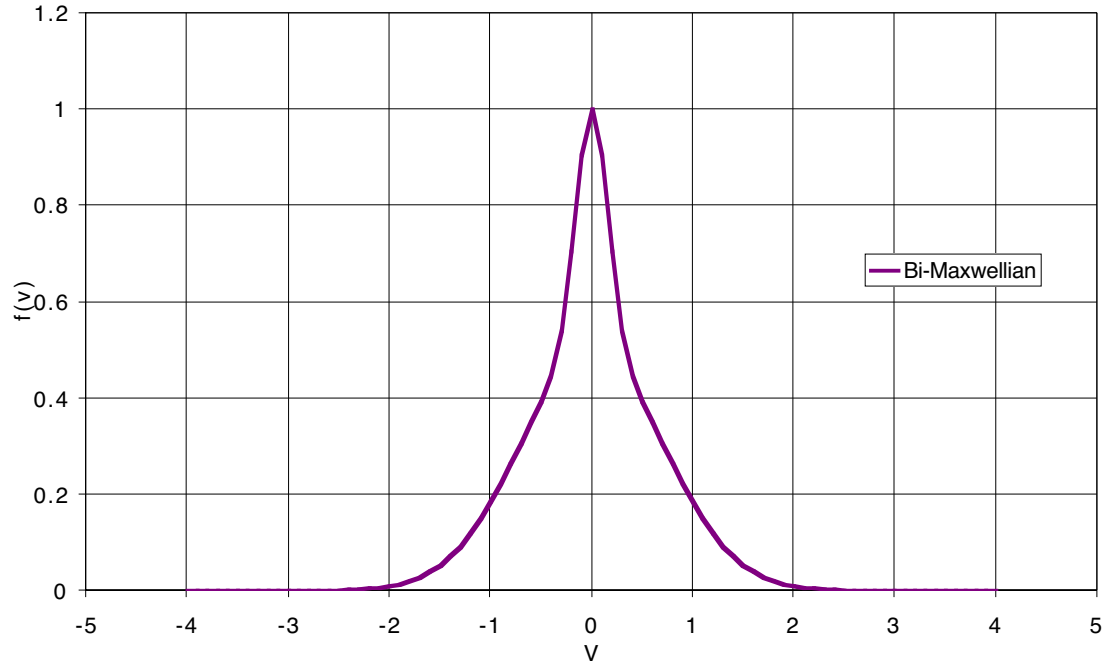
$$f(v) = n \left(\frac{m}{2\pi kT} \right)^{1/2} \exp \left[\frac{-m(v)^2}{2kT} \right].$$

This is what happens if there are enough collisions to evenly distribute the energy. In other cases, however, we will be working in with a system in which the rate of collisions is not large enough to evenly distribute the energy. Some typical examples are:

The bi-Maxwellian Distribution

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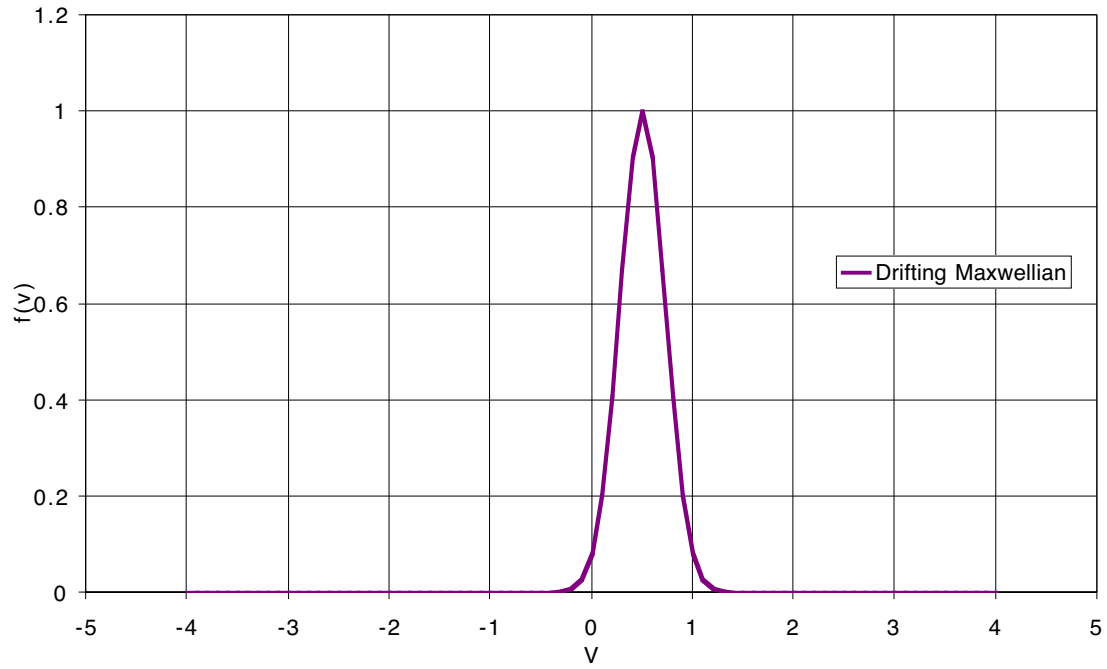
Where

