Optimized Integrated Harmonic Filter Inductor for Dual-Converter Fed Open-End Transformer Topology

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Abstract—Many high power converter systems are often connected to the medium voltage network using a step-up transformer. In such systems, the converter-side windings of the transformer can be configured as an open-end and multi-level voltage waveforms can be achieved by feeding these open-end windings from both ends using the two-level dual-converter. An LCL filter with separate converter-side inductors for each of the converter is commonly used to attenuate the undesirable harmonic frequency components in the grid current. The magnetic integration of the converter-side inductors is presented in this paper, where the flux in the common part of the magnetic core is completely canceled out. As a result, the size of the magnetic component can be significantly reduced. A multi-objective design optimization is presented, where the energy loss and the volume are optimized. The optimization process takes into account the yearly load profile and the energy loss is minimized, rather than minimizing the losses at a specific operating point. The size reduction achieved by the proposed inductor is demonstrated through a comparative evaluation. Finally, the analysis is supported through simulations and experimental results.

Index Terms—Voltage source converters (VSC), dual-converter, filter design, open-end transformer topology, multi-objective optimization, integrated inductor, harmonic filter, magnetic integration

I. INTRODUCTION

Many high power converter systems are often connected to the medium voltage network and a step-up transformer is used to match the voltage levels of the converter with the medium voltage grid. In some applications, the transformer is also required for providing galvanic isolation. In such systems, the converter-side windings of the transformer can be configured as an open-end. This open-end transformer winding can be fed from both the ends using the two-level Voltage Source Converters (VSCs) [1], as shown in Fig. 1. The number of levels in the output voltages is the same as that of the three level Neutral point clamped (NPC) converter and each of the two-level VSC operates with the half of the dc-link voltage than the dc-link voltage required for the three level NPC. This enables simple and proven two-level VSC to be used in medium voltage applications. For example, 3.3 kV converter system can be realized using the dual-converter with two-level VSCs having a switching voltage rating of 4.5 kV. However, Common Mode (CM) circulating current flows through the closed path if both the VSCs are connected to a common dc-link. The CM circulating current can be suppressed either by using the CM choke [2], [3] or by employing a proper Pulse Width Modulation (PWM) scheme to ensure complete elimination of the CM voltage [4], [5]. However, in many applications, isolated dc-links can be derived from the source itself and such extra measures for CM circulating current suppression may not be required. For example, the isolated dc-links can be obtained in

1) PhotoVoltaic (PV) systems by dividing the total number of arrays into two groups to form separate dc-links [1].
2) Wind Energy Conversion System (WECS): isolated dc-links can be obtained by using a dual stator-winding generator [6].

Therefore the analysis presented in this paper is mainly focused on the dual converter fed open-end transformer topology with two separate dc-links.

For the grid-connected applications, utilities impose stringent harmonic current injection limits. In order to obey this limits, high-order harmonic filter is often employed. An LCL harmonic filter is commonly used in high power grid-connected applications [7] and one of the possible arrangement of the LCL filter for the dual-converter fed open-end transformer topology is shown in Fig. 1. The leakage inductance of the transformer is considered to be a part of the grid-side inductor of the LCL filter. Two VSCs (denoted as High-Side Converter (HSC) and Low-Side Converter (LSC) in Fig. 1) are connected to a common shunt capacitive branch of the LCL filter through the converter-side inductors $L_{fH}$ and $L_{fL}$,
respectively. The magnetic integration of the $L_{fH}$ and $L_{fL}$ is presented in this paper. As a result of this magnetic integration, the flux in the common part of the magnetic core is completely canceled out. This leads to substantial reduction in the size of the converter-side inductor.

The detailed design procedure for the proposed integrated inductor is also presented in this paper. For proper design, it is important to investigate the design trade-offs and choose an optimum solution. The optimization can be performed with an objective to minimize losses, volume or cost. An analytical optimization to minimize winding losses is presented in [8]–[10]. The procedure to optimize the inductor volume is presented in [11]. However, the volume optimized design may have higher losses. Therefore it is important to investigate the trade-off between losses and volume, which can be achieved by performing multi-objective optimization. The multi-objective optimization of the $LCL$ output filter is presented in [12], where the optimization is carried out for a specific loading condition. However the obtained solution may not be optimal when the load varies in a large range. Therefore, the mission profile based multi-objective optimization approach for the proposed integrated inductor has been adopted in this paper.

This paper is organized as follows: The operation principle of the dual-converter fed open-end transformer topology is briefly discussed in Section II. The magnetic structure of the proposed integrated inductor is described in Section III. Section IV discusses the design procedure in general and the multi-objective optimization of the integrated inductor. The size reduction achieved by the magnetic integration is also demonstrated by comparing the volume of the integrated inductor with the separate inductor case for the 6.6 MVA, 3.3 kV WECS and it is presented in Section V. The simulation and the experimental results are finally presented in Section VI.

II. DUAL-CONVERTER FED OPEN-END TRANSFORMER TOPOLOGY

The operation of the dual-converter fed open-end transformer converter is briefly described in this section. The dual-converter system consists of the HSC and the LSC is shown in Fig. 1. A Two-level VSC is used for both the HSC and the LSC.

