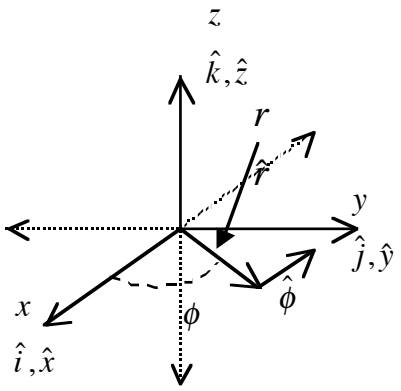
Fundamentals of Fluid Mechanics

Matthew Goeckner
Mechanical Engineering
University of Texas at Dallas

Useful equations

Cylindrical: (r,ϕ,z)



Transformations

Scalar:

$x = r\cos(\phi);\ y = r\sin(\phi);\ z = z$

$r = \sqrt{x^2 + y^2};\ \phi = \arctan\left(\frac{y}{x}\right);\ z = z$

Volume:

$dx\ \ dy\ \ dz = rdr\ \ d\phi\ \ dz$

Vector:

$\hat{r} \bullet \hat{i} = \cos(\phi);\ \hat{r} \bullet \hat{j} = \sin(\phi);\ \hat{r} \bullet \hat{k} = 0$

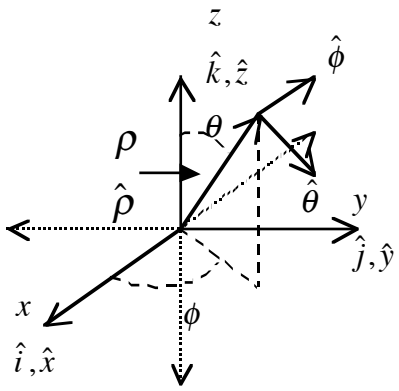
$\hat{\phi} \bullet \hat{i} = -\sin(\phi);\ \hat{\phi} \bullet \hat{j} = \cos(\phi);\ \hat{\phi} \bullet \hat{k} = 0$

$\hat{z} \bullet \hat{i} = 0;\ \hat{z} \bullet \hat{j} = 0;\ \hat{z} \bullet \hat{k} = 1$

Area elements (standard surfaces):

$$d\mathbf{s} = \begin{matrix} rd\phi\ \ dz\hat{\mathbf{r}} \\ dr\ \ dz\hat{\phi} \\ rd\phi\ \ dr\hat{\mathbf{z}} \end{matrix}$$

Spherical: (ρ,θ,ϕ)



Transformations

Scalar:

$x = \rho\cos(\phi)\sin(\theta);\ y = \rho\sin(\phi)\sin(\theta);\ z = \rho\cos(\theta)$

$\rho = \sqrt{x^2 + y^2 + z^2};\ \phi = \arctan\left(\frac{y}{x}\right);\ \theta = \arctan\left(\frac{\sqrt{x^2 + y^2}}{z}\right)$

Volume:

$dx\ \ dy\ \ dz = \rho^2 d\rho\ \ \sin(\theta)d\theta\ \ d\phi$

Vector:

$\hat{\rho} \bullet \hat{i} = \cos(\phi)\sin(\theta);\ \hat{\rho} \bullet \hat{j} = \sin(\phi)\sin(\theta);\ \hat{\rho} \bullet \hat{k} = \cos(\theta)$

$\hat{\phi} \bullet \hat{i} = -\sin(\phi);\ \hat{\phi} \bullet \hat{j} = \cos(\phi);\ \hat{\phi} \bullet \hat{k} = 0$

$\hat{\theta} \bullet \hat{i} = \cos(\phi)\cos(\theta);\ \hat{\theta} \bullet \hat{j} = \sin(\phi)\cos(\theta);\ \hat{\theta} \bullet \hat{k} = -\sin(\theta)$

Area elements (standard surfaces):

$$d\mathbf{s} = \begin{matrix} \rho\sin\theta d\phi\ \ \rho d\theta\hat{\rho} \\ d\rho\ \ \rho\sin\theta d\phi\hat{\theta} \\ d\rho\ \ \rho d\theta\hat{\phi} \end{matrix}$$