$$f(v) = (1 - \alpha)n \left(\frac{m}{2\pi kT_1} \right)^{1/2} \exp \left[\frac{-m(v)^2}{2kT_1} \right] + (\alpha)n \left(\frac{m}{2\pi kT_2} \right)^{1/2} \exp \left[\frac{-m(v)^2}{2kT_2} \right]$$

The drifting Maxwellian Distribution

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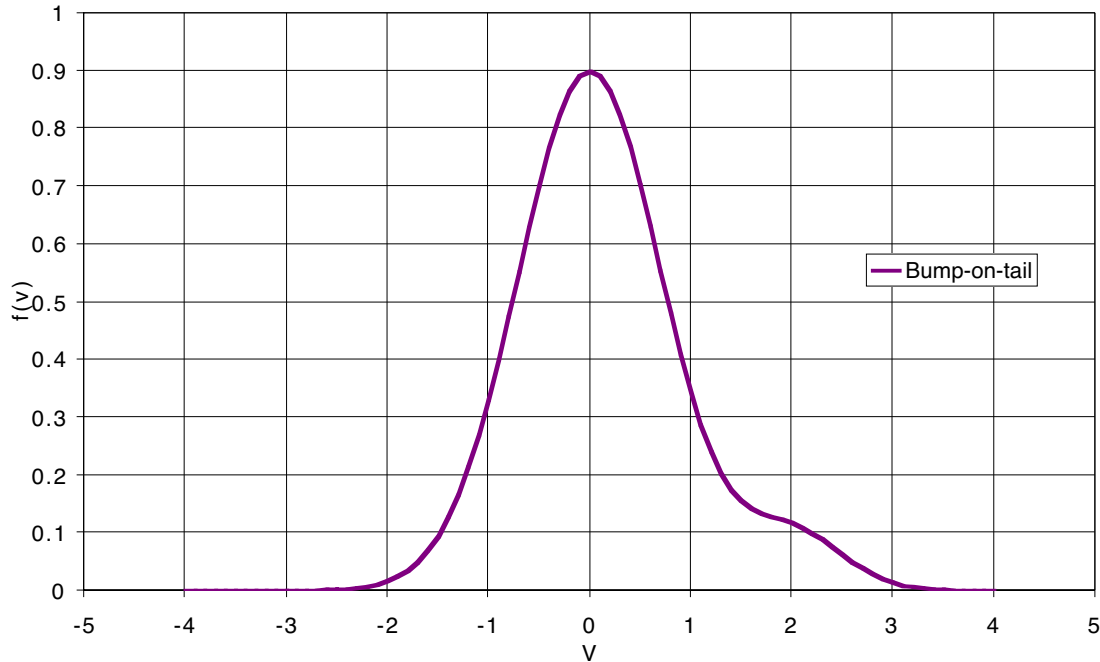
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Where

$$f(v) = n \left(\frac{m}{2\pi kT} \right)^{1/2} \exp \left[\frac{-m(v - v_0)^2}{2kT} \right].$$

and the 'bump-on-tail'



Where

$$f(v) = f_{Maxwell} + f_{drift}.$$

While other distributions are certainly possible, these four comprise most of what is observed in laboratory plasmas.