The reference voltage space vector $\bar{V}^2$ is synthesized by modulating the HSC and the LSC. The magnitude of the reference voltage space vectors of the HSC and the LSC is half than that of the desired reference voltage space vector $\bar{V}^2\left([V^H_L] = [V^L_L] = [\bar{V}]^2/2\right)$. The reference voltage space vector angle of the HSC is the same as that of the desired voltage space vector ($\psi_H = \psi$), whereas the reference voltage space vector angle of the LSC is shifted by an angle 180° ($\psi_L = \psi_H + 180°$), as shown in Fig. 2.

From Fig. 1, the voltage across the shunt capacitive branch $C_f$ of the $LCL$ filter is given as

$$V_{xx'} = (V_{xH0H} - V_{xL0L}) - L_{fH} \frac{dI_{xH}}{dt} - L_{fL} \frac{dI_{xL}}{dt} + V_{xH0L}$$  \hspace{1cm} (1)

where the subscript $x$ represents the phases $x = \{a, b, c\}$. As the dc-links are separated, the common-mode components of the voltages in (1) do not drive any common-mode circulating current. As a result, only the differential mode current would flow through the inductors. From Fig. 1, the converter-side current of the HSC can be obtained as

$$I_{xH} = I_{xg} + \Delta I_x$$  \hspace{1.5cm} (2)

where $I_{xg}$ is the current through the low-voltage side of the transformer winding and $\Delta I_x$ is the current through the shunt branch of the $LCL$ filter. Similarly, the converter-side current of the LSC is given as

$$I_{xL} = I_{xg} + \Delta I_x$$  \hspace{1.5cm} (3)

From (2) and (3), it is evident that the converter-side currents of the HSC and the LSC are equal:

$$I_{xH} = I_{xL} = I_x$$  \hspace{1.5cm} (4)

where $I_{xH}$ and $I_{xL}$ are the phase $x$ currents of the HSC and LSC, respectively. Assuming $L_{fH} = L_{fL} = L_f/2$ and using (1) and (4), the voltage across the converter-side inductor is given as

$$L_f \frac{dI_x}{dt} = V_{xHx'} + V_{x'xL} = (V_{xH0H} - V_{xL0L}) - V_{xx'} + V_{xH0L}$$  \hspace{1cm} (5)

where $L_f$ is the equivalent converter-side inductance of the $LCL$ filter. A single magnetic component with the inductance $L_f$ is realized by the magnetic integration of the $L_{fH}$ and $L_{fL}$ and the structure is discussed in the following section.

III. INTEGRATED INDUCTOR

The size and weight of the magnetic components can be reduced through magnetic integration [13]–[17]. For a three-phase three-wire systems, the sum of the phase currents is always zero. This property is exploited in three-phase magnetic structure, where three single-phase magnetic components are combined to achieve smaller three-phase single magnetic structure. The principle of magnetic integration in a power electronic circuit was introduced for the DC-DC converters in [18], where two inductors are magnetically integrated into single magnetic component without compromising the functionality, while significantly reducing the size, weight, and losses. The integration of the converter-side inductor and the grid-side inductor of the harmonic $LCL$ filter is proposed in [19] and the volume reduction is demonstrated. In many grid-connected applications, transformers are used to match the
The simplified reluctance model of this magnetic structure is shown in Fig. 3(b). The flux components in the common yoke \( (\phi_1 \text{ and } \phi_2) \) are zero and therefore the common yoke can be completely removed, as shown in Fig. 3(c). The integrated inductor has only two yokes, compared to four in the case of the separate inductors. As a result, substantial reduction in the volume of the inductor can be achieved through magnetic integration of \( L_{fiH} \) and \( L_{fiL} \). For the integrated inductor, the induced voltage across the coil \( a_H \) is given as

\[
V_{a_H} = L_{a_H a_H} \frac{dI_{a_H}}{dt} - L_{a_H b_H} \frac{dI_{b_H}}{dt} - L_{a_H c_H} \frac{dI_{c_H}}{dt} + L_{a_H a_L} \frac{dI_{a_L}}{dt} - L_{a_H b_L} \frac{dI_{b_L}}{dt} - L_{a_H c_L} \frac{dI_{c_L}}{dt} \quad (9)
\]