Normalization of the velocity distribution.

The velocity distribution is normalized so that if all particles of all velocities are accounted for then one has the local particle density. This means that

$$\int_{-\infty}^{\infty} f(v)dv = n. \text{ Thus in one dimension}$$

$$f(v) = n \left(\frac{m}{2\pi kT} \right)^{1/2} \exp \left[\frac{-m(v)^2}{2kT} \right] \text{ or in three-dimensions}$$

$$f(v) = n \left(\frac{m}{2\pi kT} \right)^{3/2} \exp \left[\frac{-m(v)^2}{2kT} \right]$$

Concept of Thermal Velocity

We know that

$$f(v) = n \left(\frac{m}{2\pi kT} \right)^{1/2} \exp \left[\frac{-mv^2}{2kT} \right].$$

The central part of the distribution, \pm sigma, will have ~69% of all the particles at or below a given v. This velocity is known as the thermal velocity, v_{th} . Thus we can ‘derive’

$$f(v) = n \left(\frac{m}{2\pi kT} \right)^{1/2} \exp \left[\frac{-(v - v_0)^2}{2v_{th}^2} \right]$$

thus

$$v_{th}^2 = \frac{kT}{m}.$$

Note that in some texts $v_{th}^2 = \frac{2kT}{m}$. We will not use this definition.

Average velocity

$$\begin{aligned} \langle v \rangle &= \frac{\int_{-\infty}^{\infty} v f(v) dv}{\int_{-\infty}^{\infty} f(v) dv} \\ &= \frac{1}{n} \int_{-\infty}^{\infty} v f(v) dv \\ &= \frac{1}{n} \left(\frac{m}{2\pi kT} \right)^{1/2} \int_{-\infty}^{\infty} \underbrace{v}_{\text{odd function}} \underbrace{\exp\left[\frac{-v^2}{2v_{th}^2}\right]}_{\text{even function}} dv \\ &= 0 \text{ (Odd functions integrated from } -\infty \text{ to } \infty \text{ is zero.)} \end{aligned}$$

This is very different than the average speed, which is

$$\begin{aligned} \langle |v| \rangle &= \frac{1}{n} \int_{-\infty}^{\infty} |v| f(v) dv \\ &= \left(\frac{m}{2\pi kT} \right)^{1/2} \int_{-\infty}^{\infty} |v| \exp\left[\frac{-v^2}{2v_{th}^2}\right] dv \\ &= \frac{1}{v_{th}} \left(\frac{1}{2\pi} \right)^{1/2} 2 \int_0^{\infty} v \exp\left[\frac{-v^2}{2v_{th}^2}\right] dv \\ &= v_{th} \left(\frac{1}{2\pi} \right)^{1/2} 2 \int_0^{\infty} \frac{v}{v_{th}} \exp\left[\frac{-v^2}{2v_{th}^2}\right] d\left(\frac{v}{v_{th}}\right) \\ &= v_{th} \left(\frac{1}{2\pi} \right)^{1/2} 2 \int_0^{\infty} \exp\left[\frac{-u^2}{2}\right] d\left(\frac{u^2}{2}\right) \\ &= v_{th} \left(\frac{1}{2\pi} \right)^{1/2} 2 \int_0^{\infty} \exp[-y] dy \\ &= \left(\frac{2kT}{\pi m} \right)^{1/2} \end{aligned}$$

in one-dimension. In three-dimension the average speed is

$$\begin{aligned} \langle |v| \rangle &= \frac{1}{n} \int_{-\infty}^{\infty} |v| f(v) 4\pi v^2 dv \\ &= \left(\frac{8kT}{\pi m} \right)^{1/2} \end{aligned}$$

Average energy in one-dimension

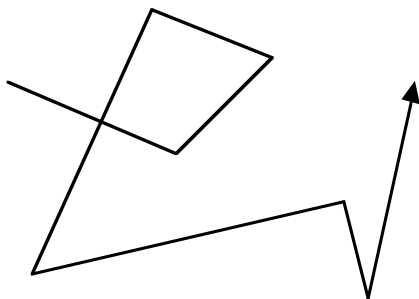
$$\begin{aligned}
 \langle E \rangle &= \frac{1}{2} m \langle v^2 \rangle = \frac{1}{n} \int_{-\infty}^{\infty} v^2 f(v) dv \\
 &= \frac{1}{2} m \left(\frac{m}{2\pi kT} \right)^{1/2} \int_{-\infty}^{\infty} v^2 \exp\left[\frac{-v^2}{2v_{th}^2} \right] dv \\
 &= \frac{1}{2v_{th}} m \left(\frac{1}{2\pi} \right)^{1/2} 2 \int_{-\infty}^{\infty} v^2 \exp\left[\frac{-v^2}{2v_{th}^2} \right] dv \\
 &= kT \left(\frac{1}{8\pi} \right)^{1/2} \int_{-\infty}^{\infty} \left(\frac{v}{v_{th}} \right)^2 \exp\left[\frac{-v^2}{2v_{th}^2} \right] d\left(\frac{v}{v_{th}} \right) \\
 &= kT \left(\frac{1}{8\pi} \right)^{1/2} \int_{-\infty}^{\infty} y^2 \exp\left[\frac{-y^2}{2} \right] dy \\
 &= kT \left(\frac{1}{8\pi} \right)^{1/2} \left[y \exp\left[\frac{-y^2}{2} \right] \Big|_{-\infty}^{\infty} + \int_{-\infty}^{\infty} \exp\left[\frac{-y^2}{2} \right] dy \right] \\
 &= kT \left(\frac{1}{8\pi} \right)^{1/2} [(2\pi)^{1/2}] = \frac{kT}{2}
 \end{aligned}$$

In three dimensions the average energy is

$\langle E \rangle = \frac{3kT}{2}$. The derivation of this is left homework.