For the magnetic structure shown in Fig. 3(c), the reluctance of the central limb is \( 2(\mathcal{R}_g + \mathcal{R}_L + \mathcal{R}_Y) \), whereas the reluctance of the side-limbs is \( 2(\mathcal{R}_g + \mathcal{R}_L + \mathcal{R}_Y) \). This introduces some asymmetry in three-phase system. However, the reluctance of the air gap \( \mathcal{R}_g \) is very large compared to the reluctance of
the magnetic path (both $\mathbb{R}_l$ and $\mathbb{R}_y$). Moreover, the length of the yoke is also small compared to the length of the limbs for the commercially available cores [21]. For the magnetic structure shown in Fig. 3(c), $\mathbb{R}_L / \mathbb{R}_y$ is typically in the range of 5-7. As a result, $\mathbb{R}_y$ can be neglected and the magnetic structure can be assumed symmetrical. With this assumption, the self-inductance of each of the coils is obtained as

$$L_s = N^2 \left[ \frac{1}{\mathbb{R}_a (1 + \frac{\mathbb{R}_L}{\mathbb{R}_a} + \frac{\mathbb{R}_y}{\mathbb{R}_a})} + \frac{1}{3 \mathbb{R}_a (1 + \frac{\mathbb{R}_L}{\mathbb{R}_a} + \frac{\mathbb{R}_y}{\mathbb{R}_a})(1 + \frac{\mathbb{R}_y}{\mathbb{R}_a})} \right]$$

(10)

The mutual inductance between the two coils of the same phase can be obtained as

$$L_{xHxL} = k_{xHxL} L_s = L_s \left[ \frac{1 + \frac{\mathbb{R}_L}{\mathbb{R}_a} + \frac{\mathbb{R}_y}{\mathbb{R}_a}}{1 + 3 \frac{\mathbb{R}_L}{\mathbb{R}_a} + 3 \frac{\mathbb{R}_y}{\mathbb{R}_a}} \right]$$

(11)

where $k_{xHxL}$ is the coupling coefficient between the coils that belong to the same phase. The mutual inductance between the two coils of the different phases are given as

$$M = L_{aHbH} = L_{bHC} = L_{cHA} = \frac{1}{2} L_s \left[ \frac{1 + \frac{\mathbb{R}_L}{\mathbb{R}_a} + \frac{\mathbb{R}_y}{\mathbb{R}_a}}{1 + 3 \frac{\mathbb{R}_L}{\mathbb{R}_a} + 3 \frac{\mathbb{R}_y}{\mathbb{R}_a}} \right]$$

(12)

Substituting the inductance values in (9) yields

$$V_{aH} = L_s (1 + k_{xHxL}) \frac{dI_a}{dt} - 2M \left( \frac{dI_b}{dt} + \frac{dI_c}{dt} \right)$$

(13)

Using (6) and (13), the induced voltage across the coil $a_H$ is given as

$$V_{aH} = [L_s (1 + k_{xHxL}) + 2M] \frac{dI_a}{dt}$$

(14)

Similarly, the induced voltage across the coil $a_L$ is given as

$$V_{aL} = [L_s (1 + k_{xHxL}) + 2M] \frac{dI_a}{dt}$$

(15)

Using (5), (14), and (15), the equivalent inductance is obtained as

$$L_f = 2[L_s (1 + k_{xHxL}) + 2M]$$

(16)

Assuming $\mathbb{R}_a >> \mathbb{R}_L$, $\mathbb{R}_a >> \mathbb{R}_y$, and $\mathbb{R}_y >> \mathbb{R}_L$, the simplified expression for the $L_f$ is obtained as

$$L_f \approx \frac{2N^2}{\mathbb{R}_y} \approx \frac{2\mu_0 N^2 A_y'}{l_y}$$

(17)

where $\mu_0$ is the permeability of the free space, $l_y$ is the length of the air gap of one limb and $A_y'$ is the effective cross-sectional area of the air gap after considering the effects of the fringing flux. The effective cross-sectional area of the air gap $A_y'$ is obtained by evaluating the cross-section area of the air gap after adding $l_y$ to each dimension in the cross-section.

IV. DESIGN OF THE INTEGRATED INDUCTOR

A design methodology is demonstrated in this section by carrying out the design of the integrated inductor for the high power WECS. The system specifications of the WECS is given in Table I. The WECS operates with the power factor close to unity. In this case, the 60° Discontinuous Pulse-Width Modulation (commonly referred to as a DPWM1 [22]) scheme could result up to 50% switching loss reduction compared to the continuous modulation scheme. Therefore DPWM1 is used to modulate the HSC and the LSC. Using this specifications, design of the integrated inductor is carried out and the design steps are illustrated hereafter.

A. Value of the converter-side inductor $L_f$

The harmonic spectra of the switched output voltage of the HSC is shown in Fig. 5(a). The major harmonic components in the switched output voltage of the individual converter appears at the carrier frequency (900 Hz), whereas these components are substantially reduced in the resultant voltage, as shown in Fig. 5(b). The spectrum comprises the maximum values of the individual voltage harmonic components of the resultant voltage, over the entire operating range is obtained and it is defined as a Virtual Voltage Harmonic Spectrum (VVHS) [23]. The LCL harmonic filter is designed such that the enough impedance is offered to the harmonic frequency components so that the individual harmonic components of the injected grid-currents remain within the specified limits.