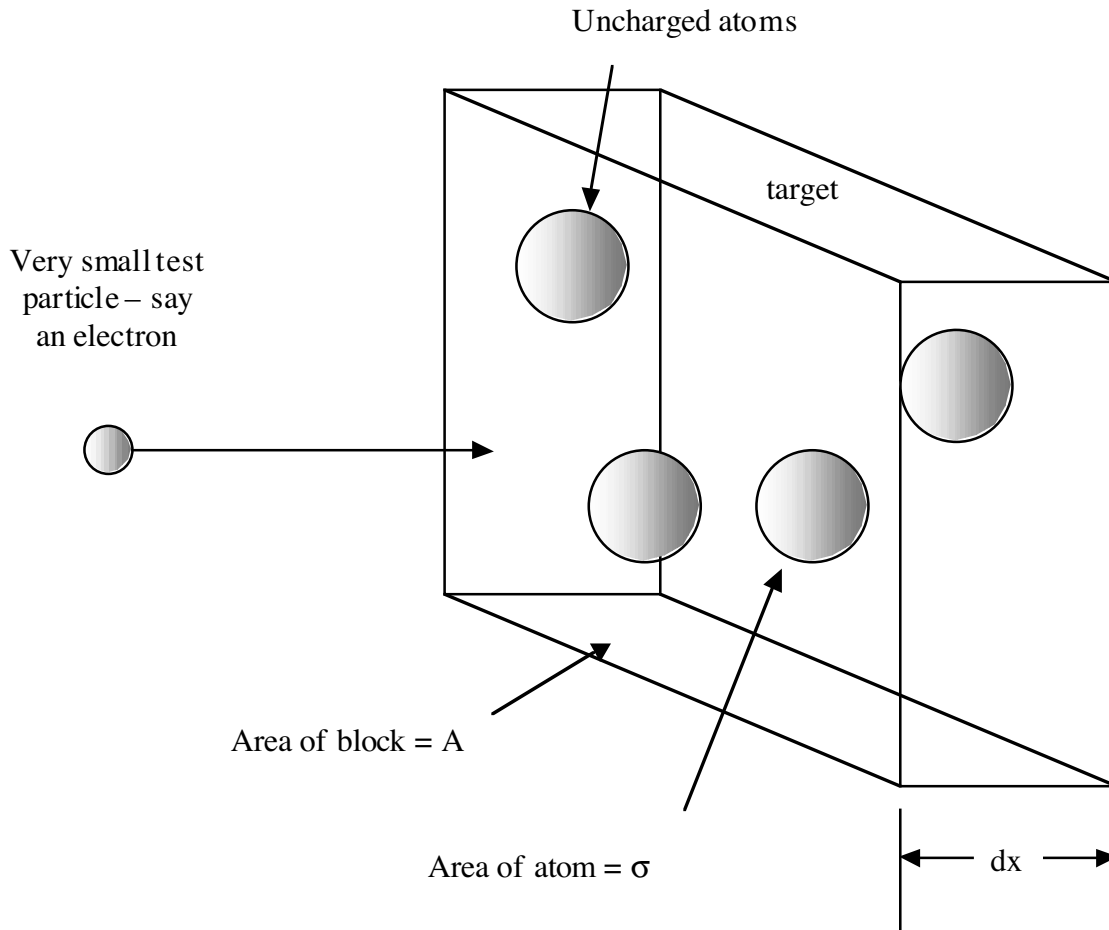
Concept of Mean-free path

As atoms/molecules pass through a volume, they collide with other atoms/molecules. A prime example of this is Brownian motion – the motion of a dust particle in air that moves in disjointed fashion.



also known as
a random walk

We can calculate the mean-free path, mfp, if we consider the following picture.



If we were to repeatedly fire test particles at the target, a fraction of the test particles will be scattered by collisions with atoms in the target. The fraction scattered is simply the area ratio.

$$\text{Fraction scattered} = \frac{N\sigma}{A}$$

where N is the number of atoms in the target. Assuming that our target is a part of a larger piece of material, in which we know the density, n , of the material then

$$\text{Fraction scattered} = \frac{nAdx\sigma}{A} = n\sigma dx.$$

Now let us send a continuous flux, Γ , of test particle – or a current density \mathbf{J} , at the target then the change in \mathbf{J} across the target is

$$\begin{aligned} \Delta\mathbf{J} &= \mathbf{J}_{\text{after}} - \mathbf{J}_{\text{before}} \\ &= (1 - n\sigma dx)\mathbf{J}_{\text{before}} - \mathbf{J}_{\text{before}} \\ &= -\mathbf{J}n\sigma dx \end{aligned}$$

or

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$$\frac{d\mathbf{J}}{dx} = -\mathbf{J}n\sigma$$

↓

$$\mathbf{J} = \mathbf{J}_0 \exp(-x / \lambda_{mfp})$$

where $\lambda_{mfp} = \frac{1}{n\sigma}$.

We can make approximations of the cross section of the target atom, based on the radius of the target atom. This is known as the hard-sphere approximation, which is at best imprecise. The reality is that the interaction, and hence collisions, is related to the target atom electron orbitals and particle energies, i.e. it is quantum mechanical in nature. From our above discussion, we can calculate the

Mean frequency of collisions

$$\nu = \frac{v}{\lambda_{mfp}} = vn\sigma$$

And the mean period of collisions

$$\tau = \frac{1}{\nu}$$

Of course the average collision frequency for a distribution of velocities is

$$\langle \nu \rangle = n \langle \nu \sigma \rangle \text{ noting } \sigma = \sigma(v).$$

Gas pressure

Lets assume that we have a particle in a box of sides l . Assume that the particle is reflected with

no loss of energy (why?) when it hits wall A1. Thus $\Delta E = 0$ but

$$\Delta M = \underbrace{-mv_x}_{\text{final momentum}} - \underbrace{mv_x}_{\text{initial momentum}} = -2mv_x$$

Where we have assumed that $v_y = 0 = v_z$. Now let us assume that the particle travels across the volume without striking anything else and reflects off side A2. The round trip time is $\Delta t = \frac{2l}{v_x}$

The force applied to side A1 is

$$F = -\frac{\Delta M}{\Delta t} = \frac{2mv_x}{2l/v_x} = \frac{mv_x^2}{l}$$

and the pressure is

$$P = \frac{F}{l^2} = \frac{mv_x^2}{l^3}$$

Putting in more particles we find

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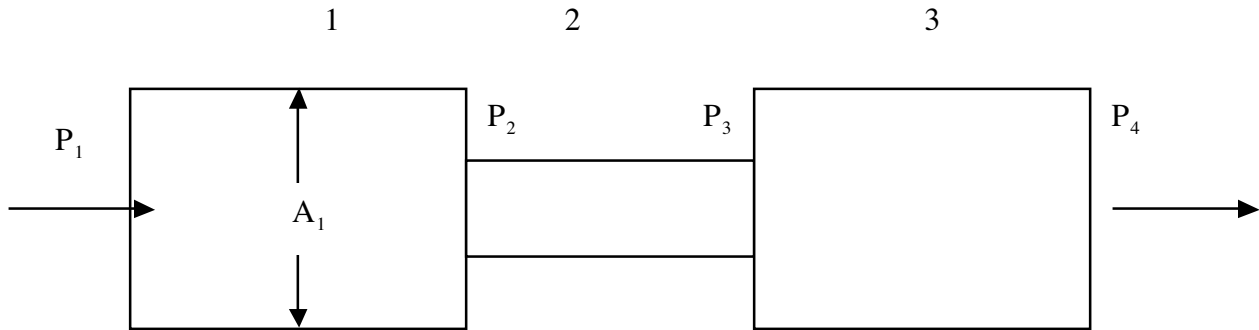
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$$\begin{aligned}
 P &= \sum_{\text{all particles}} \frac{mv_x^2}{l^3} = m \frac{N}{l^3} \sum_{\text{all particles}} \frac{mv_x^2}{N} \\
 &= mn \langle v_x^2 \rangle \\
 &= 2n \frac{1}{2} n \langle v_x^2 \rangle = 2n \langle E_x \rangle \\
 &= nkT - \text{the ideal gas law!}
 \end{aligned}$$

Lecture 2 Gas Flow

Conservation of Flux, which we will prove later

If a gas passes through a series of pipes with various cross sectional areas, then the flux is conserved. Physically this should be obvious. Think of this as the same as saying that we do not have rarefaction/compression of the gas, nor do we have a source/sink for the gas.



$$\Gamma = A_1 n_1 \langle v_1 \rangle = A_2 n_2 \langle v_2 \rangle = A_3 n_3 \langle v_3 \rangle \dots$$

Flow resistance/conductance

If a gas flow through a pipe, we expect some pressure differential to cause the flow. The ratio of the pressure to the flow is known as the resistance, R.

$$R = \frac{\Delta P}{\Gamma}$$

The conductance is simply the inverse of the R,

$$C = \frac{1}{R} = \frac{\Gamma}{\Delta P} \quad \frac{\# / s / \text{area}}{\text{Force / area}}$$

These equations are very similar to the resistance in a circuit, where we replace voltage with pressure. Thus for a series of pipes

$$\Delta P_{total} = \frac{\Gamma}{C_{total}} \text{ but}$$

$$\Delta P_{total} = \sum_i \Delta P_i \text{ and so}$$

$$\Delta P_{total} = \frac{\Gamma}{C_{total}} = \sum_i \frac{\Gamma}{C_i} = \Gamma \sum_i \frac{1}{C_i}$$

⇓

$$\frac{1}{C_{total}} = \sum_i \frac{1}{C_i}$$

or in a similar vane,

$$R_{total} = \sum_i R_i.$$

For Parallel pipes we find

$$C_{total} = \sum_i C_i \text{ and}$$

$$\frac{1}{R_{total}} = \sum_i \frac{1}{R_i}$$

Pumps

We can think of pumps as simply another conductance, S. After all we are trying to have a certain Γ go through the pump and the pump acts as if the back side has a pressure of zero. Here however, the pump conductance is given a special name, S, for 'speed'.

If a pump has a pipe connecting it to the chamber, we get an effective pump speed from

$$\frac{1}{S_{eff}} = \frac{1}{S} + \frac{1}{C_{pipe}}$$

Example:

Given a pipe with a conductance $C = 100$ L/s and a pump speed of $S = 200$ L/s we find that the effective pump speed is 66.6 L/s. This means that we using only 33% of our possible pumping ability. This can become critical in the design of a process system.

Types of gas flow.