The harmonic current injection limit for a generator con-
TABLE II
BDEW HARMONIC CURRENT INJECTION LIMITS FOR THE WECS CONNECTED TO THE 10 KV MEDIUM VOLTAGE NETWORK

<table>
<thead>
<tr>
<th>Harmonic Order $h$</th>
<th>Current Injection Limit (A/MVA/SCR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.019</td>
</tr>
<tr>
<td>7</td>
<td>0.027</td>
</tr>
<tr>
<td>11</td>
<td>0.017</td>
</tr>
<tr>
<td>13</td>
<td>0.013</td>
</tr>
<tr>
<td>17</td>
<td>0.007</td>
</tr>
<tr>
<td>19</td>
<td>0.006</td>
</tr>
<tr>
<td>23</td>
<td>0.004</td>
</tr>
<tr>
<td>25</td>
<td>0.003</td>
</tr>
<tr>
<td>odd-ordered $25 &lt; h &lt; 40$</td>
<td>0.075 / $h$</td>
</tr>
<tr>
<td>Even-ordered $h &lt; 40$</td>
<td>0.02 / $h$</td>
</tr>
<tr>
<td>$40 &lt; h &lt; 180$</td>
<td>0.06 / $h$</td>
</tr>
</tbody>
</table>

connected to the medium-voltage network, specified by the German Association of Energy and Water Industries (BDEW) [23]–[25], is considered in this paper. The permissible harmonic current injection is determined by the apparent power of the WECS and the Short-Circuit Ratio (SCR) at the Point of the Common Coupling (PCC). The maximum current injection limit of the individual harmonic components up to 9 kHz is specified in the standard and the limits for the WECS connected to the 30 KV medium-voltage network are given in Table II. Special limits are set for the odd-ordered integer harmonics below the 25th harmonic, as given in Table II. The SCR is taken to be 20 and the allowable injection limits of individual harmonic components on the low voltage side (3300 V) for the 6.6 MVA WECS are calculated.

Using VVHS and the specified values of the permissible harmonic injection, the required admittance for the $h$-th harmonic component is obtained as

$$Y^*_{h,BDEW} = \frac{I^*_{h,BDEW}}{V_{h,VVHS}}$$

(18)

where $I^*_{h,BDEW}$ is the specified BDEW current injection limit of the $h$-th harmonic component (refer to [24]) and $V_{h,VVHS}$ is the maximum values of the $h$-th harmonic components over the entire operating range. The value of the filter parameters are then chosen such that the designed filter has a lower admittance than the required value of the filter admittance for all the harmonic frequency components of interest (upto 180th harmonic frequency component in case of the BDEW standard) [26], [27].

For the $LCL$ filter, the filter admittance is given as

$$Y_{LCL}(s) = \left. \frac{I_p(s)}{V_{PWM}(s)} \right|_{\alpha=0} = \frac{1}{L_f L_g C_f s^2 + \omega^2_{CL}}$$

(19)

Using this expression the filter admittance for individual harmonic frequency component is evaluated and the values of the $LCL$ filter components are obtained. Once the value of $L_f$ is obtained, an optimized design can be carried out.

B. Core Loss Modeling

The Improved Generalized Steinmetz Equation (IGSE) [28], [29] is used to calculate the core losses. The core losses per unit volume is given as

$$P_{f,c,v} = \frac{1}{T} \int_0^T k_i |dB(t)|^\alpha |DB(t)|^{\beta-\alpha} dt$$

(20)

where $\alpha$, $\beta$ and $k_i$ are the constants determined by the material characteristics. $\Delta B$ is the peak-to-peak value of the flux density and $T$ is the switching interval. The flux waveform has major and minor loops and these loops are evaluated separately.

1) Major Loop: Assuming the inductance value to be constant, the flux density in the limb is given as

$$B_z(t) = \frac{L_f I_{x,f}(t)}{2NA_c}$$

(21)

where $A_c$ is the cross-sectional area of the limb, $I_{x,f}$ is the fundamental component of the current.

2) Minor Loop: The reference space vector $\mathbf{V}_H^*$ and $\mathbf{V}_L^*$ are synthesized using active and zero voltage vectors and the voltage-second balance is maintained by choosing appropriate dwell time of these vectors. The application of the discrete vectors results in an error between the applied voltage vector and the reference voltage vector, as shown in Fig. 6 for the HSC. The error voltage vector during the $k$th state in a switching cycle is given as

$$\mathbf{V}_{H,err,k} = \mathbf{V}_k - \mathbf{V}_H$$

(22)

where $\mathbf{V}_H$ is the reference space vector and $\mathbf{V}_k$ is the VSC voltage vector during $k$th state. When the space voltage vector is in sector 1 ($0^\circ \leq \psi \leq 60^\circ$), $\mathbf{V}_k = \{\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3\}$. Similarly, the error voltage vectors for the LSC also exists due to the finite sampling. These error voltage vectors lead to the minor loop in the flux density waveform and it is evaluated by performing time integral of the error voltage vector.

The time integral of the error voltage vector is known as the harmonic flux vector [30], [31] and the difference of the harmonic flux vectors of the HSC and the LSC are directly proportional to the flux in the integrated inductor. In the reference frame, rotating synchronously at the fundamental frequency, the instantaneous error voltage vectors can be decomposed into $d$-axis and the $q$-axis components as (see Fig. 6).
Similarly, the \( d \)-axis and the \( q \)-axis components of instantaneous error voltage vectors of the LSC are also obtained. Then the difference of the \( d \)-axis components of the harmonic flux vectors of the HSC and LSC and the difference of the \( q \)-axis components of the harmonic flux vectors of the HSC and LSC are evaluated separately as

\[
B_{ac,d}(t) = \frac{1}{2N_{Ac}} \int (\vec{V}_{H,\text{err},d} - \vec{V}_{L,\text{err},a}) dt
\]

\[
B_{ac,q}(t) = \frac{1}{2N_{Ac}} \int (\vec{V}_{H,\text{err},q} - \vec{V}_{L,\text{err},q}) dt
\]

(24)

Using the \( d \)-axis and the \( q \)-axis components, the ripple component of the flux density in the limb corresponding to the phase \( a \) is obtained as

\[
B_a = B_{ac,d} \cos \psi - B_{ac,q} \sin \psi
\]

(25)

The VSCs are assumed to be modulated using the asymmetrical regularly sampled PulseWidth Modulation (PWM), where the reference voltage space voltage vector is sampled twice in a carrier cycle.

Using this information, the core loss calculations have been carried out for the major loop and each of the minor loops using (20). Then, the total core losses are obtained as

\[
P_{fe} = P_{fe,v} V_{fe}
\]

(26)

where \( V_{fe} \) is the volume of the magnetic core.

C. Copper Loss Modeling

The copper loss is evaluated by considering the ac resistance of the winding, which takes into account the skin and proximity effects [32]. The total winding losses of all six coils are [33]

\[
P_{cu} = 6R_{dc} \sum_{h=1}^{\infty} k_{ph} I_{xh}^2
\]

(27)

where

\[
k_{ph} = \sqrt{h} \Delta \left[ \frac{\sin(2\sqrt{h} \Delta) + \sin(2\sqrt{h} \Delta)}{\cosh(2\sqrt{h} \Delta) - \cos(2\sqrt{h} \Delta)} \right] + \frac{2}{3} \left( m^2 - 1 \right) \frac{\sin(\sqrt{h} \Delta) - \sin(\sqrt{h} \Delta)}{\cosh(\sqrt{h} \Delta) + \cos(\sqrt{h} \Delta)}
\]

(28)

and \( \Delta = T_e / \delta \) and \( R_{dc} \) and \( R_{ac} \) are the dc and the ac resistance of the coil, respectively. \( m \) is the number of layers in the coil, \( T_e \) is the thickness of the conductor, and \( \delta \) is the skin depth. \( I_{xh} \) is the \( h \)th harmonic frequency component of the line current \( I_x \). The harmonic spectrum of the resultant voltage is obtained analytically [14] and the \( h \)th harmonic frequency component of the line current \( I_x \) is obtained as

\[
I_{xh} = Y_{H,LCL} V_h
\]

(29)

D. Thermal Modeling

The liquid cooling for the inductor is considered and the cooling arrangement is shown in Fig. 7(a). The semi-circular aluminum cooling plates with the duct to carry the coolant is considered. This cooling plate is electrically insulated using the epoxy resin. As the heat transfer is anisotropic for the laminated steel, two cooling plates along the edges that are perpendicular to the lamination direction are considered. The hot spot temperature in both the core and the coil (\( T_{fe} \) and \( T_{cu} \), respectively) is estimated using the equivalent thermal resistance network [34], shown in Fig. 7(b). For the simplicity of the analysis, the temperature in the core and the coil is assumed to be homogeneous.

The heat transfer mechanism due to the convection and the radiation is considered, where \( R_{cw}(f-w) \) and \( R_{cu}(c-w) \) are the convection thermal resistance between the core and coolant (water) and between the coil and coolant, respectively. Similarly, \( R_{r}(f-a) \) and \( R_{r}(c-a) \) represent the radiation thermal resistance between the core and the ambient and between the coil and the ambient, respectively. The radiation thermal resistance value is obtained using the formulas presented in [34].

The thermal resistance between the cooling plate and the coolant is given as

\[
R_{(c-w)} = \frac{1}{h_{c-w} A_{c-w}}
\]

(30)

where \( h_{c-w} \) is the heat transfer coefficient and \( A_{c-w} \) is the coolant contact surface. The heat transfer coefficient is

\[
h_{c-w} = 3130 \left( \frac{q}{785.4 D_d^4} \right)^{0.87} (100 D_d)^{-0.13}
\]

(31)

where \( q \) is the coolant flow rate in [l/s] and \( D_d \) is the diameter of the duct in [m]. The thermal resistance between the core and duct surface is given as

\[
R_{(f-c-w)} = \frac{2 \rho_c}{\lambda_{cp} (A_{cp} f + \pi D_d L_d)} + \frac{T_i}{\lambda_i A_{cp} f}
\]

(32)
where $L_{eq}$ is the equivalent distance from the cooling surface to the duct, $A_{eq}$ is the contact area of the cooling plate with the core, $L_d$ is the length of the duct, and $\lambda_{ep}$ is the thermal conductivity of the aluminum. $T_i$ is the thickness of the insulation and $\lambda_i$ is the thermal conductivity of the insulation. Using (30) and (32), the thermal resistance between the core and the coolant is obtained as

$$R_{eq(f-w)} = R_{eq(f-p)} + R_{eq(c-w)}$$  \hspace{1cm} (33)

In a similar manner, the thermal resistance between the coil and the coolant $R_{eq(c-w)}$ can be also obtained. However, in the heat flow path of the copper losses, there is an additional layer of the insulation material, which is represented as $R_{eqi}$ in Fig. 7(b).