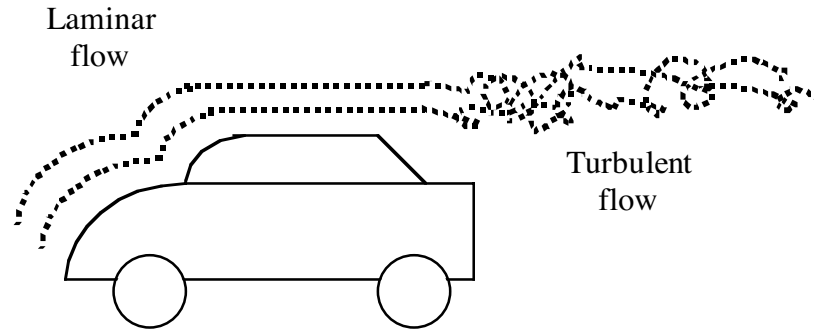
There are three radically different types of gas flow. They are:

Laminar fluid (viscous) flow

Turbulent fluid flow

Molecular flow

The distinction between Laminar ('layer') and turbulent is often seen in wind tunnel experiments on new cars.

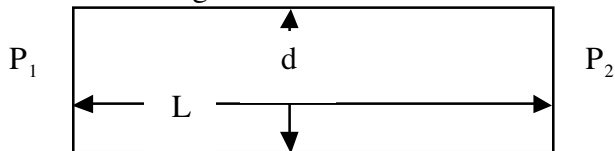


One can determine if one is operating in fluid flow or molecular flow by determining the Knudsen number, K_n .

$$K_n = \frac{\overbrace{\lambda_{mfp}}^{\text{mean-free path for all collisions}}}{\underbrace{d}_{\text{characteristic distance across system}}} = \begin{cases} < 1 & \text{Molecular} \\ \sim 1 & \text{Transition} \\ \gg 1 & \text{Fluid} \end{cases}$$

We can calculate the conductance in molecular and fluid flow regimes from the following equations. (Note these equations are at least partly experimental approximations.)

Fluid Flow through a round tube



with the requirements $P_1 > P_2$ and $L \gg d$.

Laminar Flow (the most common for us)

$$C = \frac{\pi}{128\eta} \frac{d^4}{L} \frac{P_1 + P_2}{2}; \quad \eta = \text{gas viscosity}$$

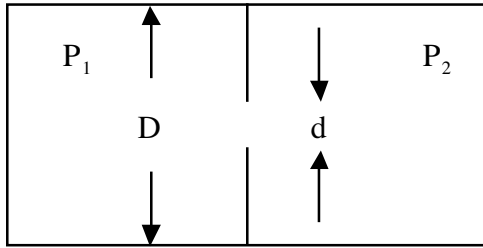
Here P_1 and P_2 are in mbar and L and d are in cm, giving C in L/s.

Turbulent Flow

$$C = \frac{d}{P_1 - P_2} \left(\frac{4}{\pi\eta} \right)^{1/7} \left[\frac{\pi^2}{3.24} \frac{5}{4} d^3 \frac{P_1^2 - P_2^2}{2L} \right]^{4/7} \left(\frac{\overbrace{\tilde{R}}^{\text{gas constant}}}{\underbrace{M_{molar}}_{\text{Molar mass}}} T \right)^{3/7}$$

Here, M_{molar} is in g/mole, R is 83.14 mBar L/Mole/K and T is in K.

Fluid Flow through an orifice



with the requirements $P_1 > P_2$.

$$C = \frac{P_1}{P_1 - P_2} \frac{\pi d^2}{4} \left(\frac{2RT}{M_{molar}} \right)^{1/2} \left(\frac{P_2}{P_1} \right)^{1/\chi} \left[\left(\frac{\chi}{\chi - 1} \right) \left(1 - \frac{P_2}{P_1} \right) \right]^{(1-\chi)/\chi}^{1/2}$$

where $\chi = \frac{C_p}{C_v}$ is the adiabatic constant

Molecular flow through a round tube

$$C = \frac{\pi d^2}{4} \frac{\left[\frac{RT}{2\pi M_{molar}} \right]^{1/2}}{\left(1 + \frac{3L}{4d} \right) \zeta}; \text{ where } 1 \leq \zeta \leq 1.12 \text{ is a fudge factor}$$