### E. Loading Profile and Energy Yield

The typical wind profile and the power output of a wind turbine over an one year span is shown in Fig. 8. As it is evident from Fig. 8, the power processes by the converter varies in large range and optimizing the inductor for a specific loading condition may result in the suboptimal overall performance. Therefore, instead of optimizing the inductor efficiency at specific loading condition, the energy loss is minimized. In addition to the energy loss minimization, the volume of the magnetic material is obtained as

$$V_{cu} = 6Nl_{int}A_{cu}$$  \hspace{1cm} (39)

where $N$ is the number of turns, $l_{int}$ is the mean length of the turn, and $A_{cu}$ is the cross-sectional area of the coil. The parameters that are optimized are

$$X = [N B_m J m W_c]^T$$  \hspace{1cm} (40)

where $B_m$ is the maximum flux density and $J$ is the current density. $W_c$ is the width of the coil (refer Fig. 3(c)). Once the system specifications and the constraints are defined, the optimization has been carried out. As the number of turns $N$ and the number of layers $m$ only take the integer values, mixed-integer optimization problem has been formulated. The steps followed for optimizing the system specified in Table I is shown in Fig. 9 and explained briefly hereafter.

1) **Step 1: Value of the converter-side inductor $L_f$:** The leakage inductance of the transformer is considered as a part of the grid-side inductor $L_g$ and the use of any additional inductor is avoided. Therefore, the value of the $L_g$ is fixed and the values of the $L_f$ and $C_f$ are obtained while observing the following constraints:

1) $I_h < I_R$ where $h$ is the harmonic order ($2 \leq h \leq 180$)
2) Reactive power consumption in shunt branch \leq 15\% of rated power.

The required value of the filter admittance is obtained using (18) and the values of the LCL harmonic filter is chosen to ensure that the admittance offered by the designed filter is lower than the required value of the filter admittance for all the individual harmonic components, as shown in Fig. 10. The calculated values of the $L_f$ and $C_f$ are listed in Table III.

### F. Optimization Process

The multi-objective optimization has been performed, which minimize a vector of objectives $F(X)$ and returns the optimal parameters values of $X$.

$$\min F(X)$$  \hspace{1cm} (34)

where

$$F(X) = [F_1(X), F_2(X)]$$  \hspace{1cm} (35)

where $F_1(X)$ returns the energy loss (kWh). The total loss ($P_{fe} + P_{cu}$) are evaluated for each of the loading conditions and the total energy losses (kWh) are obtained as

$$F_1(X) = \frac{1}{1000} \sum_{i=1}^{j} (P_{fe} + P_{cu})T_i$$  \hspace{1cm} (36)

where $T_i$ is the time in hours during which the WECS output power is $P_i$ and associated losses are ($P_{fe} + P_{cu}$).

$F_2(X)$ returns the volume of the active parts of the inductor (litr.) and it is given as

$$F_2(X) = (V_{fe} + V_{cu}) * 1000$$  \hspace{1cm} (37)

where $V_{fe}$ is the volume of the magnetic material and $V_{cu}$ is the total volume of all the coils. The volume of the magnetic material is obtained as

$$V_{fe} = A_c(6W_t + 4W_w + 3(H_w - l_g))$$  \hspace{1cm} (38)

where $A_c$ is the cross-sectional area of the core, $W_t$ is the width of the limb, $W_w$ is the width of the window, $H_w$ is the height of the window, and $l_g$ is the length of the air gap. The copper volume is given as

$$V_{cu} = 6Nl_{int}A_{cu}$$  \hspace{1cm} (39)

### Table III

VALUES OF $L_f$ AND $C_f$ OF THE LCL FILTER

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_f$</td>
<td>1200 $\mu$H (0.22 pu)</td>
</tr>
<tr>
<td>Shunt capacitance $C_f + C_d$</td>
<td>289 $\mu$F (0.15 pu)</td>
</tr>
</tbody>
</table>

The cross-sectional area of the core is obtained as

$$A_c = \frac{L_f I_{max}}{2NB_m}$$  \hspace{1cm} (41)
where \( I_{\text{max}} \) is the rated current. The dimensions of the core is then obtained as \( W_l D_l = A_c / k_s \), where \( k_s \) is the stacking factor. For simplicity, \( W_l = B_l \) is assumed in this study. The cross-section area of the conductor is obtained as

\[
A_{cu} = W_c T_c = \frac{I_{\text{max}}}{J}
\]  

(42)
The inlet temperature of the coolant is assumed to be 20 °C and it is given as 0.01 m. The coolant flow in each of the duct and the copper losses at the full load conditions are 7.09 kW and 9.67 kW, respectively. The coolant flow in each of the duct and the copper losses at the full load conditions are 7.09 kW, and the duct diameter $D_d$ is 0.01 m. The inlet temperature of the coolant is assumed to be 20 °C.

The non-inferior (Pareto optimal) solutions for the separate inductor case and the integrated inductor case. The converter-side inductance of both the HSC and the LSC are taken to be the same ($L_{fH} = L_{fL} = 600 \mu H$). The value of each of the converter-side inductor is taken to be 600 \mu H to ensure same attenuation of the harmonic frequency components in both the separate inductor case and the integrated inductor case.

The core temperature at the rated load is calculated to be 74 °C, whereas the temperature of the coil is found to be 86 °C.

### B. Volumetric comparison

The magnetic integration leads to a reduction in the size of the inductor. This has been demonstrated by comparing the volume of the integrated inductor with the volume of the inductors in a separate inductor case. The multi-objective optimization for the separate inductor case has been also carried out with an objective to minimize the energy loss and the volume. The mission profile and the solution space (range of the parameters that are optimized) are taken to be the same in both the cases. The converter-side inductance of both the HSC and the LSC are taken to be the same ($L_{fH} = L_{fL} = 600 \mu H$). The value of each of the converter-side inductor is taken to be 600 \mu H to ensure same attenuation of the harmonic frequency components in both the separate inductor case and the integrated inductor case.

The non-inferior (Pareto optimal) solutions for the separate inductor are shown in Fig. 14. Out of many possible solutions, one design is selected with the volume of 132.8 ltr. and the energy loss of 39693 kWh. The comparison of the various performance parameters of both the separate inductor case and the integrated inductor case is given in Table VI.

The volume of the magnetic material of the integrated

\begin{align*}
\text{Volume (ltr.)} & \quad \begin{array}{cccc}
100 & 150 & 200 & 250 \\
\text{Energy loss (kWh)} & \begin{array}{cccc}
2.46 & 3.93 & 5.39 & 6.86 \\
\end{array}
\end{array} \\
\end{align*}

\text{TABLE V}

\begin{align*}
\text{Losses (kW)} & \quad \begin{array}{cccc}
10 & 15 & 20 & 25 \\
\text{Load (pu)} & \begin{array}{cccc}
0.2 & 0.4 & 0.6 & 0.8 \\
\end{array}
\end{array} \\
\end{align*}

\begin{align*}
\text{TABLE V}
\begin{array}{cccc}
\text{Coil} & a_H & b_H & c_H & a_L & b_L & c_L \\
\text{Volume (ltr.)} & \begin{array}{cccc}
19 & 19 & 19 & 19 \\
\end{array}
\end{array}
\end{align*}

\text{INDUCTANCES VALUES OBTAINED USING THE FINITE ELEMENT ANALYSIS. ALL VALUES ARE IN} \mu H.

\begin{align*}
\text{TABLE V}
\begin{array}{cccc}
\text{Coil} & a_H & b_H & c_H & a_L & b_L & c_L \\
\text{Volume (ltr.)} & \begin{array}{cccc}
19 & 19 & 19 & 19 \\
\end{array}
\end{array}
\end{align*}

\text{VOLUME VALUES ARE IN} \mu H.
TABLE VI
VOLUME AND ENERGY LOSS COMPARISON.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Separate inductor ((L_{IH} + L_{IL}))</th>
<th>Integrated inductor</th>
<th>Reduction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume of active materials (ltr.)</td>
<td>(2 \times 132.8 = 265.6)</td>
<td>187.1</td>
<td>29.5 %</td>
</tr>
<tr>
<td>Volume of magnetic materials (ltr.)</td>
<td>(2 \times 88.65 = 177.3)</td>
<td>132.2</td>
<td>25.4 %</td>
</tr>
<tr>
<td>Volume of copper (ltr.)</td>
<td>(2 \times 44.15 = 88.3)</td>
<td>54.9</td>
<td>37.8 %</td>
</tr>
<tr>
<td>Energy loss (kWh)</td>
<td>(2 \times 39.93 = 79.586)</td>
<td>71291</td>
<td>10.1 %</td>
</tr>
<tr>
<td>Total losses at full load (kW)</td>
<td>(2 \times 8.38 = 16.76)</td>
<td>14.86</td>
<td>11.3 %</td>
</tr>
</tbody>
</table>

Fig. 15. Simulated flux density waveform in the limb of phase \(a\).

Fig. 16. Simulation results. (a) Flux density waveform in the limb of phase \(a\), (b) Output current of the high-side converter \(I_{HA}\), (c) Current through the shunt branch of the \(LCL\) filter, (d) Current through the open-end transformer windings.

The time domain simulations have been carried out using PLECS. The parameters used in the simulations are specified in Table I. The integrated inductor is modeled using the magnetic toolbox, which uses the permanence model.

The converter was simulated at the rated power and the simulated flux density waveform in one of the limb is shown in Fig. 15. The flux density has a dominant fundamental frequency component with major harmonic component at the 2nd carrier harmonic frequency. The output current of the HSC is shown in Fig. 16(a), which has a major harmonic component at 2nd carrier frequency harmonic. The leg current in the \(HSC\) and the \(LSC\) is the same \((I_{aH} = I_{aL})\), only the current of the \(HSC\) \(I_{aH}\) is shown. The shunt branch of the \(LCL\) filter offers low impedance path to the harmonic components, as shown in Fig. 16(b). As a result, the injected grid currents have the desired waveform quality, as shown in Fig. 16(c).

A small scale prototype has been built to verify the effectiveness of the proposed inductor. The specifications of the small scale system is given in Table I. The dc-link voltages for the \(HSC\) and the \(LSC\) were derived from two separate dc power supplies. The converter system was connected to the \(ac\) power source from the California Instruments (MX-35), which was used to emulate harmonic free grid. The filter arrangement is shown in Fig. 1, where the \(LCL\) filter, along with the \(R_d/C_d\) damping branch is used. The values of the filter parameters are given in Table VII. A 11 kVA, 230/400 V three-phase transformer is employed where the 230 V star-connected windings are reconfigured to obtain the open-end transformer windings.

The integrated inductor is realized using the standard lami-

TABLE VII
VALUES OF \(L_f\) AND \(C_f\) OF THE \(LCL\) FILTER.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_f)</td>
<td>(9 \text{ mH (0.195 pu)})</td>
</tr>
<tr>
<td>Shunt capacitance (C_f)</td>
<td>(14 \mu F (0.065 pu))</td>
</tr>
<tr>
<td>Damping capacitance (C_d)</td>
<td>(10 \mu F (0.045 pu))</td>
</tr>
<tr>
<td>Damping resistor (R_d)</td>
<td>(30 \Omega)</td>
</tr>
</tbody>
</table>

Fig. 17. Photos of the converter-side inductor of the \(LCL\) harmonic filter. (a) 4.6 mH inductor used in the separate inductor case, where two such inductors are required. (b) 9 mH integrated inductor which magnetically integrates two separate inductors.
TMS320F28346 floating-point digital signal processor. The converters are controlled to inject the rated current to the grid and the experimental waveforms during the steady-state condition are shown in Fig. 18. The output current of LSC of phase a is also shown in Fig. 18(a). The converter-side currents have a major harmonic component, concentrated around the second carrier harmonic frequency. The transient performance has been verified by applying the step change in the reference current from 0.5 pu to 1 pu and the corresponding current waveforms are shown in Fig. 19. The harmonic spectra of the injected current is shown in Fig. 20, where it is evident that the magnitude of the individual harmonic components of the injected current is within the specified BDEW limits.

The power losses of the integrated inductor at different loading conditions have been measured using the Voltech PM3000 power analyzer. The measured losses are shown in Table VIII. The losses of the integrated inductor at the full load condition is around 1% of the rated power.

The magnetic integration of the converter-side inductor of the HSC and the LSC leads to the substantial reduction in the volume and the energy loss without compromising the harmonic performance. This has been verified by comparing the harmonic spectra of the line current in the case of the
\( \text{LCL} \) filter integrated inductor with that of the \( \text{LCL} \) filter with separate inductors. Off the shelf 4.6 mH, 16A three-phase three-limb inductors are used in the separate inductor case \((L_{f1} = L_{f2} = 4.6 \text{ mH})\) and the results are compared with the designed integrated inductor with \( L_f = 9 \text{ mH} \). The harmonic spectra of the line current in both the cases are shown in Fig. 21, which demonstrates that the harmonic performance is not compromised by the magnetic integration. The magnitude of the harmonic components are slightly smaller in the case of the separate inductor, as the effective value of \( L_f \) is 9.2 mH, as against the 9 mH in the case of the integrated inductor.

**VII. Conclusion**

An integrated inductor for the dual-converter fed open-end transformer topology is proposed. The dual-converter system often comprises of two identical VSCs. These two VSCs use two separate converter-side inductors for the \( \text{LCL} \) filter implementation. For the dual-converter system, the output currents of the given phase of both VSCs are equal. This property of the dual-converter system is exploited to cancel out the flux in one of the yokes of both the inductors through the magnetic integration. Moreover, a multi-objective optimization has performed to identify best possible solutions which leads to the minimization of the energy loss and minimization of the volume of the inductor. The size reduction achieved through magnetic integration is demonstrated by comparing the volume of the proposed solution with the separate inductor case. The integrated inductor leads to 25.4\% reduction in the volume of the magnetic material. This translates to 314 kg reduction in the weight of the magnetic component for the 6.6 MVA, 3.3 kV WECS system. The performance of the filter has been verified by simulation and experimental studies.

**Acknowledgments**

The authors would like to thank the Innovation Foundation through the Intelligent Efficient Power Electronics (IEPE) technology platform for supporting the related research.

**References**


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